Experiment

VII The Ideal Gas Law and Absolute Zero

I. Purpose:

To verify that $PV=nk_BT$ for an ideal gas and find the absolute zero of temperature.

II. References:

Serway and Jewett, Vol. 1, Chapter 19.

III. Prelab Questions (turn in at the start of lab)

1. Suppose we have 100 cm^3 of nitrogen at standard temperature and pressure (room temperature and sea level pressure; i.e., 300 K and 1 atmosphere = 1.013×10^5 Pascal (Pa)). How many moles of nitrogen are there?

2. Consider the Boyle's Law Apparatus. Suppose that the volume of the gas is 20 cc and the pressure is 30 lbs/in^2. If the plunger is depressed so that the volume is decreased to 10 cc, what will the pressure be?

3. Give an example from daily life which shows that the Celsius temperature scale is not an absolute temperature scale.

IV. Equipment

Boyle's Law Apparatus
Charles' Law apparatus
community ice bath
wall thermometer
steel beaker on a hot plate

V. Introduction

The pressure P, volume V and temperature T of an ideal gas are related by

$$PV = nk_BT, \qquad \text{[VII.1]}$$

where n is the number of molecules of the gas, and k_B is Bolztmann's constant. We can also write this as

$$PV = NRT, \qquad \text{[VII.2]}$$

where N is the number of moles of gas, and R is the gas constant. The ideal gas law is obeyed fairly well provided the density of the gas is not too high. If the density is too high, the gas may condense to form a liquid, and the law really fails! In the first part of this lab, you will keep the temperature fixed and show that as you vary the volume of a fixed quantity of gas, the pressure changes so that PV remains constant. This is Boyle's law and is a direct consequence the ideal gas law.

In the second part of the lab you will explore the absolute temperature scale. The ideal gas law only works if you use the "right" temperature scale for T. You have to measure T on an absolute temperature scale. What does this mean? The absolute temperature of an ideal gas is proportional to the kinetic energy of its molecules. The lowest value that the kinetic energy can reach is zero. On an absolute temperature scale, this happens at T=0, or what is called "absolute zero". The Celsius and Fahrenheit temperature scales are examples of temperature scales which are **not** absolute temperature scales, so the ideal gas law will seem to be wrong if you use these scales. If you are measuring temperature on the Celsius scale, then the ideal gas law must be modified to the form

$$PV = nk_B\,(t_c - t_0), \qquad [VII.3]$$

where t_o is the "absolute zero of temperature" on the Celsius scale, i.e. the temperature at which the kinetic energy of an ideal gas vanishes. We can rewrite this equation in the form

$$t_c = t_o + PV/nk_{B.}\;. \qquad [VII.4]$$

In the second part of this lab, you will determine t_o by seeing how the pressure of gas in a fixed volume V varies with temperature.

Water should not be brought to boil at all setups simultaneously. Hot plates should remain OFF unless being used to boil water.

VI. Experiment

Note for TA's: In order to avoid tripping circuit breakers and interrupting computers we suggest either of two available strategies: 1) Ask half of the students to perform the Boyle's Law experiment while the other half are performing the Charles' Law experiment;
2) Run just one hotplate per bank of lab tables. Look at the power lines on the ceiling to determine which outlet boxes would be connected to a common breaker.

Part A: Verify Boyle's Law: PV=constant

1. Set up your spreadsheet with the format shown for Table 1 on the Spreadsheet Data Table Page. Include the units needed for each column.

2. Check that the hose is securely connected to the gauge and cylindrical volume, see Figure VII.1. Check with your instructor if you do not know how to disconnect the Swagelok airline connector.

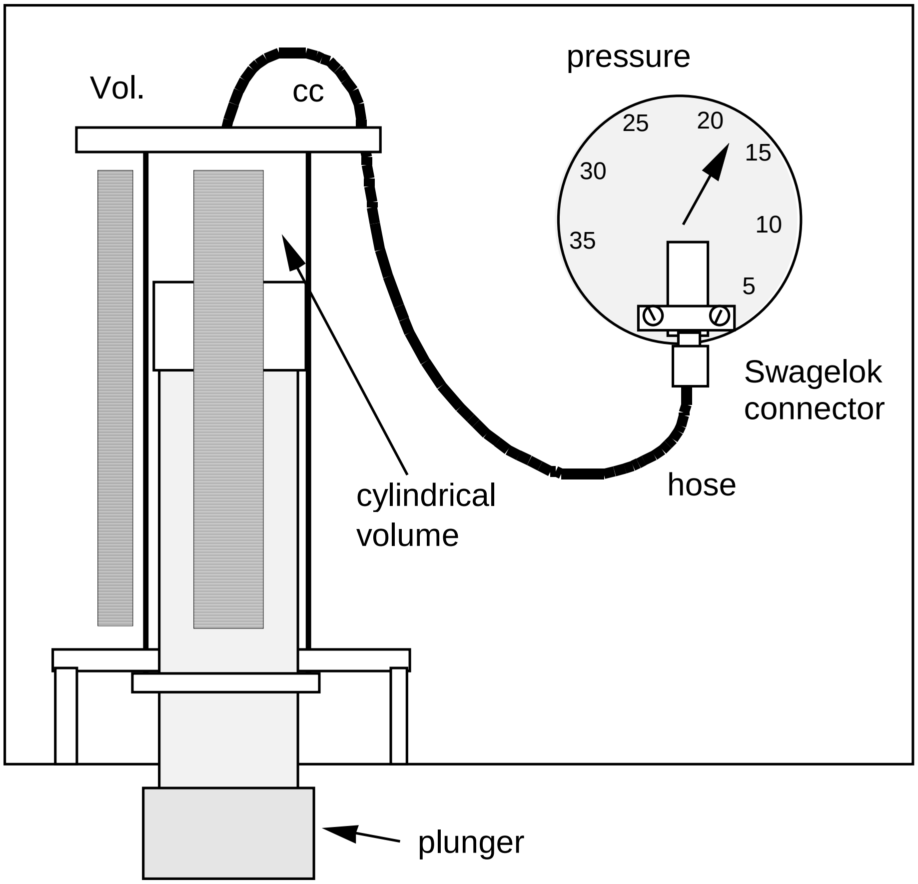

Figure VII.1 Boyle's Law Apparatus.

3. Check for air leaks by disconnecting the air hose at the gauge, pushing the plunger in to the 7cc mark, reconnecting the air hose, pushing the plunger all the way in and holding it there for at least 30 seconds. If the pressure drops appreciably, you have a leak in either the hose or the plunger seal and you will need to notify your TA to have it fixed before taking data. **NEVER remove the plunger completely from its housing.**

4. To obtain accurate measurements, you will need to recognize that the markings on the cylinder only record the volume of the cylinder and do not account for volume in the hose and gauge. This extra volume must be taken into account to obtain good agreement with Boyle's law. You are about to encounter the Swagelok connector. To operate it, merely push the collar toward the gauge. Do not remove the hose from its connectors (either to the collar or to the cylindrical volume). To calibrate the volume of the hose and gauge, remove the hose connector from the pressure gauge connector and push the plunger all the way in, so that it reads zero volume. Re-attach the hose and record the pressure. Now pull the plunger out until the pressure falls to one-half its original value. Record the volume V_{HG} at which this happens in your spreadsheet in the appropriate cell. If Boyle's law is correct, then this volume will be equal to the volume of the hose and gauge. **Do not at any time remove the plunger completely from the cylinder as this may allow breakage by accidental dropping or rolling off the table. The replacement cost for the syringe assembly is over $100.**

5. Estimate ΔV_{HG}, the experimental uncertainty in V_{HG}, and record in the appropriate cell in your spreadsheet.

6. Check for air leaks as specified in part 3. If the system is leak tight, record P, the pressure, and V_c, the volume of air in the cylinder, in your spreadsheet.

7. Estimate the uncertainties ΔP and ΔV_c. Record in the appropriate cell in your spreadsheet. The uncertainties in P and V should not change the same for all of your measurements since scale on the pressure gauge and the linear scale on the syringe have constant reading uncertainties.

8. Without removing the gauge or hose, vary the position of the plunger and again record the pressure and volume in your spreadsheet. Repeat this for six additional pressures.

Part B: Charles' Law: P/T = constant

1. Set up another spreadsheet with the format given by Table 2 in the spreadsheet data table page.

2. Carefully examine the Charles' Law apparatus depicted in figure VII.2, but do not allow your hands to warm the copper sphere. Open the valve (valve lever parallel to direction of air flow through the valve) so that the pressure inside the apparatus can equalize with the external pressure. **You must be gentle with this apparatus because (a) the gauge may have a glass front, (b) the copper sphere dents easily, and (c) calibration of the instrument can be lost. If you lack elbow room on the lab table and might accidentally damage the instrument, ask your TA for a safe place to store it while not it use.**

3. Record the pressure in your new spreadsheet. Also, measure the temperature of the air in the room and record in the spreadsheet.

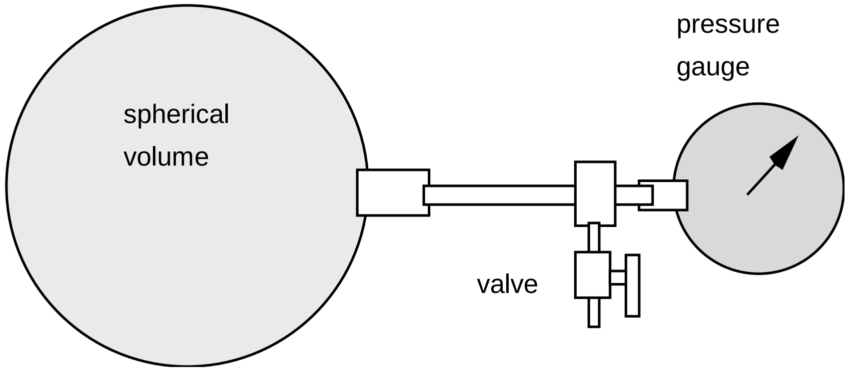

Figure VII.2 Charles' Law apparatus. The valve is shown in the open position. (Note: Pressure gauge reads in Torr and 1 Torr = 1mm Hg.)

4. Estimate the uncertainty in the pressure and temperature and record in the spreadsheet. Your uncertainties in the pressure should be about the same for all of the measurements, since you will use the same gauge. However, for the rest of the measurements you will not use the thermometer to get the temperature, but rather used the fixed temperatures 0^oC and 100^oC found in an ice water bath and in boiling water, respectively. The uncertainty in the temperature will be negligible for the rest of the measurements.

5. Bring a half full beaker of water to a rapid boil, close the Charles' Law apparatus valve, immerse the copper sphere in the boiling water for about 30 seconds, tap the gauge lightly with a pencil, and then, while keeping the apparatus vertical, take a pressure reading and record it in your spreadsheet at a temperature of 100 ^oC. **Safety: (a) Under no circumstances is it safe to place the apparatus in boiling water assuming it will remain upright and not spill boiling water onto you or your partner; (b) As soon as you have recorded your data, set the hotplate to "off" and unplug it.**

6. Next place the sphere in ice water. Allow about 30 seconds for the gas in the sphere to cool down and then record the pressure in your spreadsheet, at a temperature of 0 ^oC.

VII. Analysis

Part A (Spreadsheet Table 1)

1. For each volume measurement V_c in your spreadsheet Table 1, use your spreadsheet to compute the total volume $V = V_c + V_{HG}$ by taking into account the volume of the hose and gauge.

2. Using the data from your Table 1 spreadsheet, make a plot of your measured P versus V^{-1}. Be sure to label the axes properly including units.

3. Now place error bars on your plot, using your estimated error in P which you recorded in your Table 1 spreadsheet. You can also place error bars on V^{-1} in the horizontal direction.

4. For each pressure and volume, use the spreadsheet to compute PV.

5. Use the spreadsheet to Compute $\langle PV \rangle$, the average value of PV.

6. Now use the spreadsheet to compute $\Delta(PV)$ for each of your data points.

7. Use the spreadsheet to compute χ^2 for the theory that PV is equal to the constant value $\langle PV \rangle$. What approximate value of χ^2 would you expect, if you assume that your error estimates are reasonable.

8. Compute the **reduced value** of χ^2 and $P(\chi^2,\nu)$ record in your spreadsheet Table 1. Here the number of degrees of freedom, ν, is equal to the number of data points minus one since you used your data to compute <PV>.

Part B (Spreadsheet Table 2)

1. Using your data in spreadsheet Table 2, make a linear-linear plot of T versus P. Make sure that you choose P as the independent variable (x-axis) and T as the dependent variable. For the next part of the analysis you will need to extrapolate the data down to about -300 ^{o}C, so be sure to plot the graph with the temperature axis extending down at least this far.

2. Place error bars on your T versus P plot using your estimated error in P. Also place an error bar on your measurement of room temperature.

Use your spreadsheet to do a least-square straight line fit through your T versus P data. This line should intercept the temperature axis at the absolute zero of temperature, T_o. You should use the LINEST function in Excel so that you calculate the uncertainties in the slope and intercept for your data set.

To do this with Excel, use the Statistical Function LINEST from the function list in the Insert menu. This requires that you select the y values, the x values, and two flags set to non-zero values (say to 1). The output of the fit is directed to a 2x2 array by dragging over a 2x2 set of cells to the right and down from where the function is inserted. Place the cursor on the function display line and do a CTRL-SHIFT-ENTER key stroke. The numbers in the 2x2 array are:

Slope	Intercept
Error in Slope	Error in Intercept

4. Use your spreadsheet to get the best fit line for your data:

$$T = m P + T_o .$$

Plot this line on the same graph as the data. You could use the Excel TREND function to generate this line. Be sure to display the line function on your graph. The slope and intercept of the TREND line should agree with those from the preceding step, step 3.

5. Use the spreadsheet to compute $\langle P/(T-T_o)\rangle$, the average value of $P/(T-T_o)$.

6. Now use the spreadsheet to compute $\Delta T = m\Delta P$ for each of your data points. This is the error in the temperature predicted by theory due to errors in P, but neglecting errors in T and T_o.

7. Compute χ^2 for the theory that the measured T are equal to $T = mP + T_o$ and use the errors in the predicted T found in step 6.

8. Compute the reduced value of χ^2 and record in your spreadsheet. The number of degrees of freedom is $\nu=1$ since you computed m and T_o (two parameters) from your three data points. Also record $P(\chi^2,\nu)$.

VIII. Final Questions

Write your name, date, section, title of the experiment, and a brief (no more than four sentences!) description of the experiment. Then include a hard copy of your spreadsheet Data Table Page and brief answers to the Final Questions. Be sure to include your full page plots of P versus V^{-1} and T versus P. Make sure that the axes are labeled properly and have units.

1. Plot a theoretical curve for P versus V^{-1}. Do this on the same plot as your data from part A. Now Fit the data for P versus V^{-1} and plot it on the same graph. Be sure to label which curve is theory and which is data.

2. How does your value for T_o compare with the expected value of -273.15 ^oC.

3. (a) From Part A of the experiment, does your data agree with Boyle's Law? Explain using χ^2, χ^2/ν, or $P(\chi^2,\nu)$. .
   (b) From Part B of the experiment, does your data agree with Charles' Law? Explain using χ^2, χ^2/ν, or $P(\chi^2,\nu)$.

Spreadsheet Data Tables for Experiment VII: The Ideal Gas Law

Name: **Lab partners:** **Lab Section:**

Date:

Table 1. Experimental parameters for Boyle's Law.

V_{HG} (cm^3)	ΔV_{HG} (cm^3)	ΔP (Torr)	ΔV_c (cm^3)	
P ( )	V_c ( )	$V=V_c+V_{HG}$ ( )	PV ( )	Δ(PV) ( )
<PV> ( )				
χ^2				
ν				
χ^2/ν				
$P(\chi^2/\nu)$				

Table 2. Experimental parameters for Charles' Law.

T (°C)	P (Torr)	$T=mP+T_o$ (°C)	$\Delta T=m\Delta P$ (C°)
ΔT (C°)		ΔP (Torr)	
T_o (°C)		m = slope (C°/Torr)	
ν	1	χ^2	
χ^2/ν		$P(\chi^2/\nu)$	