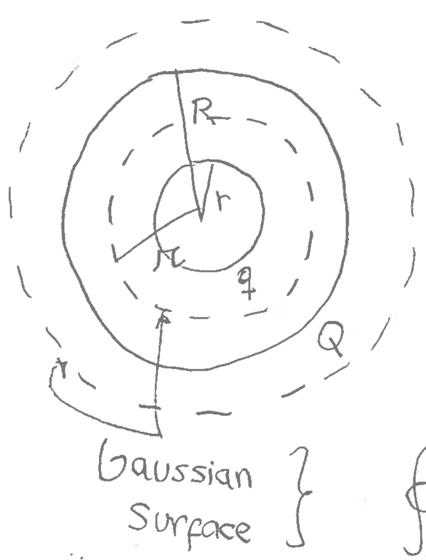
SOLUTIONS - HW#4 - PHYS 272/272H

Problem 1:



$$\Delta V = -\int_r^R \vec{E}\cdot d\vec{\ell} = V_R - V_r$$

$$d\vec{\ell} = -d\mathcal{r}\,\hat{r}$$

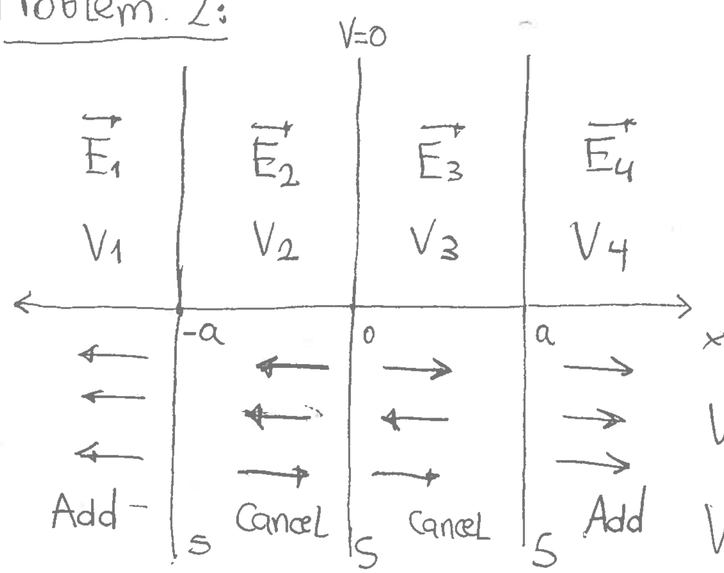
$$\Rightarrow \overrightarrow{E_{out}} = \frac{1}{4\pi\epsilon_0}\frac{(Q+q)}{\mathcal{r}^2}\hat{r}$$

$$\oiint \vec{E}\cdot d\vec{A} = \frac{q}{\epsilon_0} \Rightarrow \vec{E}_{in} = \frac{1}{4\pi\epsilon_0}\frac{q}{\mathcal{r}^2}\hat{r}$$

$$\Delta V = -\int_r^R \vec{E}_{in}\cdot d\vec{\ell} = \frac{q}{4\pi\epsilon_0}\int_r^R \frac{d\mathcal{r}}{\mathcal{r}^2} = -\frac{q}{4\pi\epsilon_0}\left(\frac{1}{R}-\frac{1}{r}\right)$$

Notice also that the problem could have been solved by considering the spheres as point charges and substracting their potentials

Problem 2:



$$\vec{E}_1 = -\frac{3S}{2\epsilon_0}\hat{x} \quad ; \quad \vec{E}_2 = -\frac{S}{2\epsilon_0}\hat{x}$$

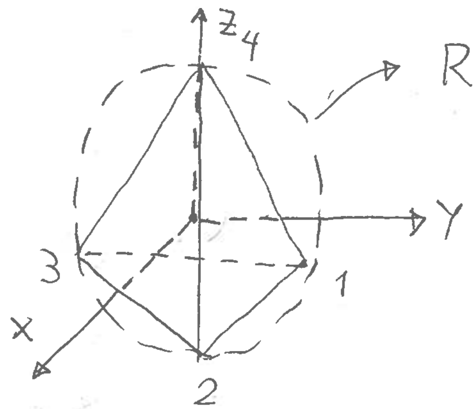
$$\vec{E}_3 = \frac{S}{2\epsilon_0}\hat{x} \quad ; \quad \vec{E}_4 = \frac{3S}{2\epsilon_0}\hat{x}$$

$\vec{E} = -\vec{\nabla} V$ thus:

$$V_1 = \frac{3S}{2\epsilon_0}x \quad ; \quad V_2 = \frac{S}{2\epsilon_0}x$$

$$V_3 = \frac{S}{2\epsilon_0}x \quad ; \quad V_4 = \frac{3S}{2\epsilon_0}x$$

Problem 3:



$R = \sqrt{\frac{3}{8}}\, a$

$$V = V_1 + V_2 + V_3 + V_4$$

$$V_1 = -\int_{\infty}^{R} \vec{E}_{proton} \cdot d\vec{\ell} \quad ; \quad d\vec{\ell} = -dr\,\hat{r}$$

$$\vec{E}_{proton} = \frac{1}{4\pi\epsilon_0} \frac{e}{r^2}\,\hat{r}$$

$$V_1 = -\frac{1}{4\pi\epsilon_0} \frac{e}{R}$$

$$W = -eV \Rightarrow \text{Energy}$$

$$V_2 = W_1 + V_{12}$$

$$V_3 = W_1 + V_{13} + V_{23}$$

$$V_4 = W_1 + V_{14} + V_{24} + V_{34}$$

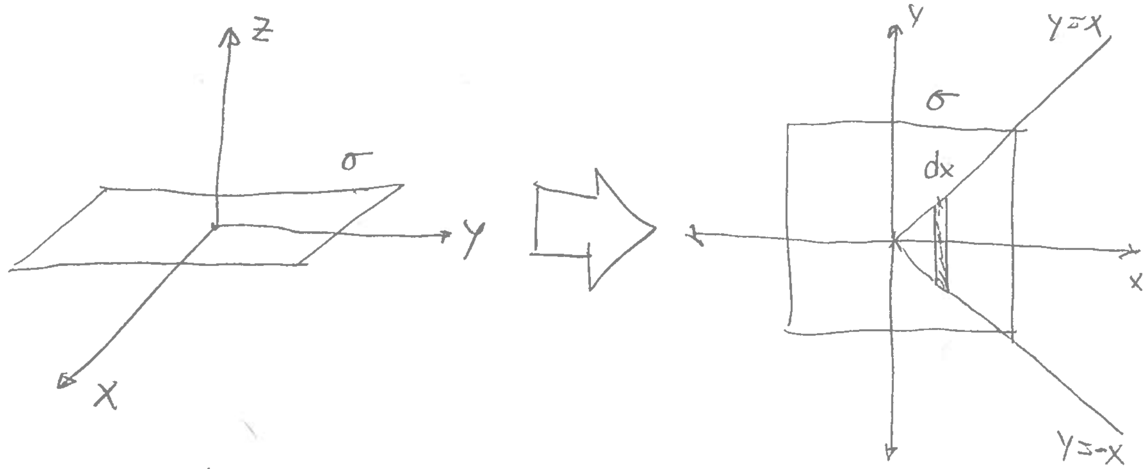
$$V_{12} = \frac{1}{4\pi\epsilon_0} \frac{e}{a} = V_{13} = V_{23} = V_{14} = V_{24} = V_{34}$$

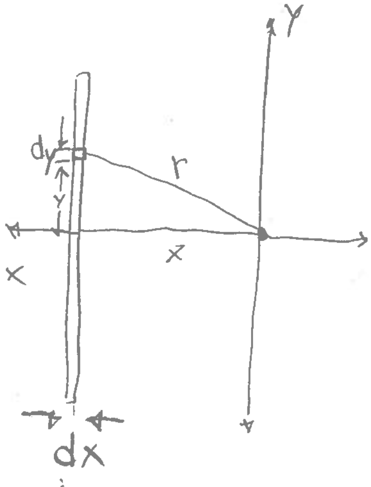
$$V = -\frac{e}{\pi\epsilon_0}\sqrt{\frac{8}{3}}\frac{1}{a} + \frac{3e}{2\pi\epsilon_0 a}$$

$$W = \frac{e^2}{\pi\epsilon_0}\sqrt{\frac{8}{3}}\frac{1}{a} - \frac{3e^2}{2\pi\epsilon_0 a} = \frac{e^2}{\pi\epsilon_0 a}\left[\sqrt{\frac{8}{3}} - \frac{3}{2}\right] > 0$$

(+) it takes this much energy to assemble this configuration, thus we can infer that an electron will experience an overall repulsive force.

Problem 4:





$$dV = \frac{4k\sigma\,dy\,dx}{r}$$

$$V = -4k\sigma \int_{0}^{b/2} dx \cdot \int_{-x}^{x} \frac{dy}{\sqrt{y^2+x^2}}$$

$$\boxed{V = -4k\sigma b \cdot \operatorname{arcsinh}(1).}$$