Experiment

IV The Vibrating String

I. Purpose:

To find the velocity of waves on a string by measuring the wavelength and frequency of standing waves.

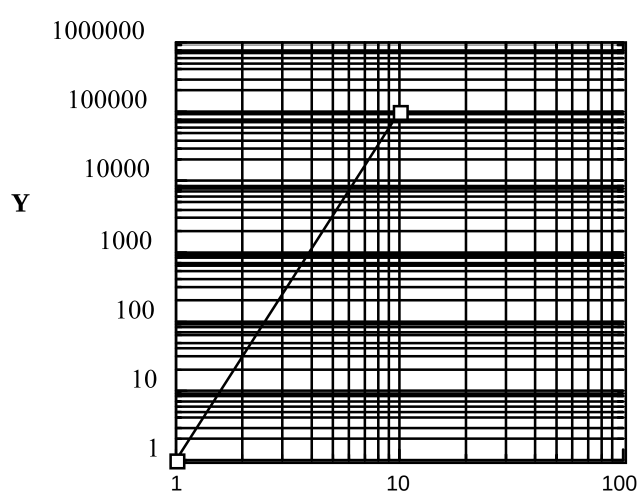
II. References:

Serway and Jewett, 6th Ed., Vol. 1, Chap. 16 and Chap 18, sections 1-4.

III. Prelab Questions (To be turned in at the start of class)

1. For a string with linear mass density of 1 gm/m, find the frequency of the fundamental for a 1.5 m long string under a tension of 0.5 N. Sketch the fundamental.

2. Plot the function $y=2x^3$ on a copy or tracing of the log-log paper below. Ignore the straight line already on the plot for this question.



3. Find the function which is plotted as a line on the graph above.

4. Consider the following pairs of x and y data. Use the Excel spreadsheet and functions to perform a least squares fit on this data, and find the slope, intercept, and uncertainty in the slope and intercept.

x	0	1	2	3	4	5
y	0.1	2.7	4.7	6.9	8.5	11.1

(To do this with Excel, use the Statistical Function LINEST from the function list in the Insert menu. This requires that you select the y values, the x values, and two flags set to non-zero values (say to 1). The output of the fit is directed to a 2x2 array by dragging over a 2x2 set of cells to the right and down from where the function is inserted. Place the cursor on the function display line and do a CTRL-SHIFT-ENTER key stroke. The numbers in the 2x2 array are:

Slope	Intercept
Error in Slope	Error in Intercept

IV. Equipment

Variable speed motor	string	pulley-clamp assembly
- with eccentric shaft	2-meter stick	weight set
- with motor controller	precision balance	C-clamp, scale and weights

Be sure that the remote/normal switch on the back of the motor power supply is set to remote, and that the oz.in./rpm switch on the front is set to rpm.

V. Introduction

A string under tension provides a medium which we can use to study the behavior of wave motion. For small amplitude waves only two characteristics determine the velocity of wave propagation (wave speed), v. They are the tension, T, and the mass/unit length, μ, of the string. In the textbook you can find the derivation of the wave speed and the wave equation. You should review this material. In this lab, we will measure the wave speed on the string by studying standing waves on a string while we vary the tension and frequency, f, of the driving force. We will attempt to show experimentally that:

$$v = \sqrt{\frac{T}{\mu}} \ .$$

When a harmonic or sinusoidal wave travels in the +x direction on a string, we can write the equation as:

$$y_1 = A\sin\left(\frac{2\pi}{\lambda}(x - vt)\right) = A\sin(kx - \omega t) \ ,$$

where A is the wave amplitude, $k=2\pi/\lambda$ is the wave number, and $\omega=2\pi f$ is the angular frequency.

If a second wave is traveling along the string at the same time, the string will respond by forming the sum of the two waves at each point. This is the principal of superposition and it will hold, provided the amplitudes of the waves are not too large. If the second wave has the same amplitude and frequency as the initial wave, but is traveling in the +x direction it can be written as:

$$y_2 = A\sin(kx + \omega t) \ .$$

The result of superimposing the two waves is:

$$y_1 = A\sin(kx - \omega t)$$
$$+$$
$$y_2 = A\sin(kx + \omega t)$$

$$y_{tot} = 2A\sin(kx)\cos(\omega t) \ .$$

The result is just a standing wave. Notice that each point on the string executes a simple oscillation cos(ωt) with an amplitude which depends on position. The amplitude of oscillation at each point is 2Asin(kx). This shows us that at the points where kx is an odd multiple of π/2 the string has amplitude 2A. These points are called "anti-nodes". At any point where kx is a multiple of π the string is fixed at y=0. These points are called "nodes". This can be summarized as follows:

Nodes (y=0 at all times):

$$kx = \frac{2\pi}{\lambda} = n\pi \ , \qquad n = 0,1,2,3\ldots$$

Antinodes ($y=y_{max}=2A$):

$$kx = \frac{2\pi}{\lambda} = 2(n+1)\pi \ , \qquad n = 0,1,2,3\ldots$$

This shows that the distance between nodes (or antinodes) is λ/2. Therefore we can measure the wavelength by measuring the distance between nodes and multiplying by 2.

The Apparatus

In this experiment you will study the behavior of standing waves on a vibrating string. The wave speed depends on the tension and the mass density of the string. You will be able to vary the tension by adding weights to one end of the string (see Fig. V.1). The string is driven by means of a motor which has an eccentric shaft that pushes up and down on one end of the string. The frequency of the wave is controlled by varying the speed of the motor. Use the remote on the box for fine adjustment of the motor speed. The propagation speed of the waves can be found by measuring the frequency and wavelength of the standing waves.

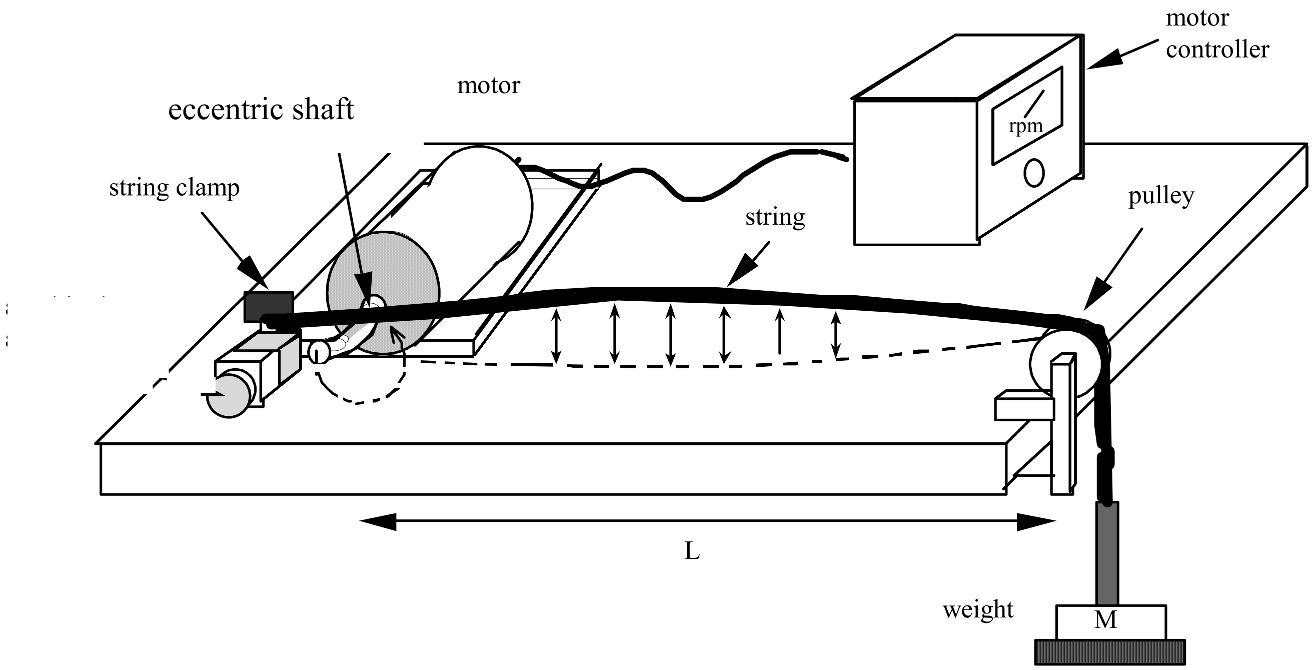


Figure V.1. Schematic of the experiment. The weight applies tension Mg to the string and the eccentric shaft on the motor pushes against the string, causing it to vibrate as the motor turns.

VI. Experiment

A. Set up your spreadsheet like Tables 1, 2 and 3 on the data table page at the end of the lab (don't forget the units).

B. Determine the linear density of the string by weighing several meters of string on the precision electronic balance. This should be a class project. From the mass m and the length z of the string, find the mass density $\mu = m/z$. Record your value for μ in your spreadsheet. (If the balance is not level or does not read zero when the pan is empty, check with the instructor. **Treat the balance gently. This is a precision device**.)

C. Determine the uncertainty in the length z and mass m. Record your values in Table 1.

D. Place a 50 gram weight on the holder. Record the *total mass of the weight and holder* in the first row of spreadsheet Table 2.

E. Make sure that the motor is firmly held in place and cannot slide around if more weight is attached.

F. Measure L, the length of the string between the eccentric shaft and the top of the pulley. Record your value in Table 1.

G. Now set the string into vibration, starting at frequency f=0 and increasing f until you obtain a maximum in the amplitude. The fundamental mode has the shape sketched in

Fig. V.1; there is a large displacement in the middle of the string and the only nodes are at each end. Be sure that your mode has the right shape. It can be a bit tricky to find the fundamental the first time. The problem is that the system has a fairly large "Q", so that it takes a few seconds for it to react and settle down to a steady vibration. The wave amplitude at resonance should be about 1 centimeter. The lowest frequency at which you find a resonance is the fundamental. The motor control reads in units of RPM or revolutions per minute. To get the frequency in Hertz divide the meter reading by 60. Record your value for the fundamental frequency in your spreadsheet Table 2. **Show your results to the TA.**

H. Estimate Δf, the uncertainty in your measured frequency f, and record in your spreadsheet Table 2.

I. Measure x, the distance between the nodes (use the length of the string and count the nodes to evaluate x). For the fundamental, this should be close to your value for L. Record in your spreadsheet. (Reminder: the distance between adjacent nodes is *half* of the wavelength).

J. Estimate Δx, the uncertainty in the internodal spacing. Record in your spreadsheet Table 2.

K. Now increase the frequency and find the next few harmonics. Providing the tension is not too large, you should be able to see at least three or four of them. It may be helpful to recognize that the higher harmonics should appear at integer multiples of the fundamental frequency. Also, you may find that the odd harmonics will start out vibrating all right, but then change to vibrating at the fundamental or some other lower frequency mode. This can happen because a real vibrating string does not exactly obey the ideal equations you were taught in class. You can stop the unwanted modes of vibration by barely touching the string at the place nearest to the motor where you expect no motion (i.e. at a node) for the mode you are interested in. This effect does not usually happen for the even harmonics. You should also look at the string and see that it looks sinusoidal. Record your values for the frequency and inter-nodal spacing for the next three harmonics in your spreadsheet Table 2.

L. Repeat steps H through L for tensions corresponding to total masses of approximately 100, 200, 300 and 400 grams. Record your values for the frequency and inter-nodal spacing of the resonances in the appropriate rows of your spreadsheet.

VII. Analysis

(1) For each tension T and frequency f, use your spreadsheet to determine the wave velocity using the relationship:

$$v = f\lambda \ .$$

Record your values in meters/seconds and **show your results to the TA**.

(2) Determine the uncertainty in the velocity v for each measurement. As in the earlier labs, this requires you to propagate errors giving

$$\Delta v = \sqrt{\left(\frac{\delta v}{\partial \lambda}\right)^2 [\Delta\lambda]^2 + \left(\frac{\delta v}{\partial f}\right)^2 [\Delta f]^2} \quad . \qquad [IV.1]$$

The derivatives are just $\partial v/\partial\lambda = f$, and $\partial v/\partial f = \lambda$. Making these substitutions and rearranging, the uncertainty in one measurement of v can be written as:

$$\Delta v = v\sqrt{\left(\frac{\Delta\lambda}{\lambda}\right)^2 + \left(\frac{\Delta f}{f}\right)^2} \quad . \qquad [IV.2]$$

Use your spreadsheet to find Δv for each measurement.

(3) You should have found in step 2 of the analysis that Δv depends on the frequency. Thus, some of your measurements of v are more accurate than others. To get the best estimate for the wave velocity $\langle v \rangle$, you will need to take this into account by using the weighted average. **For each tension,** use your spreadsheet to compute the weighted mean

$$\langle v \rangle = \frac{\dfrac{v_1}{\Delta v_1^2} + \dfrac{v_2}{\Delta v_2^2} + \dfrac{v_3}{\Delta v_3^2} + \cdots}{\dfrac{1}{\Delta v_1^2} + \dfrac{1}{\Delta v_2^2} + \dfrac{1}{\Delta v_3^2} + \cdots} \quad . \qquad [IV.3]$$

Where v_1 is the velocity of the fundamental, Δv_1 is the uncertainty in v_1, v_2 is the velocity of the second harmonic, etc. If you found n different resonant frequencies, then the numerator and denominator should each have n terms. Since the velocity should only be the same if the tension is the same, do not average velocities for measurements made with different tensions. For each tension, use your spreadsheet to find $\langle v \rangle$.

(4) For each tension, use your spreadsheet to calculate $\sigma_{\langle v \rangle}$ the standard deviation in your measurements of $\langle v \rangle$. Recall that

$$\sigma_v = \sqrt{\frac{\sum_{i=1}^{n} [v_i - \langle v \rangle]^2}{n-1}} \quad , \qquad [IV.4]$$

where n is the number of different resonant frequencies you found at a given tension. Note that you can't use the STDEV function in Excel for this because you must use the weighted mean for $\langle v \rangle$ and not the simple mean that STDEV uses. You will need an extra column of $[v_i - \langle v \rangle]$ and a column sum of it for each tension.

(5) For each tension, determine the experimental uncertainty in your value <v> given by

$$\Delta < v >= \frac{1}{\sqrt{\frac{1}{\Delta v_1^2}+\frac{1}{\Delta v_2^2}+\frac{1}{\Delta v_3^2}+\cdots}} \quad . \qquad \text{[IV.5]}$$

(6) Now use the spreadsheet to make a log-log plot of the average velocities <v> versus the tension T. Include error bars on the points. [Log-Log graphs can be made by first plotting v vs. T and then changing the x and y axis to log axes. Include error bars for all v points by right clicking on a plotted data point and choosing "Format Data Series." You need to choose both the plus and minus values of the v uncertainty.]

<u>Note:</u> If you haven't made log-log graphs before, this experiment provides a good introduction. The idea is that theoretically we expect that v depends on the tension as:

$$v = \sqrt{\frac{T}{\mu}} \quad . \qquad \text{[IV.6]}$$

If we were to make a plot of V versus T on regular graph paper, it wouldn't be a straight line, but rather a square root curve. However, on a log-log plot, you can plot v versus T and get a straight line, even though v is proportional to the square root of T! This clearly requires that the graph paper itself is weird.

To see what is going on, take the log of both sides of Eq. [VI.6]You will get:

$$\log(v) = \log\left(\sqrt{T}\right) - \log\left(\sqrt{\mu}\right) = \frac{1}{2}\log(T) - \frac{1}{2}\log(\mu) \ .$$

Notice that if you plot log(v) versus log(T) you get a straight line with a slope of 1/2. Now you could just use columns in your Excel spreadsheet to compute log(v) and log (T) and plot them up on regular graph paper to get a straight line. But this kind of thing happens so often in science and engineering that the graph paper manufacturers produce graph paper that takes the logarithm automatically by laying out the vertical and horizontal scales logarithmically instead of linearly, see Figure VI.2. Located at the left end of the x axis is the number 1 (since log1=0), at a distance of 0.301 units is the number 2 (since log2=0.301) and so on up to one unit which is labeled 10 (since log10=1). So you plot something on this paper just as you would on linear paper and it automatically comes out that you are plotting the logarithm.

As you might expect, there is a complication sometimes. This happens when you want to compute the "slope" of the "line". For our case, from Eq. [VI.6]the slope on the log-log plot is 1/2. This 1/2 comes from the fact that the velocity goes like $T^{1/2}$, so the slope on the log-log plot is really the power to which T is raised. You compute the slope in the usual way except that you must take logarithms. For example, the slope of the line in Figure [IV.2] is: $\text{Slope} = \frac{\log(3.17) - \log(1)}{\log(10) - \log(1)} = 0.5$.

(7) Now use the tension and mass density to compute the theoretically expected wave speed

$$v_i^{theory} = \sqrt{\frac{T_i}{\mu}} \qquad \text{with} \qquad T_i = Mg_i \ , \qquad [IV.7]$$

where T_i is the value of tension in the i-th row of Table 3. For each tension, use your spreadsheet to calculate v_{theory}.

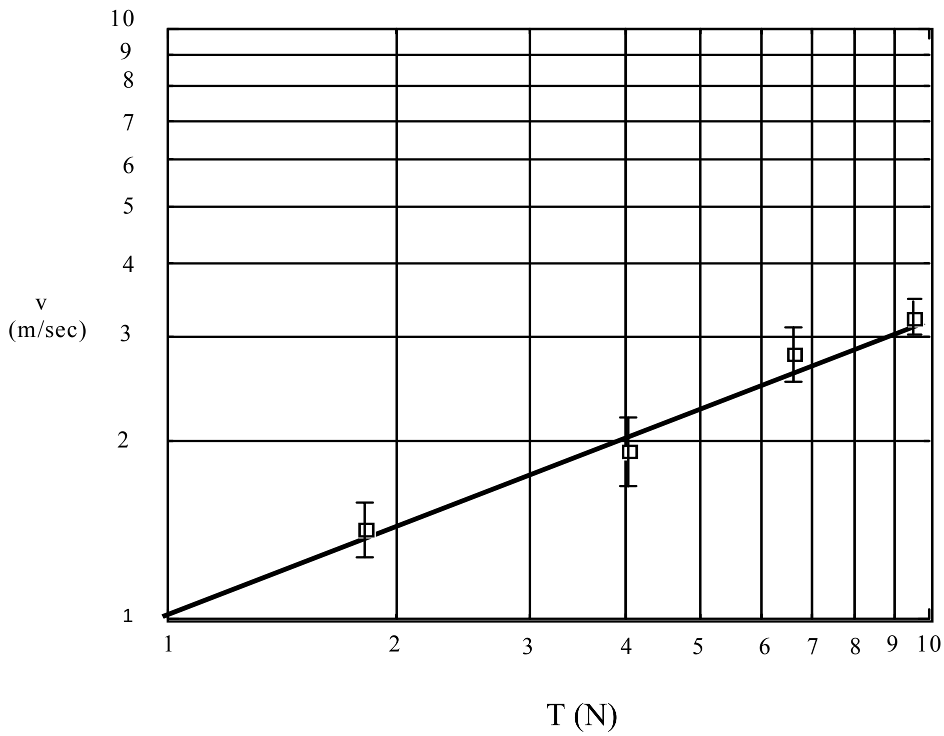


Figure VI.2. Data plot of v vs. T. Solid line shows a log-log plot of v proportional to $T^{1/2}$.

(8) Plot the theoretical values for the velocity on your log-log graph from (7) so that you can compare the theory to your <v> results. Draw a line through the v_{theory} data points that reasonably represents the data. Note that a straight line should represent the data on log-log paper! Remembering the exercise of the Prelab Question 2., estimate the power law function representing the straight line and write it down.

You can be more analytical by plotting table columns of log(v) and log(T) and using the LINEST function to find the slope and its uncertainty. The slope of the log-log plot is the exponent of T in the power law.

Excel also can fit a power law to the log axis first plot here using a power law "Trendline." You right click on a graphed point of your data, choose "Trendline," and choose Power Law as the function. You can make the fit function visible on the graph under the options tab but you do not get error estimates for the fit parameters.

VIII. Final Questions

1. **Keep your lab report brief.** Write your name, date, section, title of the experiment, and a brief (four sentences) description of the experiment. Then include a hard copy of your spreadsheet Data Table Page and brief answers to the Final Questions.

2. Include a full page copy of your plot of v versus T and the theoretical curve(s). Be sure to label the axes, include units.

3. For each tension, does your value of $\Delta\langle v\rangle$ approximately equal $\sigma_v/(n-1)^{1/2}$. In a couple of sentences, explain why it should.

4. For a given tension, does the wave speed depend on the frequency?

5. Do your experimental results confirm the theory? Compare v from $v=\lambda f$ with v from $v = \sqrt{\frac{T}{m}}$. Be sure to include your error estimates from the uncertainties in measured values used in both equations in this consideration.

6. Give an example of standing waves in musical instruments.

You may want to make a disk copy of your spreadsheet, but it is not necessary, since you will be turning in your report at the end of class.

Spreadsheet Data Tables for Experiment IV: The Vibrating String

Name: **Lab partners:** **Lab Section:**
Date:

Table 1. Experimental parameters.

m (kg)	L (m)	z (m)	Δm (kg)	Δz (m)	μ=m/z (kg/m)	Δμ (kg/m)

Table 2. Resonant frequencies and wavelengths.

M (kg)	ΔM (kg)	f (Hertz)	Δf (Hertz)	λ =2L/(n-1) (m)	Δλ (m)	v= λf (m/s)	Δv (m/s)	Δv/v	$v-v_{theory}$ (m/s)

Table 3. Results from Analysis.

T=Mg (N)	<v> (m/s)	Δ<v> (m/s)	$\sigma_{<v>}$ (m/s)	v_{theory} (m/s)	Δ v_{theory} (m/s)	<v>- v_{theory} (m/s)