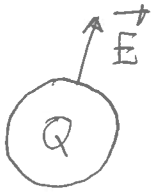
PHYS 272/272H – HW # 2 - Solutions

(1)



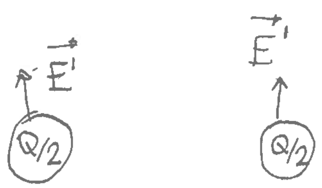
$$\oiint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$

$$d\vec{A} = dA\,\hat{r} \qquad E\,A = \frac{q_{enc}}{\epsilon_0}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{(d/2)^2}\,\hat{r}$$

$$V = \frac{4}{3}\pi\left(d/2\right)^3$$

$$V = \frac{\pi}{6}d^3$$



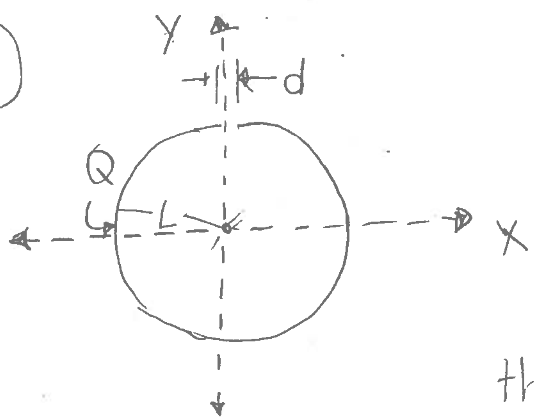
$$V' = V/2 = \frac{\pi}{12}d^3 = \frac{\pi}{6}d'^3$$

$$d' = d/2^{1/3}$$

$$\vec{E}' = \frac{1}{4\pi\epsilon_0}\frac{Q/2}{(d'/2)^2}\,\hat{r}$$

$$\boxed{\vec{E}' = \frac{2^{2/3}}{2}\vec{E} = 2^{-1/3}\vec{E}}$$

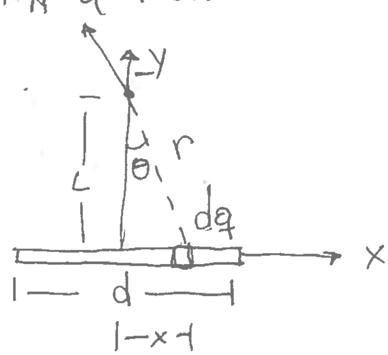
(2)



The field of a complete ring (= 0 at the center) is a superposition of the field of a charged piece and the field of the loop with a hole.

$$\vec{E}_{piece} + \vec{E}_{hole} = \vec{E}_{ring} = 0 \;;$$

$$\vec{E}_{hole} = -\vec{E}_{piece}$$



$$dE_{piece} = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2} \cos\theta \quad ; \quad dq = \frac{Q}{2\pi L} dx \qquad r^2 = L^2 + x^2$$

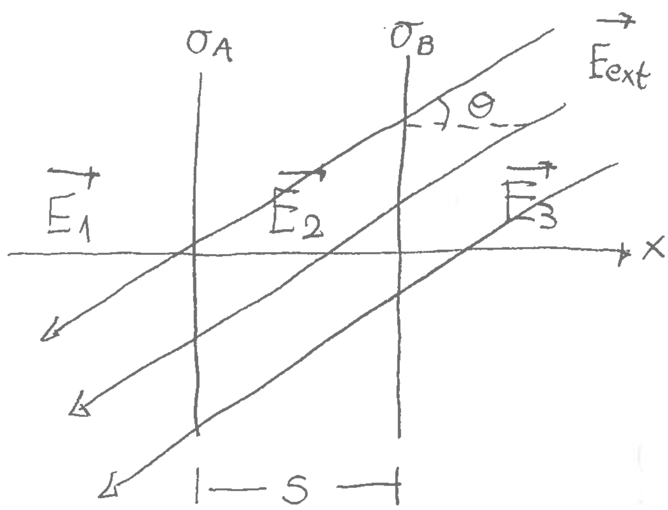
$$\cos\theta = \frac{L}{\sqrt{L^2+x^2}}$$

$$dE_{piece} = \frac{1}{4\pi\epsilon_0} \frac{Q}{2\pi} \frac{dx}{(L^2+x^2)^{3/2}} \Rightarrow E_{piece} = \frac{Q}{8\pi^2\epsilon_0} \int_{-d/2}^{d/2} \frac{dx}{(L^2+x^2)^{3/2}}$$

$$\vec{E}_{piece} = -\frac{Q}{4\pi\epsilon_0} \frac{d\sqrt{4L^2+d^2}}{\pi(4L^4+d^2L^2)} \hat{y}$$

Thus: $$\boxed{\vec{E}_{hole} = \frac{Q}{4\pi\epsilon_0} \frac{d \quad \hat{y}}{\pi L^2 (4L^2+d^2)^{1/2}} \approx \frac{Q\, d\sqrt{2}}{8\pi^2\epsilon_0 L^3} \hat{y}}$$

3



$$\vec{E}_1 = -\frac{1}{2\epsilon_0}(\sigma_A+\sigma_B)\hat{x} + \vec{E}_{ext}$$

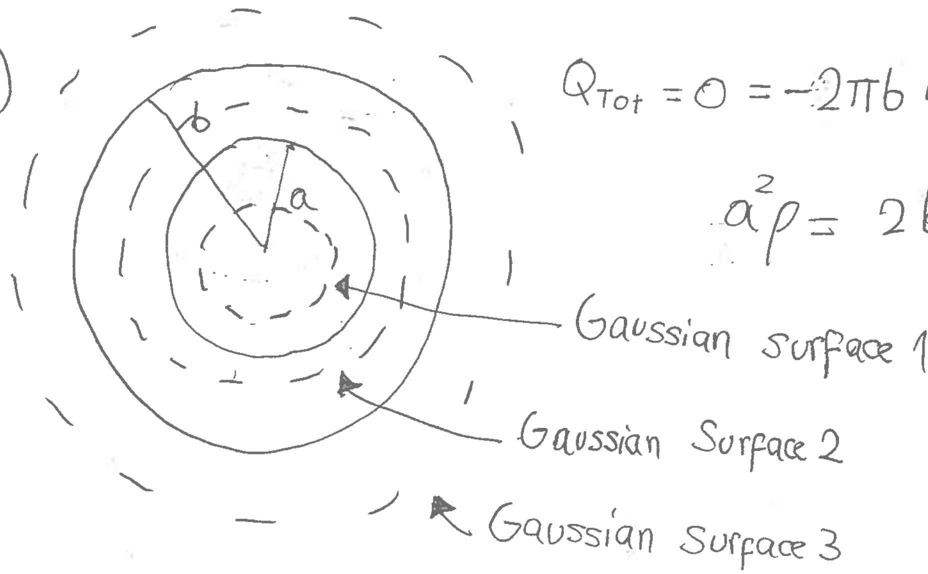
$$\vec{E}_2 = \frac{1}{2\epsilon_0}(\sigma_A-\sigma_B)\hat{x} + \vec{E}_{ext}$$

$$\vec{E}_3 = \frac{1}{2\epsilon_0}(\sigma_A+\sigma_B)\hat{x} + \vec{E}_{ext}$$

$$d\vec{F} = \sigma_A\, dA\, \vec{E}_{ext} + \sigma_B\, dA\, \vec{E}_{ext}$$

$$\frac{dF}{dA} = (\sigma_A+\sigma_B)\, E_{ext} \cos\theta$$

(4)



$$Q_{Tot} = 0 = -2\pi b L \sigma + \pi a^2 L \rho$$

$$a^2 \rho = 2 b \sigma$$

$$\oiint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0} \Rightarrow d\vec{A} = dA\,\hat{r}$$

$$E_1\, 2\cancel{\pi} r \cancel{L} = \frac{\rho \cdot \cancel{\pi} r^2 \cancel{L}}{\epsilon_0} \rightsquigarrow \boxed{\vec{E}_1 = \frac{\rho r}{2\epsilon_0}\,\hat{r}} \qquad 0 < r < a$$

$$E_2\, 2\cancel{\pi} r \cancel{L} = \frac{\rho \cdot \cancel{\pi} a^2 \cancel{L}}{\epsilon_0} \rightsquigarrow \boxed{\vec{E}_2 = \frac{\rho a^2}{2\epsilon_0 r}\,\hat{r}} \qquad a < r < b$$

$$E_3\, 2\pi r L = \frac{Q_{tot}}{\epsilon_0} = 0 \rightsquigarrow \boxed{\vec{E}_3 = 0} \qquad r > b$$