Physics 273 - Homework #4

1) a) Suppose we construct a parallel plate capacitor using two square copper plates separated by a distance of 0.0025 cm. (Maybe we use a thin piece of paper to separate the plates.) Each plate is 10 cm by 10 cm square. Look up the expression for an ideal parallel plate capacitor and calculate the expected capacitance for our device. You may assume that the material in the gap between the plates is vacuum. Please give your answer in MKS units for capacitance.

b) Now we construct an inductor by wrapping a copper wire around a plastic soda bottle. The bottle is 7 cm in diameter and 15 cm long. We use 22 gauge wire (0.064 cm diameter), which allows us to fit 234 tightly packed loops on the 15 cm length of the bottle. Look up the expression for an ideal solenoid and calculate the expected self-inductance for our device. As in part (a), assume that the interior of the bottle has the electrical properties of vacuum. Please give your answer in MKS units for inductance.

c) We charge up the capacitor by connecting it to a 9 V battery. How much charge is on each plate of the capacitor?

d) How much energy is stored in the capacitor after it is charged?

e) We now connect the capacitor and the inductor to form an LC oscillator. What is its natural frequency?

f) Copper is not a perfect conductor, so our inductor has some resistance as well as inductance. Therefore a better model for our circuit would be an RLC circuit, where the resistor, capacitor, and inductor are all connected in series. At room temperature the bulk resistivity (volume resistivity) of copper is 1.72×10^{-8} Ohm-meters. Use this number and the dimensions of the copper wire to calculate the expected resistance of our inductor. Please give your answer in Ohms. (Look up the definition of bulk resistivity if necessary.)

g) How long will it take for the energy stored in the LC circuit to decrease to 50% of its initial value?

h) What is the Q value of our circuit?

i) Suppose we want to use our circuit to detect the AM radio signal from WTOP, which broadcasts at a frequency of f = 1500 kHz. We will need to change the geometry or material of our capacitor and/or inductor to make our circuit resonate at the WTOP frequency. Suppose we decide to modify the capacitor by changing the gap between the plates. What distance should we use for the gap?

2) **Numerical solution of a forced harmonic oscillator with damping.** The equation of motion for the forced harmonic oscillator with damping is

$$\ddot{x} + \gamma\dot{x} + \omega_0^2 x = F(t)/m$$

where *F(t)* is the forcing function. (As with the damped harmonic oscillator, ($\gamma = b/m$), (*b)* is the drag coefficient, and ($\omega_0^2 = k/m$)). Suppose that the forcing function has the form of a cosine function:

$$F(t) = F_0 \cos(\omega_f t)$$

where (F_0) has units of Newtons, and (ω_f) is the forcing frequency.

Copy your numerical solution to the damped harmonic oscillator (Homework #3, problem #4) and modify it by adding in the effects of the external force. Use the following parameters in your numerical solution: k = 10 N/meter, m = 1 kg, b = 0.3 N/(meter/second), F_0 = 0.5 Newtons. Use a time step of 0.01 seconds, and calculate for 4000 time steps (a total of 40 seconds). For the initial conditions, assume that the oscillator starts from rest at its equilibrium position (x_0 = 0.0 and v_0 = 0.0). (The last parameter in the problem is the forcing frequency (ω_f), which we will vary in the following questions.)

a) Calculate by hand the expected natural frequency (ω_0) for this oscillator.
b) Calculate by hand the expected damped frequency (ω_d). (This is the frequency of the oscillator in the absence of a forcing function, but in the presence of the damping force.)
c) Set the forcing frequency to ω_f = 1 Hz, and print out a plot of the oscillator position as a function of time for the first 40 seconds. Please set the y-axis limits of your plot to +/- 0.60 meters. Also, please mark the region where the transient solution has a noticeable effect, and mark the region where the transient solution does not have a noticeable effect.
d) Now set the forcing frequency to ω_f = 0.1 Hz. In your solution you should see clearly the effect of the damped frequency (ω_d). Measure the value of the damped frequency as observed in your solution, and compare it to the expected value from part (b).
e) Now we will examine the steady-state part of the solution and how it depends on the forcing frequency. For this question, set the y-axis limits on your plot to be +/- 0.60 meters, and do not let those limits change as you alter the forcing frequency. First set the forcing frequency to ω_f = 1 Hz and measure the amplitude of the steady-state solution. Hint: if you are using Excel, you can use the MAX() function to automatically "observe" this amplitude. For example, MAX(D3011:D4011) returns the maximum value of the cells D3011 through D4011.
f) Step the forcing frequency from 1.0 Hz to 5.0 Hz in 0.1 Hz steps (a total of 41 different forcing frequencies). At each step, record the amplitude of the steady-state part of the solution. Make a plot of this amplitude as a function of the forcing frequency, and mark on your plot the value of the natural frequency which you calculated in part (a).
g) Among those forcing frequencies that you tried in part (g), which gives the largest amplitude for the steady state solution? Print out a plot of the position of the oscillator as a function of time for this particular forcing frequency.