HW 8

P21.7 (a) $PV = Nk_BT$: $N = \frac{PV}{k_BT} = \frac{1.013\times10^5 \text{ Pa}\left[\frac{4}{3}\pi(0.150 \text{ m})^3\right]}{\left(1.38\times10^{-23} \text{ J/K}\right)(293 \text{ K})} = \boxed{3.54\times10^{23} \text{ atoms}}$

(b) $\overline{K} = \frac{3}{2}k_BT = \frac{3}{2}\left(1.38\times10^{-23}\right)(293) \text{ J} = \boxed{6.07\times10^{-21} \text{ J}}$

(c) For helium, the atomic mass is $m = \frac{4.00 \text{ g/mol}}{6.02\times10^{23} \text{ molecules/mol}} = 6.64\times10^{-24} \text{ g/molecule}$

$m = 6.64\times10^{-27} \text{ kg/molecule}$

$\frac{1}{2}m\overline{v^2} = \frac{3}{2}k_BT$: $\therefore v_{\text{rms}} = \sqrt{\frac{3k_BT}{m}} = \boxed{1.35 \text{ km/s}}$

P21.9 (a) $\overline{K} = \frac{3}{2}k_BT = \frac{3}{2}\left(1.38\times10^{-23} \text{ J/K}\right)(423 \text{ K}) = \boxed{8.76\times10^{-21} \text{ J}}$

(b) $\overline{K} = \frac{1}{2}mv_{rms}^2 = 8.76\times10^{-21} \text{ J}$

so $v_{rms} = \sqrt{\frac{1.75\times10^{-20} \text{ J}}{m}}$ (1)

For helium, $m = \frac{4.00 \text{ g/mol}}{6.02\times10^{23} \text{ molecules/mol}} = 6.64\times10^{-24} \text{ g/molecule}$

$m = 6.64\times10^{-27} \text{ kg/molecule}$

Similarly for argon, $m = \frac{39.9 \text{ g/mol}}{6.02\times10^{23} \text{ molecules/mol}} = 6.63\times10^{-23} \text{ g/molecule}$

$m = 6.63\times10^{-26} \text{ kg/molecule}$

Substituting in (1) above,
we find for helium, $\boxed{v_{rms} = 1.62 \text{ km/s}}$

and for argon, $\boxed{v_{rms} = 514 \text{ m/s}}$

P21.13 We us the tabulated values for C_P and C_V

(a) $Q = nC_P\Delta T = 1.00 \text{ mol}(28.8 \text{ J/mol}\cdot\text{K})(420-300) \text{ K} = \boxed{3.46 \text{ kJ}}$

(b) $\Delta E_{\text{int}} = nC_V\Delta T = 1.00 \text{ mol}(20.4 \text{ J/mol}\cdot\text{K})(120 \text{ K}) = \boxed{2.45 \text{ kJ}}$

(c) $W = -Q + \Delta E_{\text{int}} = -3.46 \text{ kJ} + 2.45 \text{ kJ} = \boxed{-1.01 \text{ kJ}}$

P21.21 In the isovolumetric process $A \to B$, $W = 0$ and $Q = nC_V \Delta T = 500$ J

$$500 \text{ J} = n\left(\frac{3R}{2}\right)(T_B - T_A) \text{ or } T_B = T_A + \frac{2(500 \text{ J})}{3nR}$$

$$T_B = 300 \text{ K} + \frac{2(500 \text{ J})}{3(1.00 \text{ mol})(8.314 \text{ J/mol}\cdot\text{K})} = 340 \text{ K}$$

In the isobaric process $B \to C$,

$$Q = nC_P \Delta T = \frac{5nR}{2}(T_C - T_B) = -500 \text{ J}.$$

Thus,

(a) $$T_C = T_B - \frac{2(500 \text{ J})}{5nR} = 340 \text{ K} - \frac{1\,000 \text{ J}}{5(1.00 \text{ mol})(8.314 \text{ J/mol}\cdot\text{K})} = \boxed{316 \text{ K}}$$

(b) The work done on the gas during the isobaric process is

$$W_{BC} = -P_B \Delta V = -nR(T_C - T_B) = -(1.00 \text{ mol})(8.314 \text{ J/mol}\cdot\text{K})(316 \text{ K} - 340 \text{ J})$$

or $W_{BC} = +200$ J

The work done on the gas in the isovolumetric process is zero, so in total

$$W_{\text{on gas}} = \boxed{+200 \text{ J}}.$$

P21.25 (a) $$P_i V_i^{\gamma} = P_f V_f^{\gamma}$$

$$P_f = P_i\left(\frac{V_i}{V_f}\right)^{\gamma} = 5.00 \text{ atm}\left(\frac{12.0}{30.0}\right)^{1.40} = \boxed{1.39 \text{ atm}}$$

(b) $$T_i = \frac{P_i V_i}{nR} = \frac{5.00(1.013 \times 10^5 \text{ Pa})(12.0 \times 10^{-3} \text{ m}^3)}{2.00 \text{ mol}(8.314 \text{ J/mol}\cdot\text{K})} = \boxed{365 \text{ K}}$$

$$T_f = \frac{P_f V_f}{nR} = \frac{1.39(1.013 \times 10^5 \text{ Pa})(30.0 \times 10^{-3} \text{ m}^3)}{2.00 \text{ mol}(8.314 \text{ J/mol}\cdot\text{K})} = \boxed{253 \text{ K}}$$

(c) The process is adiabatic: $\boxed{Q = 0}$

$$\gamma = 1.40 = \frac{C_P}{C_V} = \frac{R + C_V}{C_V}, \; C_V = \frac{5}{2}R$$

$$\Delta E_{\text{int}} = nC_V \Delta T = 2.00 \text{ mol}\left(\frac{5}{2}(8.314 \text{ J/mol}\cdot\text{K})\right)(253 \text{ K} - 365 \text{ K}) = \boxed{-4.66 \text{ kJ}}$$

$$W = \Delta E_{\text{int}} - Q = -4.66 \text{ kJ} - 0 = \boxed{-4.66 \text{ kJ}}$$

P22.6 The heat to melt 15.0 g of Hg is $|Q_c| = mL_f = (15\times10^{-3}\text{ kg})(1.18\times10^{4}\text{ J/kg}) = 177\text{ J}$

The energy absorbed to freeze 1.00 g of aluminum is

$$|Q_h| = mL_f = (10^{-3}\text{ kg})(3.97\times10^{5}\text{ J/kg}) = 397\text{ J}$$

and the work output is $W_{\text{eng}} = |Q_h| - |Q_c| = 220\text{ J}$

$$e = \frac{W_{\text{eng}}}{|Q_h|} = \frac{220\text{ J}}{397\text{ J}} = 0.554, \text{ or } \boxed{55.4\%}$$

The theoretical (Carnot) efficiency is $\frac{T_h - T_c}{T_h} = \frac{933\text{ K} - 243.1\text{ K}}{933\text{ K}} = 0.749 = 74.9\%$

P22.7 $\text{COP(refrigerator)} = \frac{Q_c}{W}$

(a) If $Q_c = 120\text{ J}$ and $\text{COP} = 5.00$, then $\boxed{W = 24.0\text{ J}}$

(b) Heat expelled = Heat removed + Work done.

$$Q_h = Q_c + W = 120\text{ J} + 24\text{ J} = \boxed{144\text{ J}}$$

P22.11 $T_c = 703\text{ K}$ $\quad T_h = 2\,143\text{ K}$

(a) $$e_c = \frac{\Delta T}{T_h} = \frac{1\,440}{2\,143} = \boxed{67.2\%}$$

(b) $|Q_h| = 1.40\times10^{5}\text{ J}$, $W_{\text{eng}} = 0.420|Q_h|$

$$\mathcal{P} = \frac{W_{\text{eng}}}{\Delta t} = \frac{5.88\times10^{4}\text{ J}}{1\text{ s}} = \boxed{58.8\text{ kW}}$$

P22.13 Isothermal expansion at $T_h = 523\text{ K}$
Isothermal compression at $T_c = 323\text{ K}$
Gas absorbs 1 200 J during expansion.

(a) $$|Q_c| = |Q_h|\left(\frac{T_c}{T_h}\right) = 1\,200\text{ J}\left(\frac{323}{523}\right) = \boxed{741\text{ J}}$$

(b) $$W_{\text{eng}} = |Q_h| - |Q_c| = (1\,200 - 741)\text{ J} = \boxed{459\text{ J}}$$

P22.17 (a) In an adiabatic process, $P_f V_f^\gamma = P_i V_i^\gamma$. Also, $\left(\frac{P_f V_f}{T_f}\right)^\gamma = \left(\frac{P_i V_i}{T_i}\right)^\gamma$.

Dividing the second equation by the first yields $T_f = T_i \left(\frac{P_f}{P_i}\right)^{(\gamma-1)/\gamma}$.

Since $\gamma = \frac{5}{3}$ for Argon, $\frac{\gamma-1}{\gamma} = \frac{2}{5} = 0.400$ and we have

$$T_f = (1\,073\text{ K})\left(\frac{300\times10^3\text{ Pa}}{1.50\times10^6\text{ Pa}}\right)^{0.400} = \boxed{564\text{ K}}.$$

(b) $\Delta E_{\text{int}} = nC_V\Delta T = Q - W_{\text{eng}} = 0 - W_{\text{eng}}$, so $W_{\text{eng}} = -nC_V\Delta T$,

and the power output is

$$\mathcal{P} = \frac{W_{\text{eng}}}{t} = \frac{-nC_V\Delta T}{t} \text{ or}$$

$$= \frac{(-80.0\text{ kg})\left(\frac{1.00\text{ mol}}{0.039\,9\text{ kg}}\right)\left(\frac{3}{2}\right)(8.314\text{ J/mol}\cdot\text{K})(564-1\,073)\text{K}}{60.0\text{ s}}$$

$$\mathcal{P} = 2.12\times10^5\text{ W} = \boxed{212\text{ kW}}$$

(c) $e_C = 1 - \frac{T_c}{T_h} = 1 - \frac{564\text{ K}}{1\,073\text{ K}} = 0.475$ or $\boxed{47.5\%}$

Quick Quiz 22.3.

If COP is 4, you need to do 0.25 of Work to transfer heat amount of 1.

$COP \equiv \frac{Q}{W}$ ← Heat from cold to hot or from hot to cold

↖ Work done by engine