Experiment

II The Pendulum

I. Purpose:

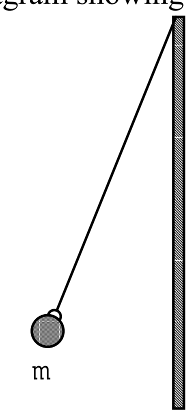
To determine the value of g, the acceleration due to gravity, using a pendulum.

II. References: (CourseTextbooks)

Serway and Jewett, 6^{th} Edition, Vol. 1, Chapter 15 Section 5
Appendix A of the 261 lab manual, "Data Reduction and Error Analysis"

III. Prelab Questions (to be turned in at the start of Lab)

1. Make a free body diagram showing the forces acting on a simple pendulum.



2. Use the free body diagram to show that there is a restoring force in the θ-direction which can be written as

$$F_\theta = -mg\,\mathrm{Sin}(\theta)$$

3. The small angle approximation says that $\sin(\theta) \approx \theta$ (this is only true if θ is measured in radians). In steps of 0.1 radians, find the percentage error this approximation introduces between θ = 0 and 1.5. That is, find $\left(\frac{\mathrm{Sin}(\theta) - \theta}{\theta}\right)$ for θ = 0.01, 0.1, 0.2, ..., 1.4, 1.5.

4. Using g = 9.80 m/sec^2, find the period for a 1 m long pendulum.

5. Suppose you can measure the length of the pendulum in part VI to an accuracy of ±0.5 cm, and you can measure the time of a set of oscillations to an accuracy of ±0.2 sec. How many periods must you measure so that the contribution of the uncertainty in time is smaller than the uncertainty in length, when calculating g?

IV. Equipment

A variable length pendulum
Two-meter stick
Electronic timer
Vernier calipers

V. Introduction

In the prelab we showed that $F_\theta = -mg\sin(\theta)$. We use the approximation that $\sin(\theta) \approx \theta$ to write that $F_\theta = -mg\theta$. The displacement S of the ball in the "θ" direction along the arc is $S=L\theta$. This implies that:

$$F_\theta = m\frac{d^2S}{dt^2} = mL\frac{d^2\theta}{dt^2}$$

where m is the mass of the ball. This can be rewritten in the standard form:

$$\frac{d^2\theta}{d^2t} = -\frac{g}{L}\theta$$

This equation has exactly the same form as Hook's Law for springs ($F = md^2x/dt^2 = -kx$) where in this case θ -> x and g/L -> k/m. The solution to this equation is:

$$\theta = \theta_o \mathrm{Cos}(\omega t)$$

where: $$\omega = \sqrt{\frac{g}{L}}$$

The period T of the oscillations is:

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$$

It is important to note that the period depends only on L and g, and is independent of the mass of the ball. This implies that all pendula of the same length will have the same period if g is the same.

We can use measurements of L and T to find g:

$$g = \frac{4\pi^2 L}{T^2} \qquad \text{[V.1]}$$

It can be shown (as we did in the previous labs) that Δg, the uncertainty in g due to the uncertainty in L and T, is just:

$$\Delta g = g\sqrt{\left(\frac{\Delta L}{L}\right)^2 + \left(\frac{2\Delta T}{T}\right)^2} \qquad \text{[V.2]}$$

VI. Experiment

1. Set up your spreadsheet with the same format as Tables 1, 2 and 3 on the Spreadsheet Data Table Page at the end of this write-up.

2. Using the two-meter stick, measure the length of the pendulum and estimate the error. Carefully consider what points you should measure. (Hint: You should measure the diameter of the pendulum weight separately with the vernier, and combine it with a measurement of the length of the string to an appropriate point. When combining lengths, remember to propagate the error to find the error in the overall length of the pendulum.) Fill in the values in your spreadsheet.

3. Measure the period T as described below and determine the experimental error in this measurement. In order to satisfy the assumption $\sin(\theta) \approx \theta$ and obtain results for g which are precise to within about ±0.1%, you must limit the amplitude of your pendulum to about ±0.08 radian or $\pm 5^{o}$.

4. To reduce both systematic and random errors in the measurement of the period, you should measure the time for ten periods, repeating that measurement ten times. Fill in your values in spreadsheet Table 1**.** For best results, you should choose the string crossing past the vertical support post as your reference point, as this is the most rapid point of the motion and is therefore most precisely judged. **Remember that the 1st time that the string crosses the support and you start the timer, you should start your counting at zero, not at one.**

5. Use your spreadsheet to compute the average time, <t>, and the standard deviation of the times σ_t.

6. Now use your spreadsheet to compute the average period for the pendulum, <T>. The measurement is the time for 10 periods . So, the period is the total time divided by 10.

7. Use your spreadsheet to compute the uncertainty in the average period Δ<T>. To do this properly, you need to remember that the uncertainty in the average of N measurements is a factor of $1/N^{1/2}$ times smaller than the uncertainty in one measurement. Also, you will need to keep the factor of 10 straight between t and T. Taking both these factors into account, we can write:

$$\Delta < T >= \frac{\Delta T}{\sqrt{N}}$$

8. Now use your spreadsheet to compute g and its uncertainty Δg using Equations [V.1] and V.2]. Note that to calculate the uncertainty in g the average period <T> you need to use the uncertainty in the mean period, Δ<T> in Equation V.2, not just the uncertainty in T.

9. Repeat the above measurements for 3 other lengths of the pendulum.

10. Now use your spreadsheet to determine the weighted mean value of g from the measurements at each length along with the propagated error for the average value of g. Compare your results with the plaque on the floor outside Room Z3211 that gives accepted values as determined by the U.S. Coast and Geodetic Survey. Remember that the point of this experiment is to make an accurate measurement of g using simple apparatus; therefore you must be careful in both your measurement and your analysis.

11. Use the spreadsheet to make a plot of the four separate values of g you obtained versus the length of the pendulum. As an alternative, plot L vs. T^2 , find the slope, and convert this slope to a value for g.

VII. Final Questions

1. Keep your lab report brief. Write your name, date, section, title of the experiment, and a brief (no more than four sentences!) description of the experiment. Then include a hard copy of your spreadsheet Data Table Pages, and brief answers to the Final Questions. Staple all of these pages together and turn them in to your TA at the end of class.

2. Discuss your results in a few sentences - Did you find agreement with the accepted values to within your estimated uncertainty?

3. How does your result compare with the average of all the measurements of g in the lab today?

4. What other sources of errors are present in this experiment? Does the size of the bob matter? What about the air resistance? How does the calibration of the timer affect your results?

5. Why should the period be independent of the mass? (Briefly explain the physics, not just that there is no mass in the period formula).

Spreadsheet Data Tables for Experiment II: The Pendulum

Physics 261, University of Maryland, College Park

Name: **Lab partners:** **Lab Section:** **Date:**

Table 1. Data for pendulum with four different lengths. For each length, make 10 measurements of the time for 10 cycles of the pendulum.

L	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}	<T>	ΔT	Δ<T>
(m)	(sec)	(sec)	(sec)	(sec)	(sec)	(sec)	(sec)	(sec)	(sec)	(sec)	(sec)	(sec)	

Notes: t_i is the ith measurement of the time for 10 periods.
<T> is the average period determined from 10 measurements t_i.
ΔT is the standard deviation of the measurements of the period.

Table 2. Table of results, <T> is the average period for one oscillation.

Length = L	ΔL	<T>	Δ<T>	g	Δg
(m)	(m)	(sec)	(sec)	(m/sec^2)	(m/sec^2)

Table 3. Summary of results.

<g>		(m/sec^2)
Δ<g>		(m/sec^2)
$g_{accepted}$		(m/sec^2)