October 25, 2007
Physics 272 Exam 2:

Name: Solution

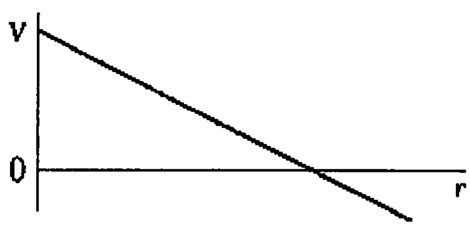
The value of the electric constant $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2/Nm^2$.
The value of the magnetic constant $\mu_0 = 4\ \pi \times 10^{-7}\ T\ m/A^2$

Solve the five problems in the exam.

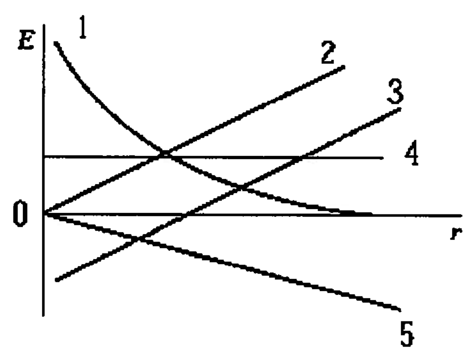
1.-

The electrostatic potential as a function of distance along a certain line in space is shown in graph (1).

a) Which of the curves in graph (2) is most likely to represent the electric field as a function of distance along the same line? 4

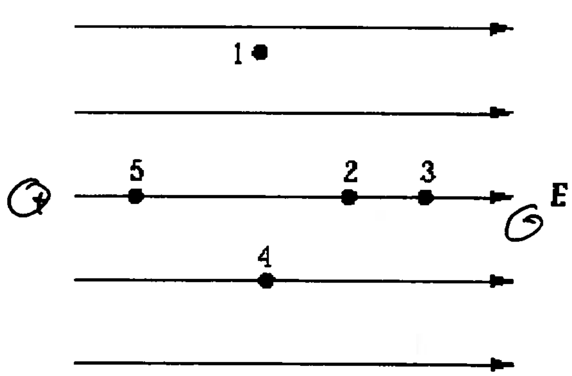


(1)



(2)

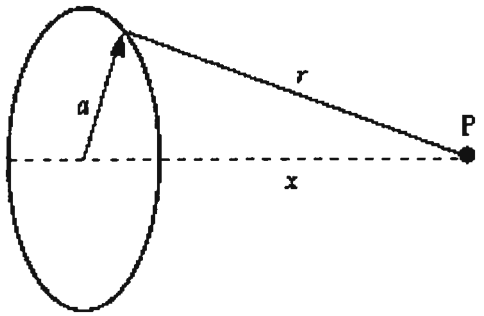
b) Which of the points shown in the diagram (3) are at the same potential? 1, 4
c) Which point in the electric field in the diagram (3) is at the highest potential? 5
d) Which point in the electric field in the diagram (3) is at the lowest potential? 3



(positive charges)

(3)

2.- A ring of radius a is in the *yz* plane with its center at the origin. The ring carries a uniform charge of Q. Find the electric potential at *x* >>a.



$E(r)$ very far, radial (sphere) symmetry!
Gauss's law.

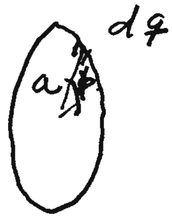
$$E \cdot 4\pi x^2 = \frac{1}{\epsilon_0} Q$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{x^2}$$

$$\int \vec{E} \cdot d\vec{l} = \frac{Q}{4\pi\epsilon_0} \int_{\infty}^{x} \frac{1}{x^2} dx = -\frac{Q}{4\pi\epsilon_0} \frac{1}{x} \Big|_{\infty}^{x} = -\frac{Q}{4\pi\epsilon_0} \frac{1}{x}$$

$$V = \frac{Q}{4\pi\epsilon_0 x}$$

or



$$Q = \lambda 2\pi a$$

$$dq = \lambda a \, d\phi$$

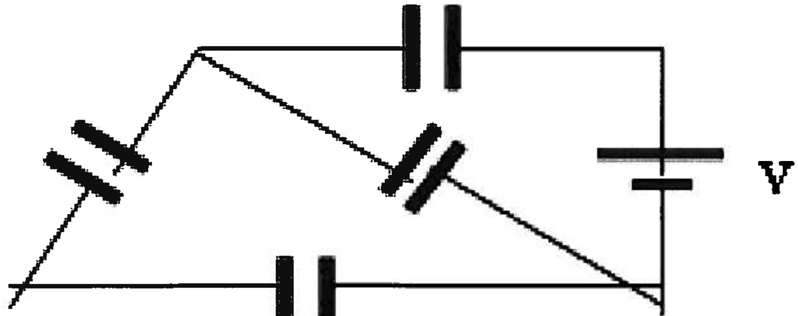
$$r = \sqrt{x^2 + a^2}$$

$$dV = \frac{1}{4\pi\epsilon_0} \frac{dq}{r}$$

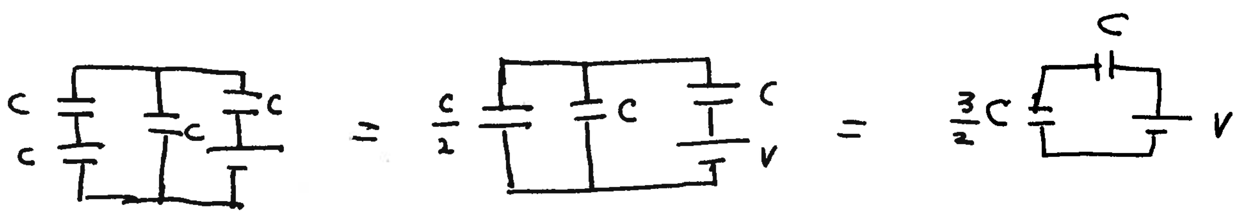
$$V = \frac{1}{4\pi\epsilon_0} \int_0^{2\pi} \frac{\lambda a \, d\phi}{\sqrt{x^2+a^2}} = \frac{1}{4\pi\epsilon_0} \frac{\lambda a}{\sqrt{x^2+a^2}} 2\pi$$

$$V = \frac{Q}{4\pi\epsilon_0} \frac{1}{|x|}$$ when $x \gg a$

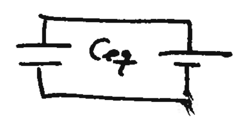
3.-



If all the four capacitors have equal values of 50 μF then calculate the equivalent capacitance of the circuit shown above.



$$\Rightarrow \frac{\frac{3}{2}}{\frac{5}{2}} C = \frac{3}{5} C$$

$=$ 

Ceq. 30 μF

4.- A 12 V automobile battery with negligible internal resistance can deliver a total charge of 16- Ampere hour

a) What is the total stored energy in the battery?
b) How long could this battery provide 150W to a pair of headlights?
c) What is the resistance of each of the headlights?

a) $P = i V$ $\qquad \Delta U = P \Delta t$

$\Delta U = 12 \times 16 = 192\ W\ h$

convert to Joules $\qquad 1\ h = 3600\ s.$

$\Delta U = 196 \times 3.6 \times 10^3 = 6,91 \times 10^5\ J$

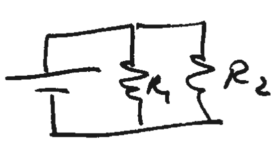
b) $P = 150$ Watts.

$$\Delta t = \frac{\Delta U}{P} = \frac{192\ W \cdot hour}{150\ W} = 1,28\ h.$$

c) $P = \frac{V^2}{R_{TOT}}$ $\qquad$ Headlights are in parallel.

$$R_{TOT} = \frac{V^2}{P} = \frac{(12)^2}{150} = \frac{144}{150} = 0,96\ \Omega$$

$R_1 = R_2$



$R_{TOT} = \frac{R_1}{2}$

$R_1 = 1,92\ \Omega$

5.- A beam of charged particles moving with a speed of 10^6 m/s enters a uniform magnetic field of 0.1 T at right angles to the direction of motion. If the particles move in a radius of 0.2 m, then calculate their period of motion.

Translation Kinetic energy = Rotational energy

$$\frac{1}{2} m v^2 = \frac{1}{2} m \omega^2 r^2$$

$$\frac{v}{r} = \omega \qquad T = \frac{2\pi}{\omega} = \frac{2\pi r}{v} = 1,26 \times 10^{-}$$

or:

$$\frac{v}{r} = \frac{qB}{m} \qquad \omega = \frac{qB}{m}$$

$$\frac{v}{r} = \omega$$