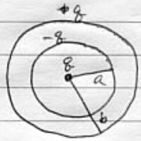
Exam 3

P1. part I



a) $r<a$ $\quad E = \frac{k_e q}{r}$

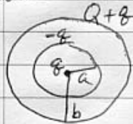
b) $a<r<b$ $\quad E = 0$ (inside conductor) $q_{in} = 0$

c) $r>b$ $\quad E = \frac{k_e q}{r}$

d) $r=a$ $\quad \sigma = -q/\pi a^2$

$r=b$ $\quad \sigma = +q/\pi b^2$

Part II



a) $r<a$ $\quad E = \frac{k_e q}{r}$

b) $a<r<b$ $\quad E = 0$

c) $r>b$ $\quad E = \frac{k_e (Q+q)}{r}$

d) $r=a$ $\quad \sigma = -q/\pi a^2$

$r=b$ $\quad \sigma = (Q+q)/\pi b^2$

P2. a) $\Delta V = E \Delta y$

b) the surface of Earth

c) Potential energy of a test charge near Earth's Surface

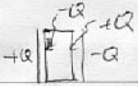
$$U_E = q\Delta V = q E \Delta y$$

Potential energy due to gravity

$$U_g = mg\Delta y.$$

They are both directly proportional to the height.

d) $F_E = F_g \Rightarrow qE = mg \Rightarrow q = \frac{mg}{E} = 4.9C$



P3. a) surface charge induced on the dielectric is

$$Q_1 = 0.3\,\mu C \quad + Q_2 = -0.3\,\mu C$$

b)

$$C = k\frac{\epsilon_0 A}{d} = \frac{3.5 \times (8.85\times10^{-12}\frac{C^2}{N\cdot m^2}) \times (0.22m)^2}{0.0018m}$$

$$= 8.33 \times 10^{-10} F$$

$$C = \frac{Q}{\Delta V} = \frac{Q}{Ed}$$

$$\Rightarrow \quad E = \frac{Q}{Cd} = \frac{0.3\times10^{-6}C}{(8.33\times10^{-10}F)(0.0018m)}$$

$$= 2\times10^{5}\ V/m$$

c) $$U = \frac{Q^2}{2C} = \frac{1}{2}C(\Delta V)^2$$

$$= \frac{(0.3\times10^{-6}C)^2}{2\,(8.33\times10^{-10}F)} = 5.4\times10^{-5}\,J$$

P4. a) $$\rho = \rho_0[1+\alpha(T-T_0)]$$

$$= (2.82\times10^{-8}\,\Omega\cdot m)[1 + 3.9\times10^{-3}/{}^\circ C\ (50^\circ C - 20^\circ C)]$$

$$= 3.15\times10^{-8}\,\Omega\cdot m$$

b) $$J = \sigma E = \frac{E}{\rho} = \frac{0.200\,V/m}{3.15\times10^{-8}\,\Omega\cdot m} = 6.35\times10^{6}\,A/m^2$$

c) $$J = \frac{I}{A} \Rightarrow I = JA$$

$$= (6.35\times10^{6}\,A/m^2)(\pi\cdot(5\times10^{-5}m)^2)$$

$$= 0.05\,A$$

d) $n = \frac{\text{density of Al}}{\text{mass per atom}} = \frac{2.70 \times 10^3 \text{ kg/m}^3}{\frac{26.95 \times 10^{-3} \text{ kg/mole}}{6.02 \times 10^{23} \text{ electrons/mole}}}$

$= 6.02 \times 10^{28} \text{ electrons/m}^3$

$v_d = \frac{J}{nq} = \frac{6.35 \times 10^6 \text{ A/m}^2}{(6.02 \times 10^{28} \text{ electrons/m}^3)(1.60 \times 10^{-19} \text{ C/electron})}$

$= 6.6 \times 10^{-4} \text{ m/s}$

e) $\Delta V = Ed = (0.200 \text{ V/m})(2.00 \text{ m}) = 0.4 \text{ V}$

P5. Potential due to a solid disk with radius a is

$$V_a = 2\pi k_e \sigma [(x^2 + a^2)^{1/2} - x]$$

Potential due to a solid disk with radius b is

$$V_b = 2\pi k_e \sigma [(x^2 + b^2)^{1/2} - x]$$

Using the method of superposition, the potential due to a disk with a hole (inner radius a, outer radius b) is

$$V = V_b - V_a$$

$$= 2\pi k_e \sigma [(x^2 + b^2)^{1/2} - (x^2 + a^2)^{1/2}]$$

with $\sigma = \frac{Q}{A} = \frac{Q}{\pi b^2 - \pi a^2}$

When $a \to 0$, expression for V becomes

$$V = V_b = 2\pi k_e \sigma [(x^2 + b^2)^{1/2} - x]$$

which is exactly the same as the potential due to a solid disk with radius b.