HW2.

1. $x = A\cos(\omega t + \phi)$

(a) $t = 0$, $x(t=0) = A\cos\phi$ (cm)

(b) $v = \frac{dx}{dt} = -A\omega\sin(\omega t + \phi)$ (cm/s)

(c) $a = \frac{dv}{dt} = -A\omega^2\cos(\omega t + \phi)$ (cm/s²)

(d) Period $T = \frac{2\pi}{\omega}$ s

Amplitude A

2. A simple harmonic oscillator takes 12.5s to undergo six complete vibrations.

(a) Period $T = \frac{12.5}{6} = 2.08$ (s)

(b) Frequency $f = \frac{1}{T}$ (Hz)

(c) Angular frequency $\omega = \frac{2\pi}{T}$ (rad/s)

3. A block of unknown mass m,

Spring constant $k = 6.00$ N/m

Amplitude $A = 10.5$ cm

When the block is halfway between its equilibrium position and the end point, speed $v = 27.0$ cm/s

$x=0$ Ⓐ $x_A = \frac{1}{2}A$ Ⓑ $x_B = A$

$v_A = 27\ cm/s$ $v_B = 0$

Energy is conserved.

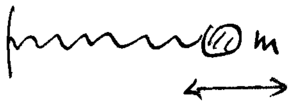
$$E_Ⓐ = \frac{1}{2} m v_A^2 + \frac{1}{2} k x_A^2 = \frac{1}{2} \cancel{m v_B^2}^{\,0} + \frac{1}{2} k x_B^2 = E_Ⓑ$$

$$\Rightarrow \quad m = \frac{2}{v_A^2}\left[\frac{1}{2}kA^2 - \frac{1}{2}k\left(\frac{A}{2}\right)^2\right]$$

$$= \frac{2}{(27\times10^{-2})^2}\left[\frac{1}{2}\times 6\times(10.5\times10^{-2})^2 - \frac{1}{2}\times 6\times\left(\frac{1}{2}\times 10.5\times\right.\right.$$

4.

$m = 65\,g$, $k = 35\ N/m$, Amplitude $A = 8\ cm$



(a)

When the spring is stretched maximum, $v=0$.

$$E_{tot} = \frac{1}{2} \cancel{m v^2}^{\,0} + \frac{1}{2} k x^2 = \frac{1}{2} k A^2$$

(b) $$E_{tot} = \frac{1}{2} k A^2 = \frac{1}{2} m v^2 + \frac{1}{2} k x^2 \qquad (x = 1.05 cm)$$

$$\Rightarrow \quad v = \sqrt{\frac{1}{m} k (A^2 - x^2)}$$

(c) $$E_{kinetic} = E_{tot} - E_{pot} = \frac{1}{2} k A^2 - \frac{1}{2} k x^2 \qquad (x = 3 cm)$$

(d) $$E_{pot} = \frac{1}{2} k x^2$$

5. Amplitude $A = 3.70$ cm

Spring Constant $k = 250 N/m$

Mass of the block $m = 0.8$ kg

(a) Mechanical energy E

$$E_{tot} = E_{kinetic} + E_{potential}$$

$$= \frac{1}{2}mV^2 + \frac{1}{2}kx^2$$

$$= \frac{1}{2}kA^2$$

(b) The maximum speed V_{max}

when the potential energy is zero, ($x = 0$),

$V = V_{max}$

$$E_{tot} = \frac{1}{2}kA^2 = \frac{1}{2}mV_{max}^2$$

$$\therefore V_{max} = \sqrt{\frac{k}{m}}A$$

(c) The maximum acceleration a_{max}

When the spring stretched maximum,
force is maximum and acceleration is also maximum.

$$F = kx = ma$$

$$\therefore a_{max} = \frac{k}{m}x_{max} = \frac{k}{m}A$$

6. Amplitude $A = 6\,cm$.

What is the position x for spring's speed to equal $3/4$ of its maximum speed V_{max}?

$$E_{tot} = \frac{1}{2}kA^2 = \frac{1}{2}mV^2 + \frac{1}{2}kx^2$$

$$= \frac{1}{2}mV_{max}^2$$

① : $\frac{1}{2}kA^2 = \frac{1}{2}mV_{max}^2 \Rightarrow V_{max} = \sqrt{\frac{K}{m}}A$

② : $\frac{1}{2}KA^2 = \frac{1}{2}m\left(\frac{3}{4}\cdot\sqrt{\frac{K}{m}}A\right)^2 + \frac{1}{2}Kx^2$

$$\therefore x = \pm\left[A^2 - \frac{9}{16}A^2\right]^{1/2} = \pm\sqrt{\frac{7}{16}}A$$

7. $\theta = (0.320\text{ rad})\cos\omega t$, $\omega = 7.93\text{ rad/s}$

(a) Period $T = \frac{2\pi}{\omega}$

(b) length of the pendulum, l

$$T = 2\pi\sqrt{\frac{l}{g}} = \frac{2\pi}{\omega}, \quad g = 9.8\,m/s^2$$

$$\therefore l = \frac{g}{\omega^2}$$

8. The period of the pendulum T

$$T = 2\pi \sqrt{\frac{l}{g_{eff}}}$$

(a) An elevator is accelerating upward at $4\ m/s^2$

$$g_{eff} = 9.8 + 4$$

(b) accelerating downward at $4\ m/s^2$

$$g_{eff} = 9.8 - 4$$

(c) $g_{eff} = \sqrt{9.8^2 + 4^2}$ horizontally accelerating at $4\ m/s^2$