

## Phys 273 – Formula Sheet #2

$$\begin{aligned}
 m\ddot{x}_1 + (k + k_{12})x_1 - k_{12}x_2 &= 0 & \rightarrow & \quad x_1(t) = a_1 e^{i\omega_1 t} + a_2 e^{i\omega_2 t} & \quad \omega_1 = \omega_S = \sqrt{k/m} \\
 m\ddot{x}_2 + (k + k_{12})x_2 - k_{12}x_1 &= 0' & & \quad x_2(t) = a_1 e^{i\omega_1 t} - a_2 e^{i\omega_2 t} & \quad \omega_2 = \omega_L = \sqrt{(k + 2k_{12})/m} \\
 \bar{x}(t) &= \sum_{n=1}^2 a_n \bar{q}_n e^{i\omega_n t}, & \bar{q}_1 &= (1, 1), \bar{q}_2 = (1, -1), & a_i = \frac{\bar{y}_0 \cdot \bar{q}_i}{|\bar{q}_i|^2}
 \end{aligned}$$

$$\begin{aligned}
 \ddot{y}_p + 2\omega_0^2 y_p - \omega_0^2 (y_{p+1} + y_{p-1}) &= 0, \quad \omega_0^2 = T/ml, \rightarrow \bar{x}(t) = \sum_{n=1}^N a_n \bar{q}_n e^{i\omega_n t} \\
 \bar{q}_n &= \left( \sin\left(\frac{n\pi}{N+1}\right), \sin\left(\frac{2n\pi}{N+1}\right), \dots, \sin\left(\frac{Nn\pi}{N+1}\right) \right), \quad A_{pn} = \sin\left(\frac{pn\pi}{N+1}\right), \quad a_i = \frac{\bar{y}_0 \cdot \bar{q}_i}{|\bar{q}_i|^2} \\
 \omega_n &= 2\omega_0 \sin\left(\frac{n\pi}{2(N+1)}\right)
 \end{aligned}$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{\rho}{T} \frac{\partial^2 y}{\partial t^2}, \quad v = \sqrt{T/\rho}$$

$$y(x, t) = \sum_{n=1}^{\infty} c_n \sin\left(\frac{n\pi x}{L}\right) e^{i\omega_n t}, \quad \omega_n = \sqrt{\frac{T}{\rho}} \frac{n\pi}{L}, \quad c_n = \frac{2}{L} \int_0^L y(x, t=0) \sin\left(\frac{n\pi x}{L}\right) dx$$

$$y(x, t) = A \sin(kx - \omega t), \quad k = \frac{2\pi}{\lambda}, \quad \omega = 2\pi f = \frac{2\pi}{T}, \quad v = \lambda f = \frac{\omega}{k}$$

$$y(x, t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dk A(k) e^{i(kx + \omega t)}, \quad A(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dx f(x) e^{-ikx}$$

$$\int_0^L \sin\left(\frac{n\pi x}{L}\right) \sin\left(\frac{m\pi x}{L}\right) dx = \frac{L}{2} \delta_{nm}, \quad \sum_{j=1}^N \sin\left(\frac{jn\pi}{N+1}\right) \sin\left(\frac{jm\pi}{N+1}\right) = \left(\frac{N+1}{2}\right) \delta_{nm}$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[ a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right) \right],$$

$$a_n = \frac{1}{L} \int_{-L}^L f(x) \cos\left(\frac{n\pi x}{L}\right) dx, \quad b_n = \frac{1}{L} \int_{-L}^L f(x) \sin\left(\frac{n\pi x}{L}\right) dx$$

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{in\pi x/L}, \quad c_n = \frac{1}{2L} \int_{-L}^L f(x) e^{-in\pi x/L} dx,$$

$$a_n = c_n + c_{-n}, \quad b_n = i(c_n - c_{-n}), \quad a_0 = 2c_0$$

$$c_n = \begin{cases} \frac{1}{2}(a_{-n} + ib_{-n}), & n < 0 \\ \frac{1}{2}a_0, & n = 0 \\ \frac{1}{2}(a_n - ib_n), & n > 0 \end{cases}$$

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dk A(k) e^{ikx}, \quad A(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dx f(x) e^{-ikx}$$

$$Z = \frac{F}{\dot{y}} = \frac{T}{v_p} = \rho v_p$$

$$\frac{A_2}{A_1} = \frac{2Z_1}{Z_1 + Z_2}, \quad \frac{B_1}{A_1} = \frac{Z_1 - Z_2}{Z_1 + Z_2}$$