The Linear Wave equation
Practice problems

$$\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y(x,t)}{\partial t^2}$$

1. (a) Evaluate A in the scalar equality (7+3)4 = A. (b) Evaluate A, B, and C in the vector equality $7.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{k}} = A\hat{\mathbf{i}} + B\hat{\mathbf{j}} + C\hat{\mathbf{k}}$. Explain how you arrive at the answers to convince a student who thinks that you cannot solve a single equation for three different unknowns. (c) What If? The functional equality or identity A + B cos (Cx + Dt + E) = (7.00 mm) cos(3x + 4t + 2) is true for all values of the variables x and t, which are measured in meters and in seconds, respectively. Evaluate the constants A, B, C, D, and E. Explain how you arrive at the answers.

2. Show that the wave function $y = e^{b(x - vt)}$ is a solution of the linear wave equation, where b is a constant.

3. Show that the wave function y = ln[b(x – vt)] is a solution to the linear wave equation, where b is a constant.

4. (a) Show that the function $y(x,t) = x^2 + v^2t^2$ is a solution to the wave equation. (b) Show that the function in part (a) can be written as f(x + vt) + g(x – vt), and determine the functional forms for f and g. (c) What If? Repeat parts (a) and (b) for the function y(x,t) = sin(x)cos(vt).

5. A traveling wave propagates according to the expression y = (4.0 cm) sin(2.0x – 3.0t), where x is in centimeters and t is in seconds. Determine (a) the amplitude, (b) the wavelength, (c) the frequency, (d) the period, and (e) the direction of travel of the wave.