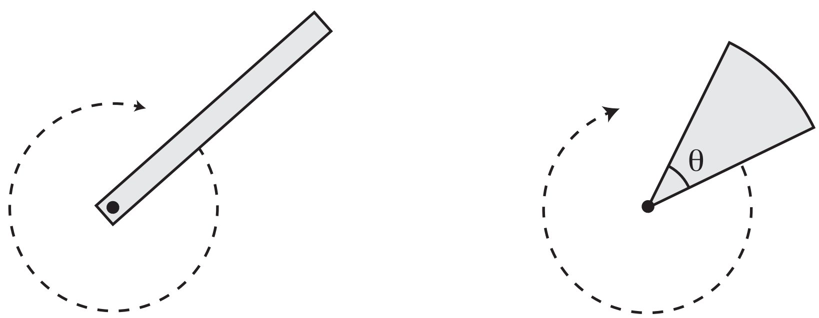
Homework #10

Due Monday, May 2

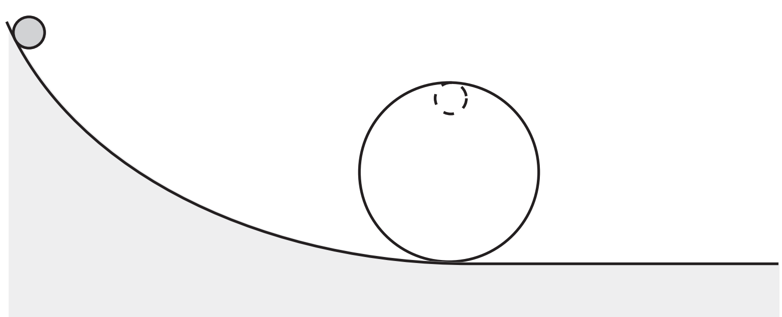
1. Compute the moment of inertia of the objects below.



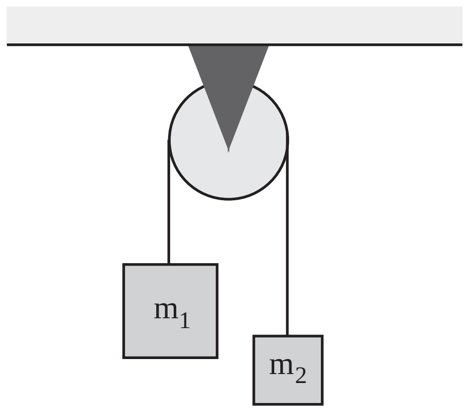
(a) Compute the moment of inertia of a thin rod of mass M and length R for rotations about its end. Explain why your result is not the same as the result for rotations about the center of the rod.

(b) Compute the moment of inertia of a thin sheet cut into the shape of a wedge of pie with radius R, opening angle θ, and total mass M, for rotations about the corner of the wedge. Your result should differ from the moment of inertia calculated in part (a), even in the limit $\theta \to 0$. Your result should be the same as one of the other shapes in table 11-2 in your book. Explain both of these facts.

2. A solid spherical marble is released from rest on a track with a "loop the loop" of radius R. At what height must the marble be released in order to make it around the loop without falling off? Assume that the marble rolls without slipping, and assume that the size of the marble can be neglected compared to the radius R.



3. *Return of the dreaded Atwood machine.* An Atwood machine consists of two masses m_1 and m_2 and a pulley shaped like a solid cylinder of mass M and radius R.



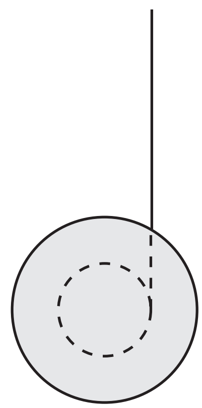
(a) Draw a free-body diagram for each of the masses and the pulley. For the pulley, which forces give rise to nonzero torques?

(b) Find the acceleration of m_1 using Newton's second law for translation and rotation.

(c) Use energy conservation to find the velocity of m_1 after it has moved a vertical distance h.

(d) Find the acceleration of m_1 from your result from part (c). Does it agree with your result of part (b)?

4. A yo-yo consists of three solid cylinders with the same uniform mass density. The two outer cylinders have a radius R and the inner cylinder has a radius $\frac{1}{2}R$. The total mass of the yo-yo is M. A string is wound around the inner cylinder and the yo-yo is released from rest with the string hanging vertically.



(a) Find the moment of inertia of the yo-yo in terms of M and R.

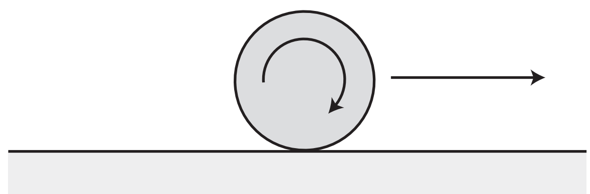
(b) Draw a free-body diagram for the yo-yo. Which forces give rise to nonzero torques?

(c) Find the acceleration of the yo-yo after it is released using using a free-body diagram and Newton's second law for rotation and translation.

(d) Use energy conservation to find the speed of the yo-yo after it has dropped a vertical distance h.

(e) Find the acceleration of the yo-yo from your result from part (d). Does it agree with your result of part (c)?

5. *The dreaded bowling ball problem.* Just after a bowler releases a bowling ball of radius 0.12 m and mass 1.2 kg, it is sliding without rolling at a speed 0.65 m/s. The coefficients of friction between the bowling ball and the ground are $\mu_k = 0.15$ and $\mu_s = 0.20$. As the bowling ball moves, friction gradually makes it rotate, until it is rolling without slipping.



(a) Draw a free-body diagram for the bowling ball, showing all forces and where they act on the ball. Which direction does friction act? Which of the forces give rise to nonzero torques about the center of mass?

(b) Find the acceleration of the bowling ball during the time that it is still slipping.

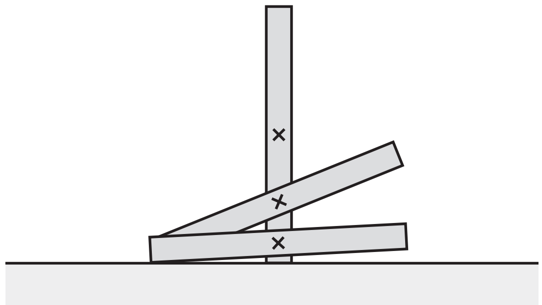
(c) Find the angular acceleration of the bowling ball during the time that it is still slipping.

(d) How long does it take for the bowling ball to spin fast enough to roll without slipping?

(e) What is the change in the total kinetic energy (translational and rotational) from the time the ball is released to the point where it starts rolling without sliding?

(f) Find the work done by friction during the time the ball was slipping. Does the work done by friction account for the change in kinetic energy found in part (e)?

6. *The dreaded falling stick problem.* A stick of mass M and length L falls from the vertical position at rest on a frictionless surface. We showed in class that the center of mass of the stick falls straight down.



(a) Draw a free-body diagram of the stick during its fall. Which of the forces on the stick do nonzero work?

(b) Just before the stick hits the ground, it can be thought of as rotating about the end that is on the ground. Use this to find the speed at which the center of mass hits the ground.