Physics 260 Homework Assignment 5

Chapter 18

1 PSE6 18.P.001

Plug in given x and t values for both functions and add them up to get the resultant position, $y = y_1 + y_2$.

2 PSE6 18.P.007

a. Since the end is fixed, the reflected wave is inverted. Thus, when they meet, they cancel and the amplitude is zero.

b. The end is not fixed. So the reflected wave is non-inverted. When they meet, they add and the amplitude is sum of the original amplitudes, 2A.

3 PSE6 18.P.009

Suppose the man's ears are at the same level as the lower speaker. The sounds from two speakers travel through distances $R_{upper} = \sqrt{L^2 + d^2}$ and $R_{lower} = L$. So the path difference is $\Delta R = R_{upper} - R_{lower} = \sqrt{L^2 + d^2} - L$. Minima(destructive interference) occur when the path difference is equal to $(2n-1)\lambda/2$, where n=1, 2, 3, $\cdots$ The wavelength can be determined from the speed of the sound and the frequency using $v = \lambda f$. Hence

$$\sqrt{L^2 + d^2} - L = \frac{(2n-1)\lambda}{2}$$

$$\sqrt{L^2 + d^2} = \frac{(2n-1)\lambda}{2} + L$$

$$L^2 + d^2 = \frac{(2n-1)^2\lambda^2}{4} + (2n-1)\lambda L + L^2$$

$$L = \frac{d^2 - (2n-1)^2\lambda^2/4}{(2n-1)\lambda}$$

This will actually give the answer to part (b). Just plug in n = 1, 2, 3, $\cdots$ and stop when L value becomes negative. In order to have positive L value, we need the numerator in the expression to be positive(the denominator is always positive). So

$$d^2 - (2n-1)^2\lambda^2/4 \geq 0$$

$$d \geq \frac{(2n-1)\lambda}{2}$$

$$n \leq \frac{d}{\lambda} + \frac{1}{2}$$

The number of minima he hears is the greatest integer solution.

4 PSE6 18.P.015

From the given frequency and speed of sound, we can calculate the wavelength, $\lambda = v/f$. The distance between adjacent nodes is half of the wavelength, and the distance between adjacent node and antinode is a quarter of the wavelength. Since the speakers are in phase, the midpoint between two speakers is an antinode. Add or subtract $1/4\lambda$ to to the position of the midpoint to get the adjacent nodes and then add and subtract half the wavelength the get the next two nodes and so on until you found all the nodes.

5 PSE6 18.P.021

a. Let n be the number of nodes in the standing wave resulting from the lesser mass. Then $n-1$ is the number of nodes for the standing wave resulting from the other mass. For standing waves, $\lambda = \frac{2L}{n}$. So, the frequency can be found using the expression

$$f = \frac{v_n}{\lambda_n} = \frac{\sqrt{T_n/\mu}}{2L/n} = \frac{n}{2L}\sqrt{\frac{T_n}{\mu}}$$

Also,

$$f = \frac{v_{n-1}}{\lambda_{n-1}} = \frac{\sqrt{T_{n-1}/\mu}}{2L/n-1} = \frac{n-1}{2L}\sqrt{\frac{T_{n-1}}{\mu}}$$

Thus,

$$\frac{n}{2L}\sqrt{\frac{T_n}{\mu}} = \frac{n-1}{2L}\sqrt{\frac{T_{n-1}}{\mu}}$$

$$\frac{n-1}{n} = \sqrt{\frac{T_n}{T_{n-1}}}$$

$$n = \frac{1}{1-\sqrt{T_n/T_{n-1}}}$$

And the tension is due to the weight of the hanging object. Once n is determined, plug it back into one of the expressions for f and calculate the frequency of the vibrator.

b. The largest mass will correspond to a standing wave of 1 loop($n = 1$). From the first expression for f in part (a) we know that

$$T = mg = 4\mu L^2 f^2$$

So, the largest object mass that can produce a standing wave is

$$m = 4\mu L^2 f^2 / g$$

6 PSE6 18.P.036

a. In a pipe open in both ends, the frequency of the lowest note is given by

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$$

b. The corresponding wavelength is v/λ and the distance between adjacent antinodes is 1/4 of the wavelength.

7 PSE6 18.P.038

An open pipe is a pipe with both ends open and a closed pipe has one end closed and the other end open. For the open pipe, $n = 1$. Therefore, $L = v/2f$. For the closed pipe, $n = 3$. So $L = 3v/4f$.

8 PSE6 18.P.046

This setup can be modeled as an air column closed at one end. So, $L_n = nv/4f$. Start out with $n = 1$ until you get two shortest lengths that are greater than the initial length.

9 PSE6 18.P.052

a. Possible frequencies are $f_1 = f_0 + f_{beat}$ and $f_2 = f_0 - f_{beat}$

b. Tightening the string increases the tension, which means the speed and frequency are also increased. Since the beat frequency increases, we must have f_1 as the original frequency. And the new frequency of the string is f_0 plus the new beat frequency f_{beat}, $f_n = f_0 + f_{beat}$.

c. From $f = \frac{1}{2L}\sqrt{\frac{I}{\mu}}$ yields

$$\frac{f_n}{f_0} = \sqrt{\frac{T_n}{T_0}}$$

$$T_n = \left(\frac{f_n}{f_0}\right)^2 T_0$$

So the percentage change is $\frac{T_0 - T_n}{T_0}$. The tension should be reduced.

10 PSE6 18.P.061

Assume the depth of the well is L. Then for some integer n,

$$L = (2n-1)\frac{v}{4f_1}$$

And for the next resonance we have

$$L = [2(n+1)-1]\frac{v}{4f_2} = (2n+1)\frac{v}{4f_2}$$

Note that f_1 corresponds to lower frequency. So set two equations equal and solve for n, yields

$$n = \frac{f_2 + f_1}{2(f_2 - f_1)}$$

Plug n back into the expression for L. If n didn't turn out to be an integer, then choose a best fitting integer and plug n into both of the expressions for L and then take the average for the depth of the well.