1 (a) $A = (7.00 + 3.00)\,4.00$ yields $\boxed{A = 40.0}$

(b) In order for two vectors to be equal, they must have the same magnitude and the same direction in three-dimensional space. All of their components must be equal. Thus, $7.00\hat{\mathbf{i}} + 0\hat{\mathbf{j}} + 3.00\hat{\mathbf{k}} = A\hat{\mathbf{i}} + B\hat{\mathbf{j}} + C\hat{\mathbf{k}}$ requires $\boxed{A = 7.00,\ B = 0,\ \text{and}\ C = 3.00}$.

(c) In order for two functions to be identically equal, they must be equal for every value of every variable. They must have the same graphs. In

$$A + B\cos(Cx + Dt + E) = 0 + 7.00\ \text{mm}\ \cos(3.00x + 4.00t + 2.00),$$

the equality of average values requires that $\boxed{A = 0}$. The equality of maximum values requires $\boxed{B = 7.00\ \text{mm}}$. The equality for the wavelength or periodicity as a function of *x* requires $\boxed{C = 3.00\ \text{rad/m}}$. The equality of period requires $\boxed{D = 4.00\ \text{rad/s}}$, and the equality of zero-crossings requires $\boxed{E = 2.00\ \text{rad}}$.

2 The linear wave equation is $\dfrac{\partial^2 y}{\partial x^2} = \dfrac{1}{v^2}\dfrac{\partial^2 y}{\partial t^2}$

If $y = e^{b(x-vt)}$

then $\dfrac{\partial y}{\partial t} = -bve^{b(x-vt)}$ and $\dfrac{\partial y}{\partial x} = be^{b(x-vt)}$

$\dfrac{\partial^2 y}{\partial t^2} = b^2v^2e^{b(x-vt)}$ and $\dfrac{\partial^2 y}{\partial x^2} = b^2e^{b(x-vt)}$

Therefore, $\dfrac{\partial^2 y}{\partial t^2} = v^2\dfrac{\partial^2 y}{\partial x^2}$, demonstrating that $e^{b(x-vt)}$ is a solution

3 The linear wave equation is $\dfrac{1}{v^2}\dfrac{\partial^2 y}{\partial t^2} = \dfrac{\partial^2 y}{\partial x^2}$

To show that $y = \ln\left[b(x-vt)\right]$ is a solution, we find its first and second derivatives with respect to *x* and *t* and substitute into the equation.

$$\frac{\partial y}{\partial t} = \frac{1}{b(x-vt)}(-bv) \qquad \frac{\partial^2 y}{\partial t^2} = \frac{-1(-bv)^2}{b^2(x-vt)^2} = -\frac{v^2}{(x-vt)^2}$$

$$\frac{\partial y}{\partial x} = \left[b(x-vt)\right]^{-1} b \qquad \frac{\partial^2 y}{\partial x^2} = -\frac{b}{b}(x-vt)^{2} = -\frac{1}{(x-vt)^2}$$

Then $\frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} = \frac{1}{v^2}\frac{\left(-v^2\right)}{(x-vt)^2} = -\frac{1}{(x-vt)^2} = \frac{\partial^2 y}{\partial x^2}$ so the given wave function is a solution.

4 (a) From $y = x^2 + v^2t^2$,

evaluate $\frac{\partial y}{\partial x} = 2x$ $\quad \frac{\partial^2 y}{\partial x^2} = 2$

$\frac{\partial y}{\partial t} = v^2 2t$ $\quad \frac{\partial^2 y}{\partial t^2} = 2v^2$

Does $\frac{\partial^2 y}{\partial t^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$?

By substitution: $2 = \frac{1}{v^2}2v^2$ and this is true, so the wave function does satisfy the wave equation.

(b) Note $\frac{1}{2}(x+vt)^2 + \frac{1}{2}(x-vt)^2 \;= \frac{1}{2}x^2 + xvt + \frac{1}{2}v^2t^2 + \frac{1}{2}x^2 - xvt + \frac{1}{2}v^2t^2$

$= x^2 + v^2t^2$ as required.

So $\boxed{f(x+vt) = \frac{1}{2}(x+vt)^2}$ and $\boxed{g(x-vt) = \frac{1}{2}(x-vt)^2}$.

(c) $y = \sin x \cos vt$ makes

$\frac{\partial y}{\partial x} = \cos x \cos vt$ $\quad \frac{\partial^2 y}{\partial x^2} = -\sin x \cos vt$

$\frac{\partial y}{\partial t} = -v \sin x \sin vt$ $\quad \frac{\partial^2 y}{\partial t^2} = -v^2 \sin x \cos vt$

Then $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$

becomes $-\sin x \cos vt = \frac{-1}{v^2}v^2 \sin x \cos vt$ which is true as required.

Note $\sin(x+vt) = \sin x \cos vt + \cos x \sin vt$

$\sin(x-vt) = \sin x \cos vt - \cos x \sin vt$.

So $\sin x \cos vt = f(x+vt) + g(x-vt)$ with

$\boxed{f(x+vt) = \frac{1}{2}\sin(x+vt)}$ and $\boxed{g(x-vt) = \frac{1}{2}\sin(x-vt)}$.

5 Compare the given wave function $y = 4.00 \sin(2.00x - 3.00t)$ cm to the general form $y = A\sin(kx - \omega t)$ to find

(a) amplitude $A = 4.00$ cm $= \boxed{0.040\,0\text{ m}}$

(b) $k = \frac{2\pi}{\lambda} = 2.00\text{ cm}^{-1}$ and $\lambda = \pi$ cm $= \boxed{0.031\,4\text{ m}}$

(c) $\omega = 2\pi f = 3.00\ \mathrm{s}^{-1}$ and $f = \boxed{0.477\ \mathrm{Hz}}$

(d) $T = \frac{1}{f} = \boxed{2.09\ \mathrm{s}}$

(e) The minus sign indicates that the wave is traveling in the $\boxed{\text{positive } x\text{-direction}}$.