

Experiment

III

Forced Harmonic Motion

I. Purpose:

To measure the resonant frequency, quality factor, and phase shift of a driven mass-and-spring oscillator.

II. References: (common textbooks)

Serway and Jewett, 6th Ed., Vol. 1, Chapter 15, Sections 6 and 7.

III. Prelab Questions (to be turned in at the start of lab)

1. Set up your spreadsheet with a format similar to Table 1 and 2 on the Spreadsheet Data Table page at the end of this lab. This gives you a starting point. In the course of your work with the data you may add columns and rows as needed. Bring a copy of your initial spreadsheet on a floppy disk to the lab. This will save you time during the lab period.
2. Explain the physical meaning of large and small Q .
3. When a 0.1 kg mass is hung from a spring, it stretches the spring by 0.03 m. Find the spring constant and resonant frequency of the system.
4. In two or three sentences, explain why the phase difference θ should be small when ω is small.
5. Using Equations III.10 and III.11, (a) find X_0 when $\omega = \omega_0$, and (b) find θ when $\omega = \omega_0$.

IV. Review of Forced Harmonic Motion

In this lab you will study the phenomenon called resonance, which occurs when you drive a system at its natural oscillation frequency. Resonance just means that the system exhibits a maximum response to a driving force when it is driven at a particular frequency. This section reviews such forced harmonic motion. If you are not familiar with the theory of forced harmonic motion, or can't remember what Q or the phase is, then read this section. Otherwise, skip to the next section to begin the experiment.

Forced harmonic motion serves as a model for many important physical systems. You will repeatedly encounter such systems as you study physics and other sciences. Examples

range from atomic interactions, to electrical circuits, to the oscillations of buildings during earthquakes, and even to models of population growth. All of these phenomena follow equations which are very similar to one another, and this is why it is useful to study the simple mechanical oscillator in such detail.

In this experiment, you will study the motion of a mass m hanging from a spring whose support moves up and down. Let us suppose first that the support is fixed. If the displacement of the mass from its equilibrium position is x , then the force, F_s , on the mass due to the spring is given by

$$F_s = -kx \quad , \quad \text{III.1}$$

where k is the spring constant. Using Newton's third law, we obtain

$$F_s = m \frac{d^2x}{dt^2} = -kx \quad . \quad \text{III.2}$$

Rearranging the above, we find

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x \quad . \quad \text{III.3}$$

If at time $t=0$ the mass is at $x=0$ and its velocity is $v=v_0$, then the solution to this differential equation is

$$x = \frac{v_0}{\omega_0} \text{Sin}(\omega_0 t) \quad , \quad \text{III.4}$$

where $\omega_0=(k/m)^{1/2}$. ω_0 is called the "natural oscillation frequency" or "resonance frequency" and the period is then

$$T = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{m}{k}} \quad . \quad \text{III.5}$$

Notice that if the mass increases or the spring gets weaker, the motion becomes slower.

Now let us consider what happens when the support holding the spring moves up and down at some frequency ω which is not the natural oscillation frequency ω_0 . This movement of the spring will act as the "driving force" in the experiment. Let the displacement of the spring support be $A=A_0\cos(\omega t)$, where A_0 is the amplitude of the motion of the support. The force on the mass due to the spring then becomes:

$$F_s = -k(x-A) \quad . \quad \text{III.6}$$

Note that if $x=A$, then the spring will not be stretched and $F_s=0$.

In order to make a realistic model of the system, we must consider the effects of friction. For our case we will assume that the force of friction, F_f , is proportional to the velocity of the mass:

$$F_f = -b \frac{dx}{dt} \quad , \quad \text{III.7}$$

where b is a positive constant and the minus sign means that friction opposes the motion. Combining all the forces in the problem we find:

$$m \frac{d^2x}{dt^2} = -k(x - A) - b \frac{dx}{dt} . \quad \text{III.8}$$

The solution to this equation, after friction has damped out the effects of the initial conditions, is:

$$x(t) = X_0 \cos(\omega t - \theta) , \quad \text{III.9}$$

where X_0 is the amplitude of the motion and θ is the difference in phase between the motion of the support and the mass. If we define the quality factor Q of the system as $Q = m\omega_0/b$, then

$$X_0 = \frac{A_0}{\sqrt{Q^2 \omega_0^2 + \left[\frac{\omega^2}{\omega_0^2} - 1 \right]^2}} \quad \text{III.10}$$

and

$$\tan(\theta) = \frac{1}{Q} \left[\frac{\omega \omega_0}{\omega_0^2 - \omega^2} \right] , \quad \text{III.11}$$

where $0 < \theta < \pi$. Note that Equation III.9 says that the mass oscillates at the same frequency ω as the support but that there is an extra phase shift, θ , in the motion. The way to think about θ is as follows. If the mass is at its lowest point when the support is at its lowest point, then the motion is "in phase" and $\theta=0$. If the mass is at its lowest point and the support is at the center of its range of motion and moving upward, then $\theta=90^\circ$. If the mass is at its lowest point when the support is at its highest point, then $\theta=\pi$ (180°).

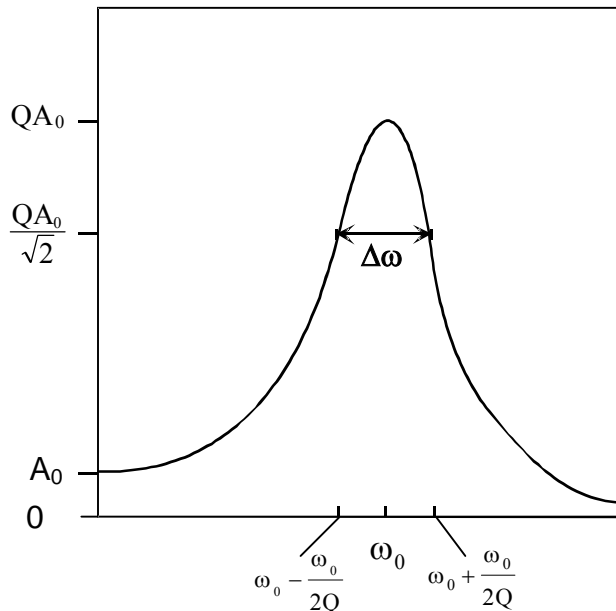


Figure III.1 Amplitude of motion, X_0 , of the mass as a function of ω . At resonance, the mass moves Q times more than the support. Here, $\Delta\omega$

is the width of the resonance, not the uncertainty estimate for ω (or ω_0)

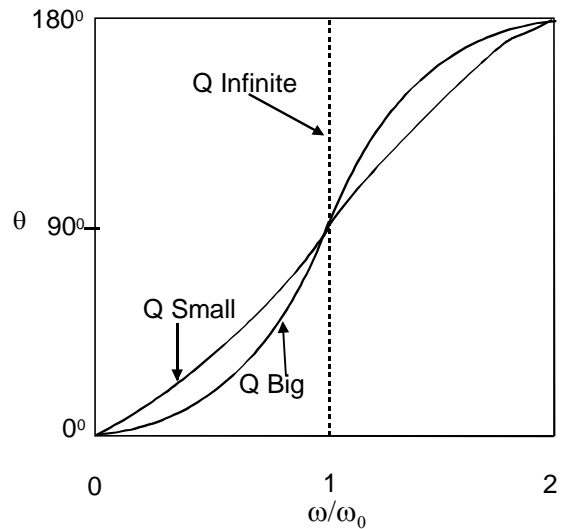


Figure III.2 Phase, θ , as a function of ω for small, large, and infinite Q . A large Q means that $Q \gg 1$.

The easiest way to understand how X_0 and θ depend on ω , is to plot out Equations III.10 and III.11. From Figure III.1, notice that the amplitude X_0 has a maximum near $\omega = \omega_0$. This amplitude maximum is called a resonance. If a system with a large Q is driven at $\omega = \omega_0$, the amplitude of the oscillation of the system, X_0 , will be much larger than the amplitude of the motion of the support, A_0 . In fact at resonance, $X_0/A_0 = Q$. Also notice that the width of the resonance, the frequency distance between the two points where X^2 is $1/2$ of X_0^2 , is $\Delta\omega = \omega_0/Q$ or $Q = \omega_0/\Delta\omega$.

From Figure III.2, we see that the phase θ approaches zero for $\omega \ll \omega_0$, and is equal to 180° for $\omega \gg \omega_0$. At resonance, the phase is 90° .

Remember: X_0 is the amplitude of motion of the mass, while A_0 is the amplitude of driving force!

V. Equipment

Forced Harmonic Motion Apparatus
Balance

VI. Experiment

Part A. Determination of the Spring Constant.

1. Set up your spreadsheets with a format similar to Tables 1 and 2 on the Spreadsheet Data Table page at the end of this write up, as you did in the Prelab.

2. For a picture of the apparatus, refer to the appendix for this experiment.
3. Measure M_{bar} the total mass of the damping rod and mass bar. Also find the mass of the spring, M_{spring} . Record your values in your spreadsheet Table 1, including your estimates of the uncertainty in each mass measurement.
4. To accurately compare theory and experiment, you must use for the mass m ,

$$m = M_{bar} + M_{spring}/3 , \quad \text{III.12}$$

for all of the Equations III.1 through III.11. The reason for this is that the above equations assume a massless spring, and a real spring is not massless. The factor of 1/3 is due to the fact that the parts of the spring close to the support don't move as much as the parts close to the mass bar. Record your value in spreadsheet Table 1.

5. Estimate the uncertainty in your value of m and record this value in your Table 1.
6. Determine the spring constant, k . Set up and align the apparatus (see the Appendix to this experiment). Hang a measured mass, m_w , of approximately 50 gm from the mass bar and measure x , the distance the mass bar moves. Record your values of x and m_w in spreadsheet Table 1, including your estimates of the uncertainty in each measurement.
7. Use $F_s = m_w g = -kx$ to determine k . Record in spreadsheet Table 1 (remember that k is a positive constant!).
8. Estimate δx , the uncertainty in your measurement of x , record the value in your spreadsheet Table 1.

Part B. Measurement of the natural oscillation frequency

1. Remove the weights from the mass bar and screw the damping magnets away from the mass bar.
2. Verify that the apparatus is aligned (see the appendix in the lab manual).
3. Leave the **DRIVE** switch off and set the function switch to **PERIOD**.
4. Pull the mass bar down a few cm below its equilibrium position and release it. The digital display will indicate the period, T , of the oscillations in seconds. Record in spreadsheet Table 1.
5. Estimate ΔT , the uncertainty in T and record in spreadsheet Table 1.
6. Calculate $\omega_0 = 2\pi/T$ and record in spreadsheet Table 1. Don't forget to put in the units for ω_0 (radians per second).

Part C. Forced Harmonic Motion.

1. Adjust the drive amplitude, A_0 , so that the support moves up and down by 2 mm. You will need to look behind the apparatus at the drive wheel to set A_0 . Record your value for A_0 in spreadsheet Table 2. Note that the value given by the Analyzer is the peak to peak distance, which corresponds to twice the amplitude A_0
2. Move the damping magnets until they are close to the mass bar to increase the damping of the motion.
3. By looking at the motion of the support and the mass, verify that the phase is 0° for low frequencies and 180° for high frequencies. (This means that at low frequencies the mass moves with the string and spring in phase i.e. the spring moves but doesn't change length. At high frequencies you should observe that the mass moves in a direction opposite to the string and that the spring changes length, but its center moves very little.)
4. Set the **DRIVE** switch on and adjust the **FREQUENCY** knob until you obtain a driving frequency, f , of 0.50 Hz.
5. Adjust the "Cord Length Fine Adjust" until a phase angle near zero is obtained. This is your final alignment of the apparatus and is justified by your answer to Pre-Lab question 4. (See Fig. III.3 in the Appendix to this experiment).
6. Adjust the driving frequency in increments of 0.2 Hz until the frequency f is well beyond the natural frequency of the system. At each setting measure the amplitude of the motion and the phase. Record in spreadsheet Table 2.
7. Explore the region near the resonance frequency (from 0.5Hz below resonance to 0.5HZ above resonance) by measuring the amplitude and phase at units of 0.1Hz. Record in spreadsheet Table 2.
8. Determine $\omega_0 = 2\pi f_0$ by locating the frequency f_0 at which the amplitude is a maximum. Record in spreadsheet Table 2.
9. By adjusting the **FREQUENCY** knob and checking the amplitude, estimate $\Delta\omega_0$, the uncertainty in your measurement of ω_0 . Record in spreadsheet Table 2.
10. **Show the TA your results.**

VII. Analysis

- (1) In part A of the experiment, you found k from $k = -m_w g/x$. This is the value of k which you recorded in spreadsheet Table 1. Now determine Δk , the uncertainty in k . This requires doing a simple error propagation:

$$\Delta k = \sqrt{\left(\frac{\partial k}{\partial m_w}\right)^2 [\Delta m_w]^2 + \left(\frac{\partial k}{\partial x}\right)^2 [\Delta x]^2}, \quad \text{III.13}$$

where Δx is the estimated uncertainty in the distance x the spring stretched. Neglecting errors in m_w and using $k = -m_w g/x$, one finds $\partial k/\partial x = m_w g/x^2 = k/x$ and thus:

$$\Delta k \approx k \left| \frac{\Delta x}{x} \right|. \quad \text{III.14}$$

Use your spreadsheet to compute Δk and record in spreadsheet Table 1.

- (2) Using your values of k and m from Table 1, calculate the theoretical value of $\omega_0 = (k/m)^{1/2}$.
- (3) Calculate $\Delta\omega_0$, the uncertainty in your theoretical estimate of ω_0 . This requires a simple error propagation:

$$\Delta\omega_0 = \sqrt{\left(\frac{\partial\omega_0}{\partial k}\right)^2 [\Delta k]^2 + \left(\frac{\partial\omega_0}{\partial m}\right)^2 [\Delta m]^2} = \frac{\omega_0}{2} \sqrt{\left(\frac{\Delta k}{k}\right)^2 + \left(\frac{\Delta m}{m}\right)^2}. \quad \text{III.15}$$

Work through the derivatives and calculate $\Delta\omega_0$ using your values of Δm and Δk from Table 1. Use the spreadsheet to calculate and record your value in the appropriate cell in Table 1.

- (4) In part B, you also found ω_0 from a measurement of the natural oscillation period T . From your uncertainty in T , find the uncertainty in $\omega_0 = 2\pi/T$. Propagating the error, we have

$$\Delta\omega_0 = \sqrt{\left(\frac{\partial\omega_0}{\partial T}\right)^2 [\Delta T]^2} = \left| \frac{\omega_0}{T} \Delta T \right|. \quad \text{III.16}$$

Record in the appropriate cell in Table 1.

- (5) From Table 2, find the frequency where the amplitude X_0 is a maximum. You can find Q by dividing this maximum amplitude at resonance by A_0 . Record your value for Q in spreadsheet Table 2.
- (6) Using your values in Table 2, use the spreadsheet to plot the amplitude X_0 versus $\omega = 2\pi f$. Be sure to label the axis properly and don't forget units.
- (7) From your plot, determine ω_0 . Label this on the plot.
- (8) Using spreadsheet Table 2, plot the phase θ versus ω .

Be sure to check whether your results agree approximately with theory.

VIII. Final Questions

Keep your lab report brief. Write your name, date, section, title of the experiment, and a brief (no more than four sentences!) description of the experiment. Then include a hard copy of your spreadsheet Data Table Page and brief answers to the Final Questions.

1. Be sure to include your plots of X_0 versus ω , and θ versus ω in your lab report. Also, this is a good time to check that the axes on your plots and all of your entries in Tables 1 and 2 have units!
2. You found three different values for ω_0 . Two are listed in Table 1 (a theoretical value found by measuring the mass and spring constant and an experimental value found by measuring the natural oscillation frequency), and one is listed in Table 2 (an experimental value found by driving the system). (a) To within your experimental uncertainties, do the two experimental values agree with each other? Explain, using the uncertainties in each quantity. (b) Does your theoretical value of ω_0 agree with the experimental values? Explain. (c) Which experimental technique is more accurate?
3. Using your values for Q and A_0 and your value for ω_0 from Part VII-8, plot a theoretical curve for X_0 versus ω on the same graphs as your experimental curve.
4. Comment on the above agreement (or disagreement) between the theoretical curve and your measurements.
5. Give a few examples of resonance effects in common phenomena.

Appendix to Experiment III – The Forced Harmonic Motion Apparatus

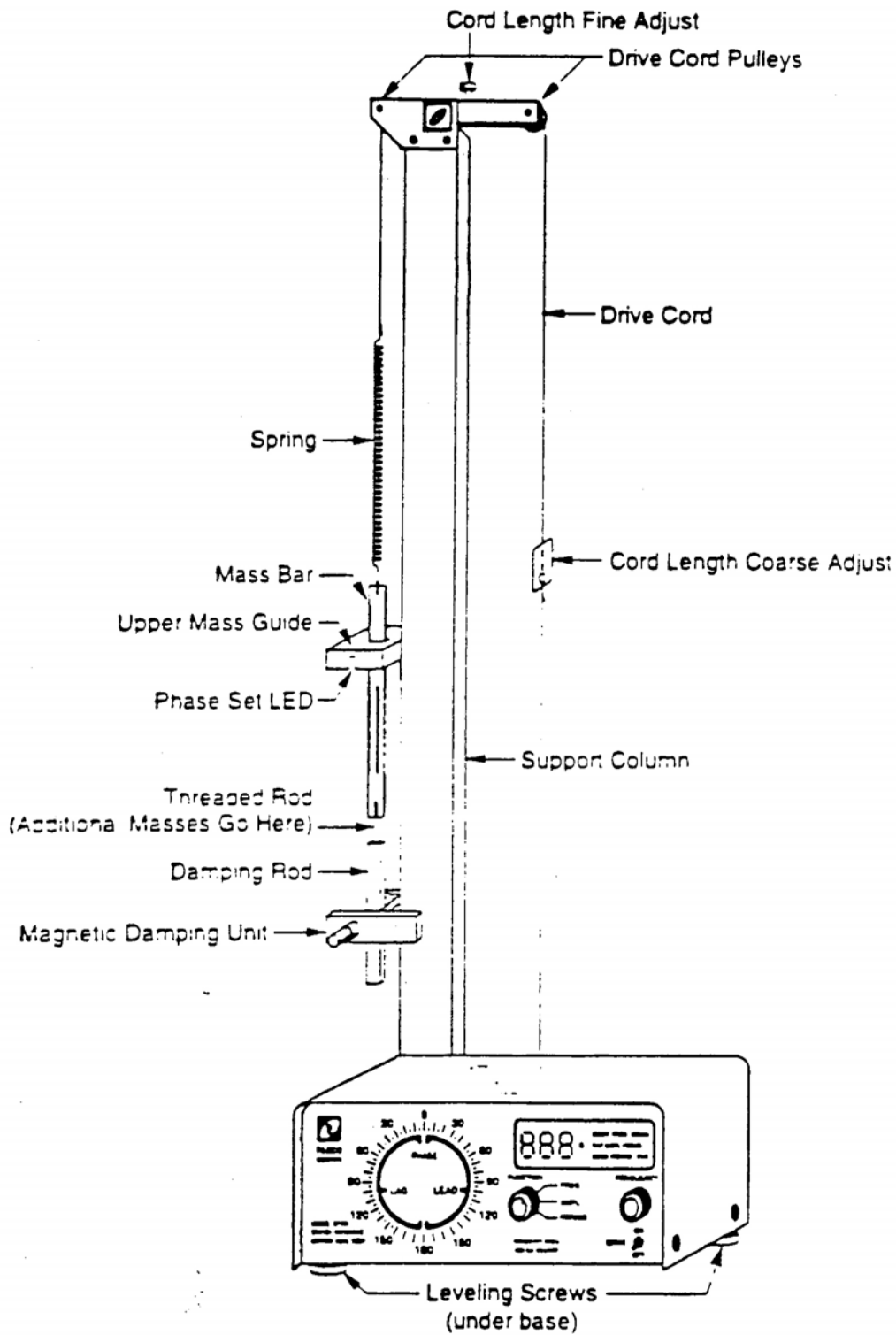


Figure III.3 - The Driven Harmonic Motion Analyzer

Aligning the Driven Harmonic Motion Analyzer (DHMA)

IMPORTANT:

1. We strongly recommend that you take the time to carefully align the Mass Bar using the two-step procedure below. Step 1 is necessary to minimize undesirable friction that can seriously impair experimental results. Step 2 is necessary to ensure that the measurements of phase and amplitude are correct.
2. If the driver amplitude, the mass, or the spring are changed, step 2 of this procedure must be repeated.

STEP 1:

When the unit is properly aligned, the Mass Bar will hang squarely in the center of the hole in the Upper Mass Guide, and will not touch any side (see Fig. III.4a). Improper alignment is shown in Fig. III.4b) and Fig. III.4c). To correct the misalignment shown in Fig. III.4b, adjust the leveling screws at the bottom of the unit (see Fig. III.3). To adjust the misalignment shown in Fig. III.4c turn the spade lug at the top of Mass Bar.

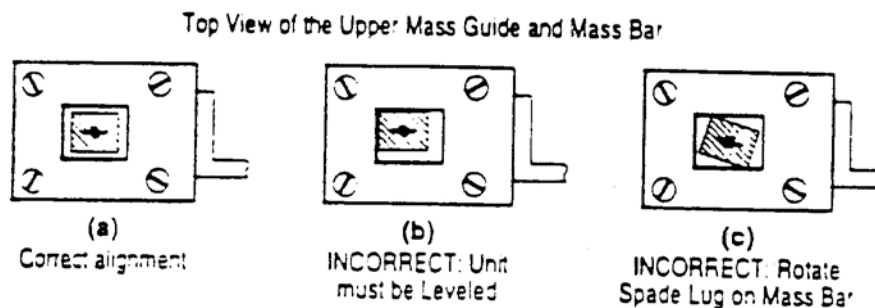


Figure III.4 – Mass Bar Alignment

STEP 2:

For correct measurements of phase and amplitude, the Mass Bar must hang so that, in its equilibrium position (when it is not oscillating), its center line is aligned with the optical sensor in the Upper Mass Guide. To accomplish this:

1. Turn the ON/OFF Switch on the rear panel of the DHMA chassis ON. Notice the Phase Set LED (Light Emitting Diode) on the front of the Upper Mass Guide. Notice that, when the center line of the Mass Bar (the 8.5 cm mark) is above the Upper Mass Guide, the LED is off. When the center line is below the guide, the LED turns on.
2. Move the Mass Bar up and down, so that its center line passes back and forth through the upper mass Guide. While doing this, watch the PHASE scale on the front panel of the chassis. Notice that, as the center line of the Mass Bar passes down through the Upper Mass Guide, a LED on the PHASE scale flashes. Rotate the Drive Wheel until this flashing LED is aligned with the zero-degree mark on the PHASE scale.
3. Now adjust the length of the Drive Cord using the plastic friction clip (labeled Cord Length Coarse Adjust in Fig. III.3). Adjust the cord length until small oscillations of the Mass Bar about its equilibrium position cause the Phase Set LED to turn on and off. At the top of the Support Column is a fine adjustment screw for cord length (see Fig. III.3). Adjust this screw until the amplitude of oscillations needed to make the Phase Set LED turn on and off are as small as possible.

A. Controls

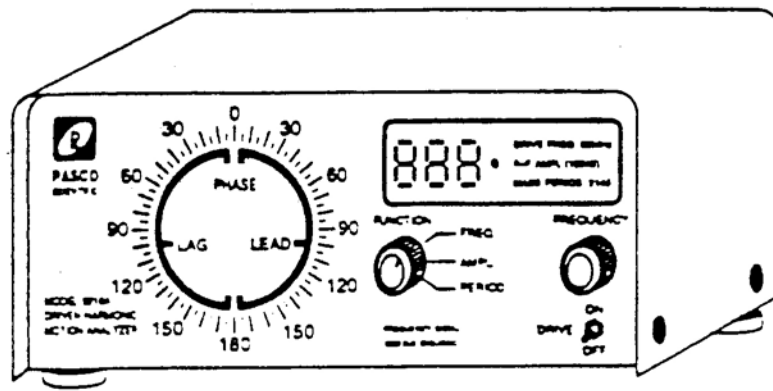


Figure III.5 – DHMA Front Panel

The Front panel of the Driven Harmonic Motion Analyzer is shown in Fig. III.5. The controls function as follows:

ON-OFF Switch: (located on rear panel) Turns power on or off.

DRIVE Switch: Engages the Drive Motor when ON, disengages the Drive Motor when OFF.

FREQUENCY Knob: Sets the frequency of rotation of the Drive Wheel. Turning clockwise increases the frequency.

FUNCTION Switch: Determines which of the following three variables is measured by the digital readout. Notice that a LED to the right of the readout lights to indicate which variable is being displayed and the units of the measurement.

FREQ: Measures the frequency of the Drive Wheel in Hertz. The frequency is determined as an average value over each one-second interval.

AMPL: Measures the peak-to-peak amplitude (total excursion) of the Mass Bar in millimeters. The measurement is made once each cycle, with the cycle beginning as the center line of the Mass Bar passes through the Upper Mass Guide moving downward.

PERIOD: Measures the period (total time for one complete oscillation) of the Mass Bar in seconds. The measurement is made once each cycle, with the cycle beginning as the center line of the Mass Bar passes through the Upper Mass Guide moving downward.

B. System Variables

The versatility of the Driven Harmonic Motion Analyzer is that it allows the experimenter to control and measure a large variety of variables.

IMPORTANT:

1. *Although normal room light should cause no problem, direct sunlight, or even brightly reflected sunlight, can interfere with the performance of the optical detectors that are used to sense the motion of the Mass Bar.*

If the amplitude, period, or phase readings are incorrect, try shielding the Upper Mass Guide from the light. If the problem persists, try reducing the room lights or moving the unit to a darker room.

2. *Whenever the Driver Amplitude, Mass, or Spring Constant are changed, Step 2 of the alignment procedure must be repeated.*