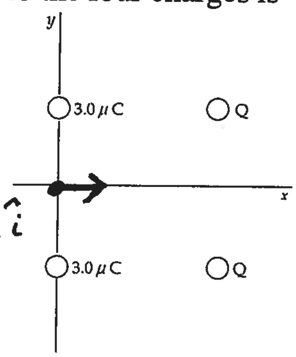
October 2, 2009
Physics 272 Exam 1:

Name: Solution

The value of the electric constant $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2/Nm^2$

Solve the four problems in the exam.

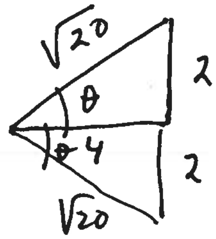
1.- Two 3.0 microC point charges are located at x=0, y=2.0 m and at x= 0 amd y=-2.0m. Two other point charges, each with charge Q, are located at x=4 m, y=2 m and at x=4 m, y = -2m as in the figure. Determine the charge Q if the electric field at the origine x=0, y=0 due to the four charges is 4.0 x 10^3 N/C **i**



$\vec{E} = 4{,}0 \times 10^3\ N/C\ \hat{i}$

By symmetry the 3μC do not enter only Q.

Since the direction is $+\hat{i}$ the charges have to be negative.



E_x

two charges ↓ same angle ↓

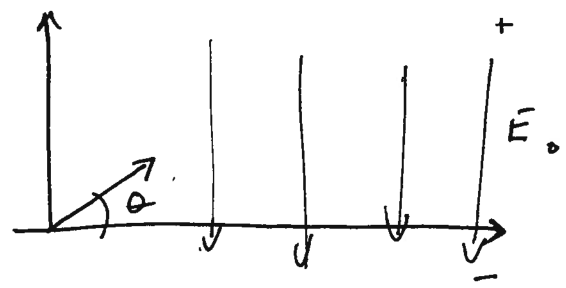
$$E_x = \frac{2\,Q}{4\pi\varepsilon_0 r^2}\cos\theta = \frac{2\,Q}{4\pi\varepsilon_0(\sqrt{20})^2}\frac{4}{\sqrt{20}} = 4\times10^3$$

$$Q = \frac{4\times10^3 \times 4\pi\varepsilon_0 \times 20 \times \sqrt{20}}{2\times4} = 497 \times 10^{-8}\ C$$

the sign of the charge is negative

$$\boxed{Q = -5\,\mu C}$$

2.- A electron (charge -1.6 x 10^{-19}C, mass 9.1 x 10^{-31} kg) leaves the origin with a speed of 3 X 10^6 m/s at an angle of 35 degrees above the x axis. A uniform electric field, given by **E**=-E_0 **j** , exists throughout the regin. Find E_0 such that the particle will cross the x axis at x=1.5 cm.



you can see that E_0 has to be negative otherwise the electron would not cross the x axis.

Neglect gravity.

$$\vec{F}_y = m\,\vec{a}_y = q\,\vec{E}$$

$$x = v_0 \cos\theta\, t \qquad y = v_0 \sin\theta\, t - \frac{1}{2} a t^2$$

$$a = \frac{qE}{m} \qquad y = v_0 \sin\theta\, t - \frac{eE}{2m} t^2$$

but $y = 0$ at $x = 1.5$ then solve for t.

$$t = \frac{x}{v_0 \cos\theta} \qquad 0 = \frac{v_0 \sin\theta\, x}{v_0 \cos\theta} - \frac{eE}{2m}\frac{x^2}{v^2\cos^2\theta}$$

$$E = \frac{v^2\, 2m \sin\theta\cos\theta}{e\,x} = -3.21 \times 10^3\ \text{V/m}$$

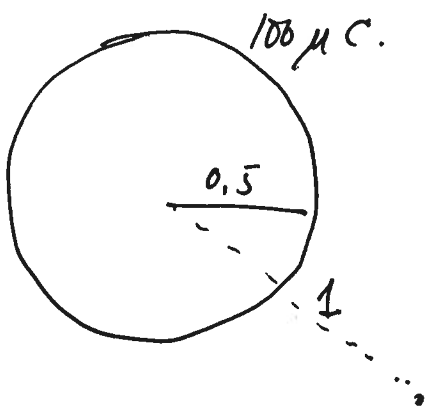
↑ consistent with what we said before.

3.- A hollowed metal spherical conductor or 50 cm radius has a total charge of 100 microCoulombs.

a) What is the magnitude of the electric field 1 m from the center of the sphere?

b) What is the surface charge density σ on the sphere?

c) What is the magnitude of the electric field at the center of the sphere?



Gauss

a) $$\oint \vec{E} \cdot d\vec{a} = \frac{Q}{\varepsilon_0}$$

take a sphere radius 1 m.

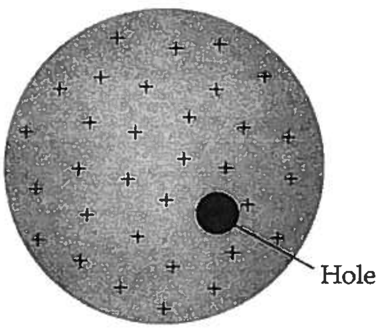
$$E \cdot 4\pi (1)^2 = \frac{100 \times 10^{-6}}{\varepsilon_0}$$

$$E = \frac{10^{-4}}{4\pi \varepsilon_0} \approx \boxed{9 \times 10^{5}\ V/m}$$

b) $$\sigma = \frac{Q}{A} = \frac{100 \times 10^{-6}}{4\pi (.5)^2} = \boxed{3.18\ 10^{-5}\ c/m^2}$$

c) conductor (closed) → 0

4. A long, thin, non-conducting uniformly charged spherical shell of radius R has a total positive charge of Q. A small circular plug of radius r is removed from the surface
a) What are the magnitude and direction of the electric field at the center of the hole?
c) Find the magnitude and direction of the electric field at the center of the sphere.



(a)

a) Decompose on two fields one closed shell and one from a "hole" with opposite charge.

On the surface: closed shell. hole

$E_{whole} = \frac{Q}{4\pi\epsilon_0 R^2}$ $E_{hole} = -\frac{\sigma}{2\epsilon_0}$

assume it small enough that it is flat.

$\sigma = \frac{Q}{4\pi R^2}$

$$E_{TOT} = \frac{Q}{4\pi\epsilon_0 R^2} - \frac{Q}{2\epsilon_0 4\pi R^2} = \frac{1}{2}\frac{Q}{4\pi\epsilon_0 R^2} = \boxed{\frac{Q}{8\pi\epsilon_0 R^2}}$$

direction outwards from hole.

c) now at the center of the sphere.

if $r \ll R$ ⟶ $E_{from\ hole} = \frac{1}{4\pi\epsilon_0}\frac{q}{R^2}$

close sphere.
↓

$$E_{TOT} = 0 + \frac{1}{4\pi\epsilon_0}\frac{Qr^2}{4R^4}$$

$q = \sigma A = \frac{Q\pi r^2}{4\pi R^2} = \frac{Qr^2}{4R^2}$

$$= \boxed{\frac{Qr^2}{16\pi\epsilon_0 R^4}}$$

pointing towards the hole.