chapter 14.

11. We can find the phase constant from the initial conditions for position and velocity:

$$\begin{cases} x_0 = A\cos\phi_0 \\ v_{0x} = -\omega A\sin\phi_0 \end{cases} \rightarrow \frac{\sin\phi_0}{\cos\phi_0} = \tan\phi_0 = -\frac{v_{0x}}{\omega x_0}$$

$x_0 = -5.0\,\text{cm} \quad v_{0x} = +36.3\,\text{cm/s}$

$\omega = 2\pi/T = \frac{4}{3}\pi\ \text{rad/s}$

$$\phi_0 = \tan^{-1}\left(-\frac{36.3\,\text{cm/s}}{(4\pi/3\ \text{rad/s})(-5.0\,\text{cm})}\right)$$

$$= \frac{1}{3}\pi\ \text{rad} \text{ or } -\frac{2}{3}\pi\ \text{rad}.$$

The tangent function repeats every 180°. so there are always two possible solutions when evaluating the arctan function. We can distinguish between them because an object with a negative position but moving to the right is in the third quadrant of the corresponding circular motion. Thus $\phi_0 = -\frac{2}{3}\pi$ rad.

(b) At time t, the phase is $\phi = \omega t + \phi_0 = (\frac{4}{3}\pi\ \text{rad/s})t - \frac{2}{3}\pi\,\text{rad}$. This gives $\phi = -\frac{2}{3}\pi$ rad, 0 rad, $\frac{2}{3}\pi$ rad, and $\frac{4}{3}\pi$ rad at respectively $t = 0\,s$, $0.5\,s$, $1.0\,s$ and $1.5\,s$.

12. $T = 2\pi\sqrt{\frac{m}{k}} = T_0 = 2.0\,s$

(a) For mass $= 2m$

$$T = 2\pi\sqrt{\frac{2m}{k}} = \sqrt{2}\,T_0 = 2.83\,s$$

(b) For mass $= \frac{1}{2}m$

$$T = 2\pi\sqrt{\frac{m/2}{k}} = \sqrt{\frac{1}{2}}\,T_0 = 1.41\,s$$

(c) The period is independent of amplitude.

Thus $T = T_0 = 2.00\,s$

(d) For a spring constant $= 2k$

$$T = 2\pi\sqrt{\frac{m}{2k}} = T_0/\sqrt{2} = 1.41\,s$$

13. $T = (12.0\,s/10) = 1.20\,s$

$$T = 2\pi\sqrt{\frac{m}{k}} \rightarrow k = \left(\frac{2\pi}{T}\right)^2 m = \left(\frac{2\pi}{1.20\,s}\right)^2 (0.200\,kg) = 5.48\ N/m$$

14. (a) $A = 2.0\,cm$.

(b) $\omega = \frac{2\pi}{T} = 10\ rad/s \rightarrow T = \frac{2\pi}{10\ rad/s} = 0.628\,s$

(c) $\omega = \sqrt{\frac{k}{m}} \rightarrow k = m\omega^2 = (0.050\,kg)(10\ rad/s)^2 = 5.0\ N/m$

(d) $\phi_0 = -\frac{1}{4}\pi$ rad.

(e) $x(t) = (2.0\ cm)\cos(10t - \frac{1}{4}\pi)$

$v_x(t) = -(-20.0\ cm/s)\sin(10t - \frac{1}{4}\pi)$

$x_0 = 1.414\,cm \qquad v_{0x} = 14.14\ cm/s$

(f) $v_{max} = A\omega = (2.0\ cm)(10\ rad/s) = 20.0\ cm/s$

(g) $E_{total} = \frac{1}{2}kA^2 = \frac{1}{2}(5.0\,N/m)(0.02\,m)^2 = 1.0\times10^{-3}\,J$

(h) $t = 0.40\,s \qquad v_{0x} = -(20.0\,cm/s)\sin[(10\,rad/s)(0.40\,s) - \frac{1}{4}\pi] = 1.4$

15. (a) $T = 1/f = 0.50\,s$

(b) $\omega = 2\pi f = 4\pi$ rad/s

(c) Using energy conservation

$$\frac{1}{2}kA^2 = \frac{1}{2}kx_0^2 + \frac{1}{2}m v_{0x}^2$$

$x_0 = 5.0$ cm. $v_{0x} = -30$ cm/s and $k = m\omega^2 = (0.2\,kg)(4\pi \text{ rad/s})^2$

we get $A = 5.54$ cm.

(d) $A\cos\phi_0 = x_0 = 5.0$ cm

$$\Rightarrow \phi_0 = \cos^{-1}\left(\frac{5.0\,cm}{5.54\,cm}\right) = 0.445 \text{ rad}$$

(e) The maximum speed $v_{max} = \omega A = (4\pi \text{ rad/s})(5.54\,cm)$

$$= 69.6 \text{ cm/s}$$

(f) $a_{max} = \omega^2 A = \omega(\omega A) = (4\pi \text{ rad/s})(69.6 \text{ cm/s}) = 875 \text{ cm/s}^2$

(g) $E_{total} = \frac{1}{2} m v_{max}^2 = \frac{1}{2}(0.200\,kg)(0.696\,m/s)^2$

$$= 0.0484\,J$$

(h) $x_{0.4\,s} = (5.54\,cm)\cos[(4\pi \text{ rad/s})(0.40\,s) + 0.445 \text{ rad}]$

$$= +3.81 \text{ cm}$$

16.) (a) $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.507\,kg}{20\,N/m}} = 1.00\,s$

(b) $\omega = 2\pi/T = 2\pi/1.0\,s = 2\pi$ rad/s

(c) $x_0 = A\cos\phi_0 \Rightarrow 0.05\,m = 0.10\,m\cos\phi_0 \Rightarrow \phi_0 = \pm\frac{1}{3}\pi$ rad

Because the mass is moving to the right at $t = 0\,s$, it's in the lower half of the circular motion diagram. Hence, $\phi_0 = -\frac{1}{3}\pi$ rad.

(d) At $t = 0\,s$, $v_x(t) = -(0.10\,m)(2\pi \text{ rad/s})\sin(2\pi t + \phi_0)$

becomes $(-0.10\,m)(2\pi \text{ rad/s})\sin(-\frac{1}{3}\pi \text{ rad}) = 0.544$ m/s

(e) $v_{max} = \omega A = (2\pi \text{ rad/s})(0.10\,m) = 0.628$ m/s

(g) $t = 1.3\,s$ $\quad x_{1.3\,s} = (0.10\,m)\cos[2\pi(1.3\,s) - \frac{1}{3}\pi \text{ rad}] = 0.0669\,m$

(h) $t = 1.3\,s$ $\quad v_{(1.3\,s)} = -(0.10\,m)(2\pi \text{ rad/s})\sin[(2\pi \text{ rad/s})(1.3\,s) - \frac{1}{3}\pi \text{ rad}]$

$$= -0.467 \text{ m/s}$$

22).

$$T_0 = 2\pi\sqrt{\frac{L_0}{g}} = 4.0\,s$$

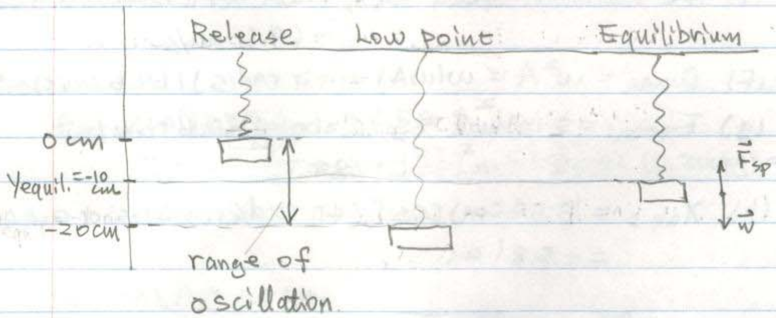
(a) $T = T_0 = 4.0\,s.$

(b) $L = 2L_0.\quad T = 2\pi\sqrt{\frac{2L_0}{g}} = \sqrt{2}\,T_0 = 5.66\,s$

(c) $L = L_0/2.\quad T = 2\pi\sqrt{\frac{L_0/2}{g}} = \frac{1}{\sqrt{2}}\,T_0 = 2.83\,s$

(d) $T = 4.0\,s.$

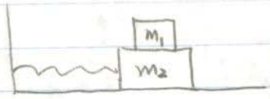
50).



The equilibrium point is the point where the mass would hang at rest, with $F_{sp} = w = mg$. At the equilibrium point, the spring is stretched by $\Delta y = 10\,cm = 0.1\,m$. Hooke's law is $F_{sp} = K\Delta y$, so the equilibrium condition is $[F_{sp} = K\Delta y] = [w = mg] \Rightarrow \frac{k}{m} = \frac{g}{\Delta y} = \frac{9.8\,m/s^2}{0.1\,m} = 98\,s^{-2}$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{98\,s^{-2}} = 1.58\,Hz$$

54).



The net force acting on the upper block m_1 is the force of friction due to the lower block m_2. The model of static friction gives the maximum force of static friction as

$f_{s\,max} = \mu_s n = \mu_s (m_1 g) = m_1 a_{max} \Rightarrow a_{max} = \mu_s g.$

$\mu_s = 0.5,\ a_{max} = \mu_s g = (0.5)(9.8\ m/s^2) = 4.9\ m/s^2.$

Two blocks will ride together if the maximum acceleration of the system is equal to or less than a_{max}.

$$a_{max} = \omega^2 A_{max} = \frac{k}{m_1+m_2} A_{max} \Rightarrow A_{max} = \frac{a_{max}(m_1+m_2)}{k}$$

$$= \frac{(4.9\ m/s^2)(1.0\ kg + 5.0\ kg)}{50\ N/m} = 0.588\ m$$

56). (a) $T = 2\pi\sqrt{\frac{L}{g}} \Rightarrow L = g\left(\frac{T}{2\pi}\right)^2 = (9.8\ m/s^2)\left(\frac{5.5\ s}{2\pi}\right)^2 = 7.51\ m$

(b) Conservation of energy:

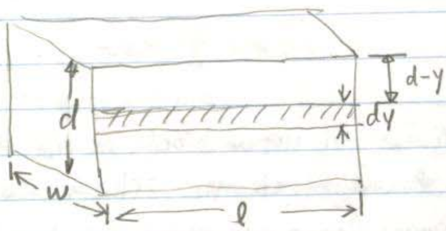
$$\frac{1}{2}mv^2 = mgh \Rightarrow V_{max} = \sqrt{2gh} = \sqrt{2gL(1-\cos 3^\circ)}$$

$$= \sqrt{2(9.8\ m/s^2)(7.51\ m)(1-\cos 3^\circ)}$$

$$= 0.449\ m/s$$

Chapter 15.

44)



(a) $F_{bottom} = P_{bottom}(lw) = (\rho_{water} g d)(lw)$

$= (1000\ kg/m^3)(9.8\ m/s^2)(0.40m)(1.0m)(0.35m)$

$= 1370\ N$

(b) $F_{total} = \int dF = \int_0^d \rho_{water}\, g L (d-y)\, dy$

$= \rho_{water}\, g l (yd - \frac{1}{2}y^2)\Big|_0^d$

$= \frac{1}{2}\rho_{water}\, g l d^2$

$= \frac{1}{2}(1000\ kg/m^3)(9.8\ m/s^2)(1.00m)(0.40m)^2$

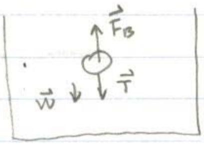
$= 784\ N$

46). Neutral buoyancy implies static equilirium, which will happen when $\rho_{avg} = \rho_w$. This means

$$\frac{m_f + m_a}{V_f + V_a} = \frac{\rho_f V_f + \rho_a V_a}{V_f + V_a} = \rho_w$$

$$\Rightarrow \frac{V_a}{V_f} = \frac{\rho_w - \rho_f}{\rho_a - \rho_w} = \frac{1000\,kg/m^3 - 1080\,kg/m^3}{1.28\,kg/m^3 - 1000\,kg/m^3} = 8.0\%$$

48).



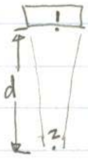
$$\sum F_y = F_B - T - W = 0 \Rightarrow F_B = T + W = \frac{4}{3} W$$

$$\Rightarrow \rho_w V_{sphere}\, g = \frac{4}{3} \rho_{sphere} V_{sphere}\, g$$

$$\Rightarrow \rho_{sphere} = \frac{3}{4} \rho_w = \frac{3}{4}(1000\ kg/m^3)$$

$$= 750\ kg/m^3$$

62).



$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

$$\Rightarrow \rho g d = \frac{1}{2}\rho (v_2^2 - v_1^2) \Rightarrow d = \frac{1}{2g}(v_2^2 - v_1^2)$$

$$v_1 A_1 = v_2 A_2 \Rightarrow v_2 = \frac{v_1 A_1}{A_2} = 2.56\ m/s$$

$$d = \frac{1}{2g}(v_2^2 - v_1^2) = 0.283\ m$$