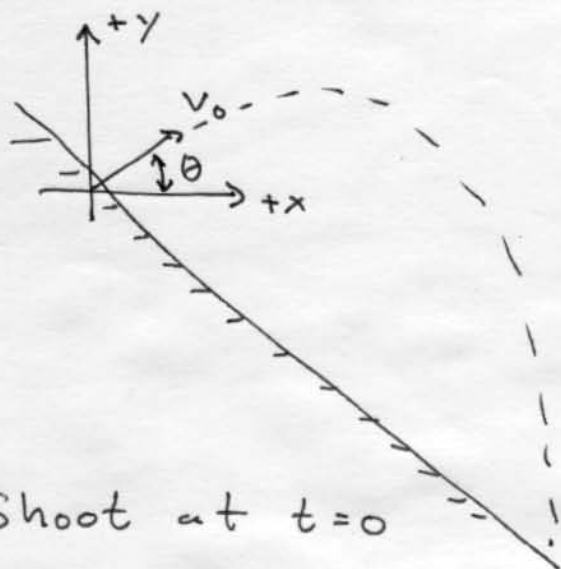


Problem 1



Shoot at $t=0$

$$x = v_0 \cos \theta \cdot t$$

$$y = v_0 \sin \theta \cdot t - \frac{1}{2} g t^2$$

} position of cannonball

Condition to land on slope: $y = -x$

$$v_0 \sin \theta \cdot t - \frac{1}{2} g t^2 = -v_0 \cos \theta \cdot t$$

Discard trivial solution $t=0 \Rightarrow$

$$v_0 \sin \theta - \frac{1}{2} g t = -v_0 \cos \theta$$

$$\frac{1}{2} g t = v_0 (\sin \theta + \cos \theta)$$

$$t = \frac{2 v_0}{g} (\sin \theta + \cos \theta)$$

To maximize time, maximize

$$f(\theta) = \sin \theta + \cos \theta$$

$$0 = f'(\theta) = \cos \theta - \sin \theta \Rightarrow \theta = 45^\circ$$

$$f''(\theta) = -\sin \theta - \cos \theta < 0 \text{ at } \theta < 45^\circ \quad \square$$