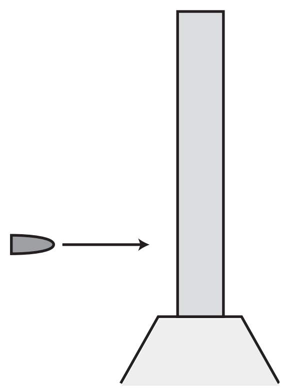
Homework #11

Due Monday, May 9

1. A beam of length 45 cm and mass 440 g is balanced vertically on a small platform as shown below. A bullet of mass 7.8 g is fired at the stick with a horizontal velocity of 730 m/s. The bullet hits the beam 15 cm above the bottom of the beam and then becomes embedded into the beam.

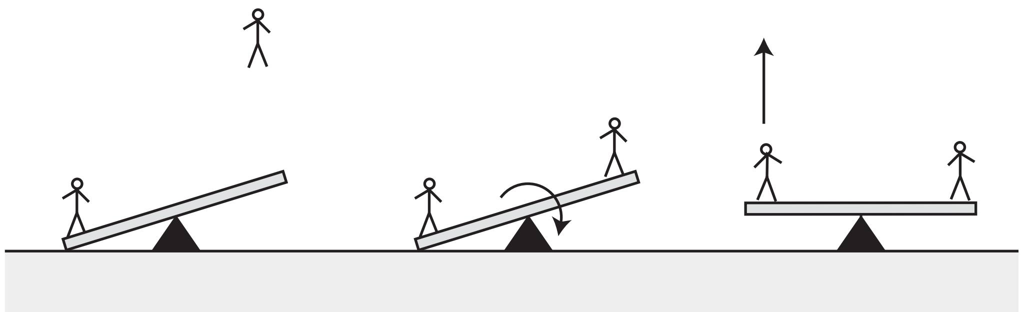


(a) The subsequent motion of the beam and bullet is a combination of translation of the center of mass and rotation about the center of mass. Sketch the subsequent trajectory of the beam and bullet.

(b) Find the velocity of the center of mass of the beam and bullet just after the collision.

(c) Find the angular velocity of the beam and bullet about the center of mass just after the collision.

2. Two circus performers perform the stunt shown below. The Great Garbanzo, who has a mass of 75 kg, drops from a vertical height of 2.5 m onto a plank. His partner Lorenzo, who has a mass of 48 kg, is standing on the other side. The plank has a mass of 14 kg and a length of 3.3 m, and the height of the support is 0.53 m. This launches Lorenzo straight into the air when the plank is horizontal, as shown. Assume that the plank pivots about the support.



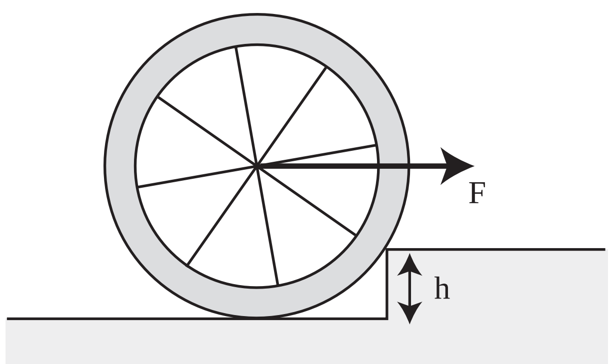
(a) What is the velocity of the Great Garbanzo just after he hits the plank? What is the rotational velocity of the plank about the support just after he hits the plank?

(b) What is the rotational speed of the plank when it is horizontal?

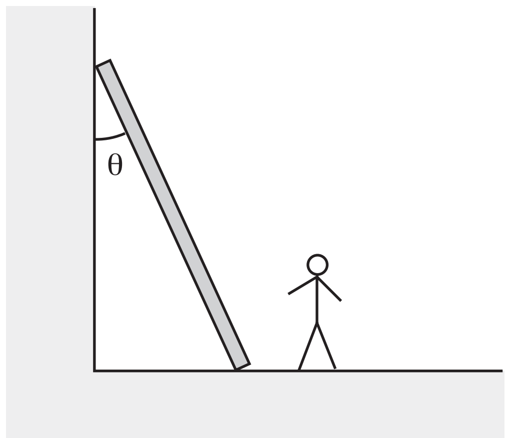
(c) Assume Lorenzo is launched straight into the air with the same velocity that he has when the plank is horizontal. How high does he go?

(d) Is mechanical energy conserved in this process? If not, how much mechanical energy was lost and where did it go?

3. A wheel of mass M and radius R is up against a barrier of height h, where $h < R$. What horizontal force F must be applied to the axle of the wheel so that it can make it over the barrier? Neglect the mass of the spokes of the wheel and axle, so that the moment of inertia of the wheel about the axle is MR^2.



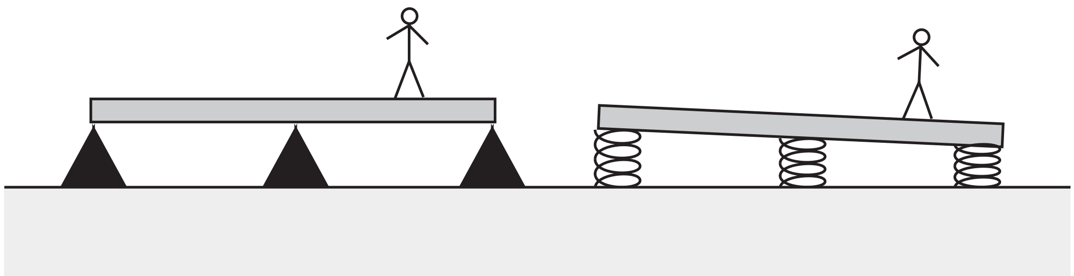
4. A ladder of length d and mass M is placed against a wall at an angle θ as shown below. A man of mass m climbs up the ladder. Neglect friction between the ladder and the wall, but assume that the coefficients of friction between the floor and the ladder μ_s and μ_k are nonzero.



(a) Draw a free-body diagram for the ladder with the man standing on it. Use the free-body diagram to show which point on the ladder it is most dangerous to stand.

(b) Find an expression for the minimum coefficient of static friction that is required so that the ladder does not slip.

5. A man of mass 65 kg is standing on a plank of mass 12 kg and length 2.3 m. The plank is supported by three rigid supports at the ends and the middle, and the man stands $\frac{3}{4}$ of the way to the right end of the plank, as shown at left.



(a) Show that if the supports are treated as perfectly rigid, the equilibrium conditions do not determine the values of the forces that the supports exert on the beam. Give two different examples of forces that would satisfy the equilibrium conditions that you might think are reasonable.

(b) In the real world, the supports are not perfectly rigid. Model them by springs with spring constant k and assume that the plank tips at an angle when the man stands on it. Find the angle and the forces exerted by the springs. You may use the small angle approximation. Now take the limit $k \to \infty$ to find the forces exerted by the rigid supports.