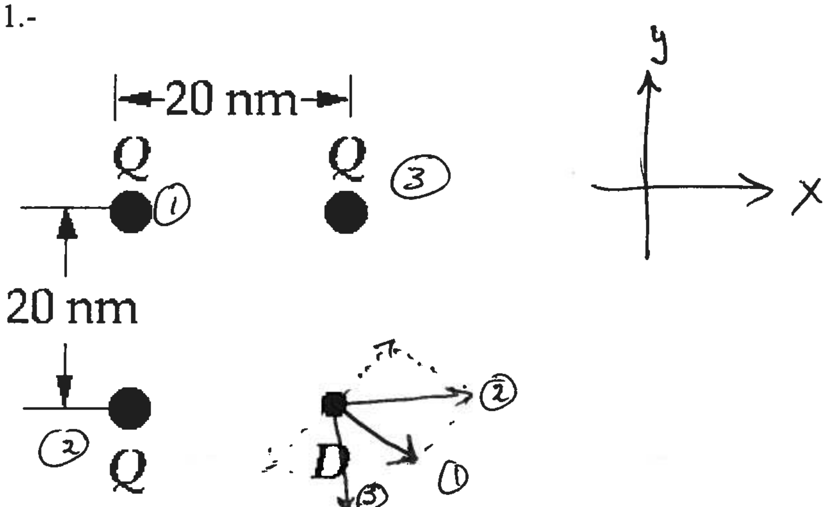
September 26, 2007
Physics 272 Exam 1:

Name: Solution;

The value of the electric constant $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2/Nm^2$

Solve the four problems in the exam.

1.-



Three charges, each of $Q = 3.2 \times 10^{-19}$ C, are arranged at three of the corners of a 20-nm square as shown. Find the magnitude of the electric field at D, the fourth corner of the square.

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{(\sqrt{2}\cdot 20)^2 (10^{-9})^2}\hat{\mu}_1 + \frac{q_2}{20^2(10^{-9})^2}\hat{\mu}_2 + \frac{q_3}{20^2(10^{-9})^2}\hat{\mu}_3\right]$$

7.2×10^{6}

$$\hat{\mu}_1 = +\frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j} \qquad \hat{\mu}_2 = \hat{i} \qquad \hat{\mu}_3 = -\hat{j}$$

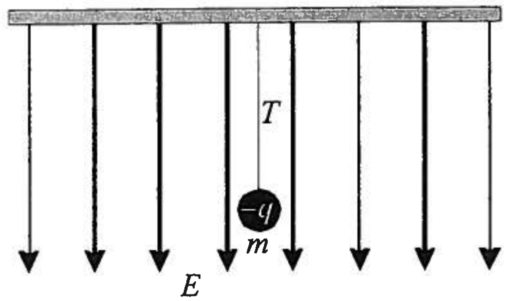
project $\hat{\mu}_{2,3}$ into $\hat{\mu}_1$ to obtain the magnitude directly as the other components cancel.

$$\vec{E} = \frac{1}{4\pi\varepsilon_0 (10^{-9})^2}\left[\frac{q_1}{(\sqrt{2}\cdot 20)^2} + \frac{1}{\sqrt{2}}\frac{q_2}{20^2} + \frac{1}{\sqrt{2}}\frac{q_3}{20^2}\right]\hat{\mu}_1$$

$$|E| = \frac{q}{4\pi\varepsilon_0}\frac{1}{20^2}\left(\frac{1}{2} + \sqrt{2}\right) = \frac{3.2\times10^{-19}}{4\pi\times 8.85\times10^{-12}} \times \frac{1}{(20\times10^{-9})^2}\cdot\left(\frac{1}{2}+\ldots\right.$$

$$= 1.38 \times 10^{7}\ N/C.$$

2.-



A conducting sphere has a net charge of $-q$ and of mass m is suspended from the ceiling by a light string. A uniform electric field E is applied vertical down on the sphere. If m = 1 g, q = 1 μC and E = 5000 N/C, Find the tension T in the string.

$$T + F_e - W = 0$$

$$T = W - F_e$$

$$= g \times 10^{-3} - 5 \times 10^{3} \times 10^{-6}$$

$$= g \times 10^{-3} - 5 \times 10^{-3}$$

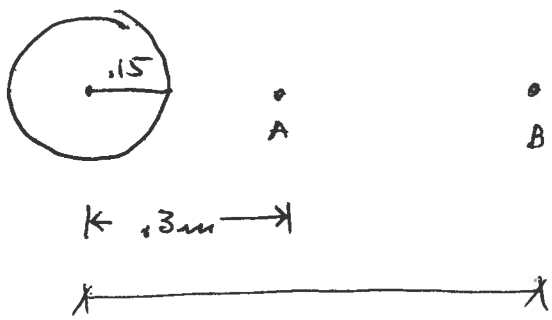
$$= (9{,}81 - 5) \times 10^{-3}$$

$$= 4{,}81 \times 10^{-3}\ N$$

3.- A solid spherical conductor has a radius of 15 cm. The electric field 30 cm from the center of this sphere has a magnitude of 800 N/C.

a) What is the magnitude of the electric field 1 m from the center of the sphere?

b) What is the surface charge density σ on the sphere?



b) charge density.

CONDUCTOR → SURFACE

~~SYMMETRY~~ SYMMETRY: SPHERE.

$$Q_{TOT} = \sigma A_{sphere}.$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{1}{\varepsilon_0} Q_{TOT}$$

$$\overbrace{E}^{800} (.3m)\, \cancel{4\pi}(.3)^2 = \frac{1}{\varepsilon_0} \sigma \cancel{4\pi}(.15)^2$$

$$\sigma = \frac{800}{\cancel{\varepsilon_0}} \frac{(.3)^2}{(.15)^2} \varepsilon_0 = 4 \cdot 800 \cdot \varepsilon_0 = 2{,}83 \times 10^{-8}\, C/m^2$$

a) $$E \cdot \cancel{4\pi}(1) = \frac{1}{\varepsilon_0} \sigma \cancel{4\pi}(.15)^2$$

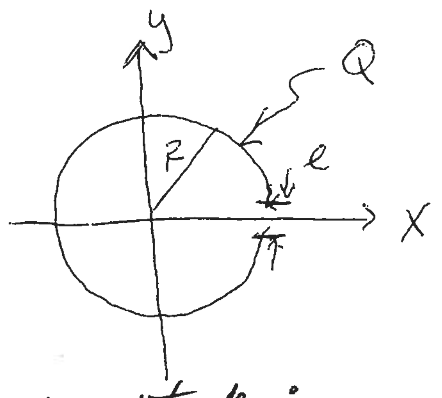
$$E = \frac{1}{\varepsilon_0} \sigma (.15)^2 = \frac{1}{\cancel{\varepsilon_0}} \times \frac{(.3)^2}{(.15)^2} \cdot 800 \cdot \cancel{\varepsilon_0}$$

or

$$E(1m) = E(.3) \cdot \frac{(.3)^2}{(1)^2} = 800 \times .09$$

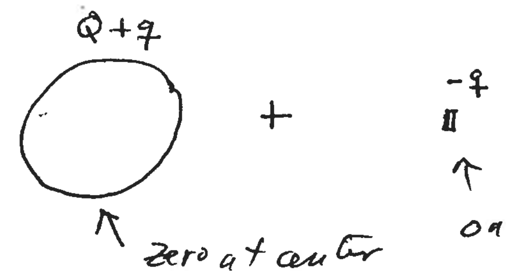
$$E = 72\ N/C$$

4. A long, thin, nonconducting plastic rod is bent into a loop with radius R. Between the ends of the rod, a small gap of length l ($l<<R$) remains. A charge Q is equally distributed on the rod.
a) Indicate the direction of the electric field at the center of the loop.
b) Find the magnitude of the electric field at the center of the loop.
c) Find the magnitude of the electric field at the gap of the loop.



a) by symmetry $\boxed{\hat{i}}$

b)



Magnitude:

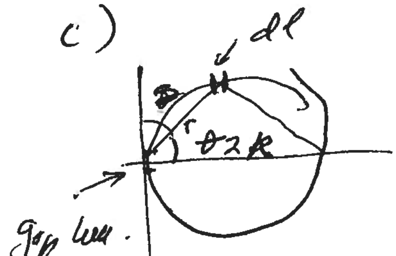
Length of the loop: $2\pi R - l = L$ $\qquad \lambda = \frac{Q}{L}$

$$q = \lambda l = \frac{Ql}{L} = \frac{Ql}{2\pi R - l} \approx \frac{Ql}{2\pi R}.$$

treat $-q$ as a point charge.

$$E = \frac{1}{4\pi\epsilon_0}\frac{\frac{Ql}{2\pi R - l}}{R^2} \approx \boxed{\frac{1}{4\pi\epsilon_0}\frac{Ql}{2\pi R^3}}$$

c)



$$d\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2}\hat{r} = \frac{1}{4\pi\epsilon_0}\frac{\lambda r\, d\theta}{r^2}\hat{r}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{\lambda\, d\theta}{r}\hat{r}$$

$r = 2R\cos\theta$

$dl = r\,d\theta \qquad dq = \lambda\, dl$

but we only need the x component by symmetry.

$$dE_x = \frac{1}{4\pi\epsilon_0}\frac{\lambda\, d\theta}{2R\cancel{\cos\theta}}\cancel{\cos\theta}$$

$$E = \frac{1}{4\pi\epsilon_0}\frac{\lambda}{2R}\int_{\frac{\pi}{2}-\delta}^{-\frac{\pi}{2}+\delta} d\theta$$

$$E = -\frac{1}{4\pi\epsilon_0}\frac{\lambda}{2R}\left[\pi - 2\delta\right] \approx \boxed{\frac{1}{8}\frac{\lambda}{R\epsilon_0}}$$