19.59. Model: Process 1 $\rightarrow$ 2 of the cycle is isochoric, process 2 $\rightarrow$ 3 is isothermal, and process 3 $\rightarrow$ 1 is isobaric. For a monatomic gas, $C_V = \frac{3}{2}R$ and $C_P = \frac{5}{2}$ R.

Visualize: Please refer to Figure P19.59.

Solve: **(a)** At point 1: The pressure $p_1 = 1.0 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$ and the volume $V_1 = 1000 \times 10^{-6} \text{ m}^3 = 1.0 \times 10^{-3} \text{ m}^3$. The number of moles is

$$n = \frac{0.120 \text{ g}}{4 \text{ g/mol}} = 0.030 \text{ mol}$$

Using the ideal-gas law,

$$T_1 = \frac{p_1 V_1}{nR} = \frac{(1.013 \times 10^5 \text{ Pa})(1.0 \times 10^{-3} \text{ m}^3)}{(0.030 \text{ mol})(8.31 \text{ J/mol K})} = 406 \text{ K}$$

At point 2: The pressure $p_2 = 5.0 \text{ atm} = 5.06 \times 10^5 \text{ Pa}$ and $V_2 = 1.0 \times 10^{-3} \text{ m}^3$. The temperature is

$$T_2 = \frac{p_2 V_2}{nR} = \frac{(5.06 \times 10^5 \text{ Pa})(1.0 \times 10^{-3} \text{ m}^3)}{(0.030 \text{ mol})(8.31 \text{ J/mol K})} = 2030 \text{ K}$$

At point 3: The pressure is $p_3 = 1.0 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$ and the temperature is $T_3 = T_2 = 2030 \text{ K}$. The volume is

$$V_3 = V_2 \frac{p_2}{p_3} = (1.0 \times 10^{-3} \text{ m}^3)\left(\frac{5 \text{ atm}}{1 \text{ atm}}\right) = 5.0 \times 10^{-3} \text{ m}^3$$

(b) For isochoric process 1 $\rightarrow$ 2, $W_{1\rightarrow 2} = 0$ J and

$$Q_{1\rightarrow 2} = nC_V \Delta T = (0.030 \text{ mol})\left(\tfrac{3}{2}R\right)(2030 \text{ K} - 406 \text{ K}) = 607 \text{ J}$$

For isothermal process 2 $\rightarrow$ 3, $\Delta E_{\text{th } 2\rightarrow 3} = 0 \text{ J}$ and

$$Q_{2\rightarrow 3} = W_{2\rightarrow 3} = nRT_2 \ln\frac{V_3}{V_2} = (0.030 \text{ mol})(8.31 \text{ J/mol K})(2030 \text{ K})\ln\left(\frac{5.0 \times 10^{-3} \text{ m}^3}{1.0 \times 10^{-3} \text{ m}^3}\right) = 815 \text{ J}$$

For isobaric process 3 $\rightarrow$ 1,

$$W_{3\rightarrow 1} = p_3 \Delta V = (1.013 \times 10^5 \text{ Pa})(1.0 \times 10^{-3} \text{ m}^3 - 5.0 \times 10^{-3} \text{ m}^3) = -405 \text{ J}$$

$$Q_{3\rightarrow 1} = nC_P \Delta T = (0.030 \text{ mol})\left(\tfrac{5}{2}\right)(8.31 \text{ J/mol K})(406 \text{ K} - 2030 \text{ K}) = -1012 \text{ J}$$

The total work done is $W_{\text{net}} = W_{1\rightarrow 2} + W_{2\rightarrow 3} + W_{3\rightarrow 1} = 410$ J. The total heat input is $Q_H = Q_{1\rightarrow 2} + Q_{2\rightarrow 3} = 1422 \text{ J}$. The efficiency of the engine is

$$\eta = \frac{W_{\text{net}}}{Q_H} = \frac{410 \text{ J}}{1422 \text{ J}} = 28.8\%$$

(c) The maximum possible efficiency of a heat engine that operates between T_{max} and T_{min} is

$$\eta_{\text{max}} = 1 - \frac{T_{\text{min}}}{T_{\text{max}}} = 1 - \frac{406 \text{ K}}{2030 \text{ K}} = 80\%$$

Assess: The actual efficiency of an engine is less than the maximum possible efficiency.