Physics 273 - Homework #6

1) Consider two coupled mechanical oscillators, like the ones we studied in class. Each of the two masses are connected to a fixed wall through a spring with spring constant (k). The masses are connected to each other with a spring with spring constant (k_{12}).

a) Suppose we hold one of the two masses fixed, so it cannot move. What is the natural frequency of the other mass?

b) Now we release the second oscillator. Compare the natural frequency from part (a) to the two normal mode frequencies of the double oscillator. Are they smaller or larger? Explain.

2) Consider again two coupled mechanical oscillators. As usual, each oscillator is connected to a wall by a spring with spring constant (k), and the two masses are coupled together by a spring with spring constant (k_{12}). Suppose that our initial conditions are that the first oscillator has an initial velocity of (v_0) and an initial position of zero, and the second oscillator starts with an initial velocity and position of zero.

a) Apply these initial conditions to the solution we found in class to find $x_1(t)$ and $x_2(t)$.

b) Let k = 1 N/m, k_{12} = 0.25 N/m, m = 1 kg, and v_0 = 1 m/s. Make a plot of the positions of both masses from t = 0 to t = 30 seconds. Please put both $x_1(t)$ and $x_2(t)$ on the same plot.

3) For the loaded string, the amplitude relationships which define the normal modes are described by the expression:

$$A_{pn} = \sin\left(\frac{pn\pi}{N+1}\right)$$

where (p) tells us which mass we are talking about, (n) tells us which normal mode we are talking about, and N is the total number of masses on the string. It is convenient to re-write this expression in a vector notation:

$$\vec{q}_n = \left(\sin\left(\frac{n\pi}{N+1}\right), \sin\left(\frac{2n\pi}{N+1}\right), \sin\left(\frac{3n\pi}{N+1}\right), \ldots, \sin\left(\frac{Nn\pi}{N+1}\right)\right)$$

In this expression, the first component of the vector describes mass #1, the second component describes mass #2, ect. Since there are N normal modes, there will be N such vectors. We call these objects 'eigenvectors'.

a) Write down in explicit numerical form the vectors $\vec{q}_1$ and $\vec{q}_2$ for the N = 2 case. Please give a numerical value for each component of the vectors, accurate to three decimal places.

b) Calculate the dot products for all pairs of vectors for the N = 2 case: $\vec{q}_1 \cdot \vec{q}_1$, $\vec{q}_1 \cdot \vec{q}_2$, and $\vec{q}_2 \cdot \vec{q}_2$.

c) Repeat part (a) for the N = 3 case, writing down the explicit numerical form for the three eigenvectors.

d) Repeat part (b) for the N = 3 case. You will need to calculate six unique dot products.

e) Repeat part (a) for the N = 4 case, writing down the explicit numerical form for the four eigenvectors.

f) Repeat part (b) for the N = 4 case. You will need to calculate the 10 unique dot products.

g) What pattern do you see in the dot products of these eigenvectors?

4) Consider a loaded string consisting of three particles of mass (m) regularly spaced on the string. At t = 0 the center particle is displaced a distance (a) from its equilibrium position. (The other two particles are located at their equilibrium positions.) We release all three particles with an initial velocity of zero.

a) Apply these initial conditions to the solution of the loaded string (which we found in class) to find the position of all three particles as a function of time.

b) Let the string tension be T = 10 N, m = 1 kg, and let the distance between the masses on the string be 0.1 m. Also let the initial displacement of mass 2 be 0.01 m. Make a plot of the positions of all three masses from t = 0 to t = 10 seconds. Please put $x_1(t)$, $x_2(t)$, and $x_3(t)$ on the same plot.