Experiment

VIII Equipotentials and Fields

I. References

Serway and Jewett, Vol. 2, Chapter 25

II. Apparatus

4 electrode boards
docking station for electrode boards
2 templates for drawing electrodes
DC power supply\
DVM, 10 volt scale

III. Pre-Lab Questions

At an arbitrary point on an equipotential curve like those in the experiment, what is the relation of electric field direction to the tangent to the equipotential curve? Sketch an example.

IV. Introduction

The concept of an electric field is a very useful and important one in the study of electricity. Replacing the fact that, according to Coulomb's Law, one charged body exerts a force on another by the idea that a charged body is surrounded by an electrical field and that another charged body placed in the field feels a force proportional to the field sounds like an unnecessary complication; and indeed it is for the simple case. Where it becomes useful is when the charged bodies have more complicated shapes than a point or sphere. This concept becomes virtually indispensable when you investigate electromagnetic waves (x-rays, light, radio).

Another idea that is closely related to the electric field is that of the electrical potential. This is useful mainly because it is usually much easier to calculate or measure than the field itself. Once you know the potential field you can then easily calculate the associated electric field. An equipotential surface (i.e., the collection of points that all have the same values of potential) **always** forms a closed surface around a charged body. A different value of potential gives a different surface. Also, no two surfaces can cross each other, because this would require that the potential have two values at the same point. We will be interested in the case when the charged body is also a conductor. In this case the surface of the body itself is an equipotential, and the equipotentials nearby have the same shape as the body and as you move away from the body the equipotentials tend to smooth out and get rounder.

Suppose you placed an irregularly-shaped charged conductor near the origin of a coordinate system; what would the equipotential surfaces look like? They would be closed surfaces around the conductor and they are hard to draw since they are three-dimensional. Instead, we will consider the intersection of these surface with the x-y plane; you get a series of closed curves around the object. Fig. (VIII.1) show the sort of thing to expect; the potential difference between each two successive curves is one volt. The magnitude of the electric field in Fig. (VIII.1) is given by the difference in potential divided by the (perpendicular) distance between two successive equipotentials (actually you have to take the limit as the distance goes to zero).

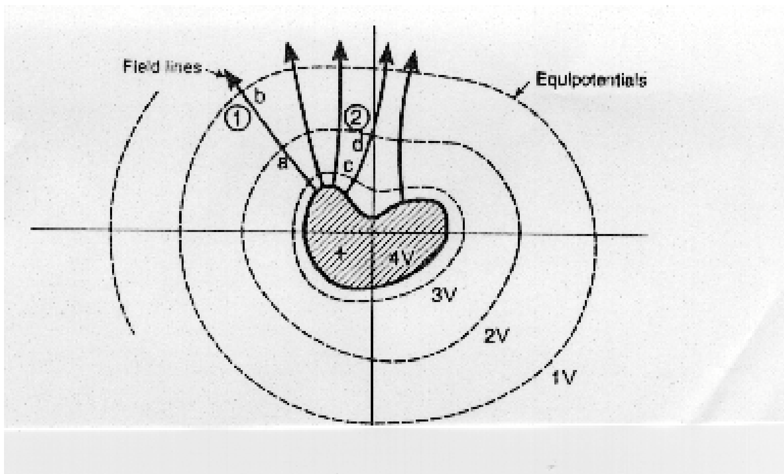


Figure VIII.1

With this in mind, you can just look at a plot like Fig. (VIII.1) and see the relative size of the electric field - where the curves are close together the field is high, and vice versa. In general, the field is high near lumps and low near depressions. Fig. (VIII.1) looks very much like a contour map of a hill, and indeed this analogy can be carried quite far. In a contour map, when the contours are close it means a steep rise (analogous to a strong field) and vice versa. Also, if you release a ball on the side of a hill it will roll along the steepest route down the slope. The direction of the steepest slope is always perpendicular to the contour line that the ball starts on. For the electrical case, the field is always perpendicular to the equipotentials, and a charge placed in the field will always move along the field lines. The analogy is very good; the main difference is that in the electrical case the “contours” are really surfaces whereas for maps they are lines. In engineering, a steep hill is said to have a large “gradient” or “grade.” Because of the above analogy, the mathematicians have taken the word, shortened it to “grad”, and used it to describe the mathematical operation of getting the electric field from the potential; (or maybe it went the other way around!) thus:

$$\overline{E} = -grad\,\psi = -\nabla\,\psi = -(i\frac{\partial\psi}{\partial\chi} + j\frac{\partial\psi}{\partial y} + k\frac{\partial\psi}{\partial z}) \qquad \text{VIII.1}$$

where **E** is the electric field and ψ is the potential. The minus sign comes from the convention that electric fields point "downhill" (toward smaller ψ). A few lines which give the direction of the electric field are drawn in Fig. (VIII.1); they must intersect the successive equipotentials at right angles.

The purpose of this experiment is to determine the electric potential in the space around two oppositely charged electrodes of various shapes. The field mapping apparatus will be used to trace out a number of equipotential surfaces (or lines, as one is restricted to a single plane) covering, in equal intervals, the potential difference between the electrodes.

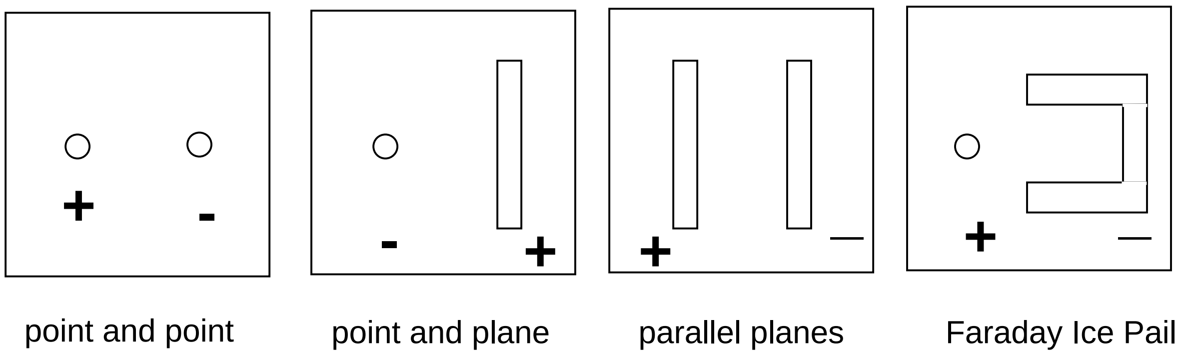
V. Procedure

The device you will use is called a "current board" or "electric field mapping board". It consists of a flat board that is painted with an electrically conducting paint on one side. If the two terminals of a dc power supply are connected to opposite ends of the painted surface, a current will flow from the positive power supply terminal to the board, spread throughout the conducting paint loop around to the other connection and return to the negative power supply terminal. It turns out that the equations describing the value of the current (or field) at each point on the board are the same as the equation that describes the field around a similar configuration on charged conductors. In either case, the potential and electric field are related by Eq. (VIII.1) and so, if you measure the equipotentials on the current board, you automatically have them for charged conductors.

You will measure the equipotentials for four pairs of conductors: (1) two points (a dipole), (2) a point and a plane, (3) two parallel planes (parallel plate capacitor), and (4) a point and a U-shaped line called the Faraday Ice Pail (to show the shielding effect of an almost-closed conductor). These conductors are actually painted on the conducting board with silver paint. Silver paint is a very good conductor whereas the other paint on the boards (carbon) is a rather poor conductor. This large difference in conductivity is what ensures that each silver electrode has the same potential throughout.

A diagram of the set-up is shown in Fig. (VIII.2). The DC power supply is a source that provides a constant voltage and current.

The power supply is hooked directly to the two silver-painted electrodes in the example shown in Fig. (VIII.2). The voltmeter is connected between one electrode and the probe. In general, if you touch the probe at an arbitrary place on the board there will be a potential difference across the voltmeter. As you move the probe around, you can find a series of points where the potential is equal. Connecting the points which are at the same potential maps the equipotential surface. .



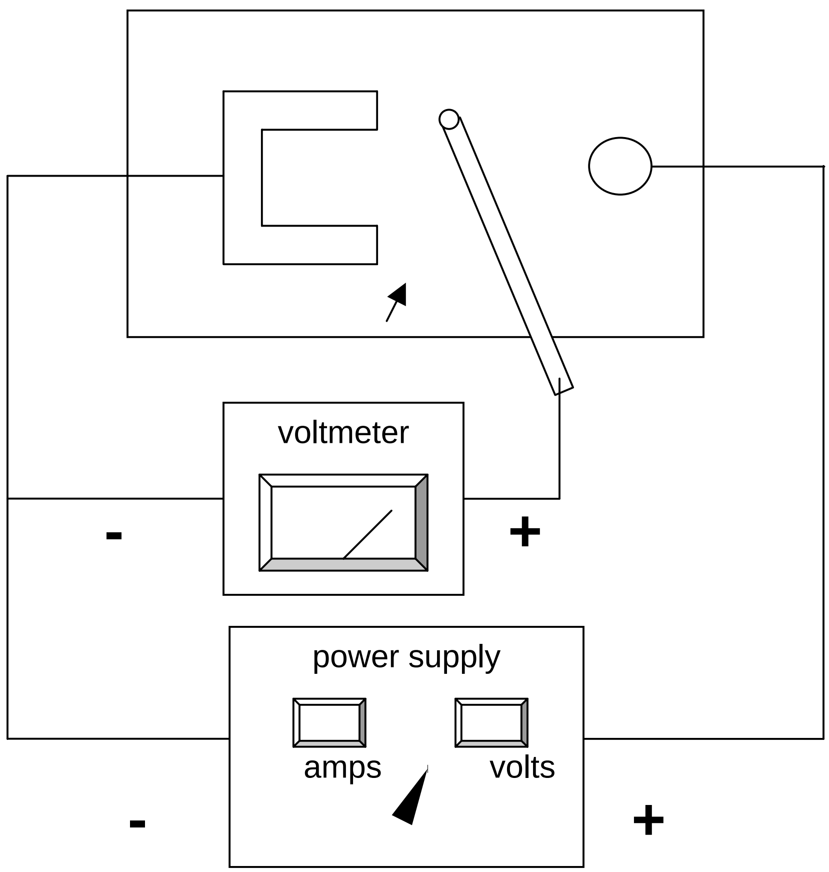


Figure VIII.2 Electrode patterns and wiring diagram. Note carefully the signs of the connections

The actual set-up is made so that you can automatically draw the equipotentials. The electrode board itself is clamped underneath a small table and, given enough hands, a piece of paper can be taped onto the top of this table. The probe is a tweezer-shaped affair that is slid on so that the tweezer end with the metal electrode presses up against the conducting paint on the electrode board and the other end presses down on the paper. There is a hole through the upper tweezer arm so that you can mark the equipotential points through it on the paper. Proceed as follows:

Select, in order, the first of the "charge configurations" mentioned above, bolt it to the bottom of the table and also secure a piece of paper on the top of the table.

Caution: In order to prevent a short circuit and possible electrode damage, do not allow the metal templates or other metal objects to get under the equipotentials apparatus.

Use the probe and meter to mark points of equal potential on your sheet, as follows: Move the probe close to the positive electrode. Set the power supply so that its meter reads just a little over 7 volts (say 7.1 V). By moving the probe over the high resistance surface, identify and mark at least 10 points that yield a DVM reading of exactly 7.0 volts. This sequence of pencil marks can then be connected by a smooth, continuous curve, which will represent an equipotential line all the points of which are at the same potential. Notice that this first equipotential, close to the electrode, follows the shape of the electrode fairly closely.

Without changing the power supply setting, repeat this process to identify equipotential lines for 6.0, 5.0, 4.0, 3.0, 2.0 and 1.0 volts.

In the same manner record equipotential lines on the remaining three electrode arrangements.

Take turns with your lab partner operating the probe and recording the equipotential points. A map needs to be done only once per electrode configuration. The map may be traced or photocopied for your partner.

At this time set the power supply to zero volts and turn it off if you have finished with it.

Notice the polarity (+ or -) of each electrode indicated in Fig. VIII.2. Wire your circuit paying attention to the polarities indicated. **Indicate these polarities on your electrode tracings.**

Now that you have the drawings there are a few more things to do with them.

The "dipole" represents the equipotentials around a pair of long straight charged wires. (Since nothing depends on the distance parallel to a long straight wire, such a wire approximates a two-dimensional system. Your plot is a section across the wires.) Now the potential around a long straight wire is proportional to $\ell n(r)$, and can be written as

$$\psi = -\psi_o \ell n(r/a) \qquad \text{VIII.2}$$

where a is the radius of the wire, and $\psi_0 = (\lambda / 4\pi\varepsilon_0)$ with λ the linear charge density on the wire in Coulomb/m. So, for two wires at opposite potentials (charge), the equipotentials are given by

$$\psi_0 \ell n(\frac{r_1}{a}) - \psi_0 \ell n(\frac{r_2}{a}) = \text{constant.} \qquad \text{VIII.3}$$

By symmetry the zero of potential is at the line halfway between the two wires. (This is a useful choice only if the wires are equal in radius.)

$$\frac{r_1}{r_2} = \text{ constant on } \textbf{each} \text{ equipotential surface.} \qquad \text{VIII.4}$$

The constant is different for different equipotentials.

VI. Final Questions

1. Check Equation VIII.4 for a few points on one of the equipotentials for the dipole pattern.

2. What happens when an equipotential gets near the edge of the current board? Why?

3. The four configurations make up a kind of a sequence. Describe, in a sentence or two for each, how the equipotentials change as you go through the sequence.

4. Finally, sketch-in some electric field lines on each of your drawings and note how they vary through the sequence of patterns.