HW 6

P19.44 (a) Initially the air in the bell satisfies $P_0 V_{\text{bell}} = nRT_i$

or $P_0\left[(2.50\text{ m})A\right] = nRT_i$ (1)

When the bell is lowered, the air in the bell satisfies

$$P_{\text{bell}}(2.50\text{ m} - x)A = nRT_f \quad (2)$$

where x is the height the water rises in the bell. Also, the pressure in the bell, once it is lowered, is equal to the sea water pressure at the depth of the water level in the bell.

$$P_{\text{bell}} = P_0 + \rho g(82.3\text{ m} - x) \approx P_0 + \rho g(82.3\text{ m}) \quad (3)$$

The approximation is good, as $x < 2.50$ m. Substituting (3) into (2) and substituting nR from (1) into (2),

$$\left[P_0 + \rho g(82.3\text{ m})\right](2.50\text{ m} - x)A = P_0 V_{\text{bell}} \frac{T_f}{T_i}.$$

Using $P_0 = 1\text{ atm} = 1.013 \times 10^5\text{ Pa}$ and $\rho = 1.025 \times 10^3\text{ kg/m}^3$

$$x = (2.50\text{ m})\left[1 - \frac{T_f}{T_0}\left(1 + \frac{\rho g(82.3\text{ m})}{P_0}\right)^{-1}\right]$$

$$= (2.50\text{ m})\left[1 - \frac{277.15\text{ K}}{293.15\text{ K}}\left(1 + \frac{\left(1.025 \times 10^3\text{ kg/m}^3\right)\left(9.80\text{ m/s}^2\right)(82.3\text{ m})}{1.013 \times 10^5\text{ N/m}^2}\right)^{-1}\right]$$

$$x = \boxed{2.24\text{ m}}$$

(b) If the water in the bell is to be expelled, the air pressure in the bell must be raised to the water pressure at the bottom of the bell. That is,

$$P_{\text{bell}} = P_0 + \rho g(82.3\text{ m})$$

$$= 1.013 \times 10^5\text{ Pa} + \left(1.025 \times 10^3\text{ kg/m}^3\right)\left(9.80\text{ m/s}^2\right)(82.3\text{ m})$$

$$P_{\text{bell}} = 9.28 \times 10^5\text{ Pa} = \boxed{9.16\text{ atm}}$$

P19.46 At 0°C, 10.0 gallons of gasoline has mass,

from $$\rho = \frac{m}{V}$$

$$m = \rho V = \left(730\ \text{kg/m}^3\right)\left(10.0\ \text{gal}\right)\left(\frac{0.003\,80\ \text{m}^3}{1.00\ \text{gal}}\right) = 27.7\ \text{kg}$$

The gasoline will expand in volume by

$$\Delta V = \beta V_i \Delta T = 9.60 \times 10^{-4}\ {}^\circ\text{C}^{-1}(10.0\ \text{gal})(20.0^\circ\text{C} - 0.0^\circ\text{C}) = 0.192\ \text{gal}$$

At 20.0°C, $10.192\ \text{gal} = 27.7\ \text{kg}$

$$10.0\ \text{gal} = 27.7\ \text{kg}\left(\frac{10.0\ \text{gal}}{10.192\ \text{gal}}\right) = 27.2\ \text{kg}$$

The extra mass contained in 10.0 gallons at 0.0°C is

$$27.7\ \text{kg} - 27.2\ \text{kg} = \boxed{0.523\ \text{kg}}.$$

P19.50 (a)

$$\frac{P_0 V}{T} = \frac{P'V'}{T'}$$

$$V' = V + Ah$$

$$P' = P_0 + \frac{kh}{A}$$

$$\left(P_0 + \frac{kh}{A}\right)(V + Ah) = P_0 V\left(\frac{T'}{T}\right)$$

$$\left(1.013 \times 10^5\ \text{N/m}^2 + 2.00 \times 10^5\ \text{N/m}^3\ h\right)$$

$$\left(5.00 \times 10^{-3}\ \text{m}^3 + \left(0.010\,0\ \text{m}^2\right)h\right)$$

$$= \left(1.013 \times 10^5\ \text{N/m}^2\right)\left(5.00 \times 10^{-3}\ \text{m}^3\right)\left(\frac{523\ \text{K}}{293\ \text{K}}\right)$$

$$2\,000h^2 + 2\,013h - 397 = 0$$

$$h = \frac{-2\,013 \pm 2\,689}{4\,000} = \boxed{0.169\ \text{m}}$$

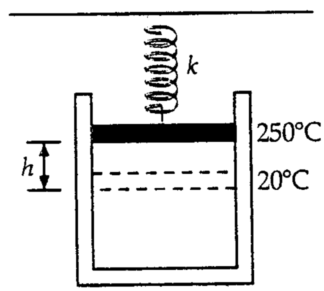

FIG. P19.50

(b)

$$P' = P + \frac{kh}{A} = 1.013 \times 10^5\ \text{Pa} + \frac{\left(2.00 \times 10^3\ \text{N/m}\right)(0.169)}{0.010\,0\ \text{m}^2}$$

$$P' = \boxed{1.35 \times 10^5\ \text{Pa}}$$

P19.68 With piston alone: $T = \text{constant, so } PV = P_0V_0$

or $P(Ah_i) = P_0(Ah_0)$

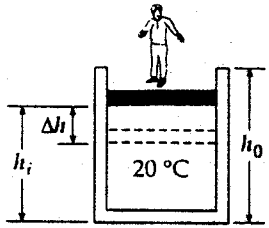

FIG. P19.68

With $A = \text{constant}$, $P = P_0\left(\frac{h_0}{h_i}\right)$

But, $P = P_0 + \frac{m_p g}{A}$

where m_p is the mass of the piston.

Thus, $P_0 + \frac{m_p g}{A} = P_0\left(\frac{h_0}{h_i}\right)$

which reduces to $h_i = \frac{h_0}{1 + \frac{m_p g}{P_0 A}} = \frac{50.0 \text{ cm}}{1 + \frac{20.0 \text{ kg}(9.80 \text{ m/s}^2)}{1.013 \times 10^5 \text{ Pa}\left[\pi(0.400 \text{ m})^2\right]}} = 49.81 \text{ cm}$

With the man of mass M on the piston, a very similar calculation (replacing m_p by $m_p + M$) gives:

$$h' = \frac{h_0}{1 + \frac{(m_p + M)g}{P_0 A}} = \frac{50.0 \text{ cm}}{1 + \frac{95.0 \text{ kg}(9.80 \text{ m/s}^2)}{1.013 \times 10^5 \text{ Pa}\left[\pi(0.400 \text{ m})^2\right]}} = 49.10 \text{ cm}$$

Thus, when the man steps on the piston, it moves downward by

$$\Delta h = h_i - h' = 49.81 \text{ cm} - 49.10 \text{ cm} = 0.706 \text{ cm} = \boxed{7.06 \text{ mm}}.$$

(b) $P = \text{const, so } \frac{V}{T} = \frac{V'}{T_i}$ or $\frac{Ah_i}{T} = \frac{Ah'}{T_i}$

giving $T = T_i\left(\frac{h_i}{h'}\right) = 293 \text{ K}\left(\frac{49.81}{49.10}\right) = \boxed{297 \text{ K}}$ (or 24°C)