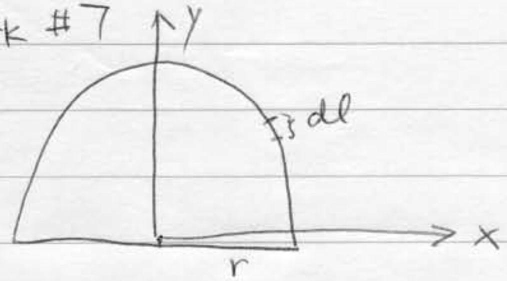
Homework #7

1.



total length: $(2+\pi)r$

mass per unit length: $\frac{m}{(2+\pi)r}$

$$\vec{r}_{CM} = \int \rho \vec{r}\, dv / m$$

$$= \frac{1}{m}\left[\int_{-r}^{r} \frac{m}{(2+\pi)r}\, x\, \hat{e}_x\, dx + \int_{-r}^{r} \frac{m}{(2+\pi)r} \left(x\,\hat{e}_x + \sqrt{r^2-x^2}\,\hat{e}_y\right) dl \right.$$

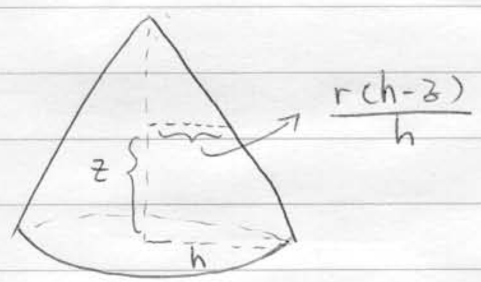
$$dl = r\,d\theta = \frac{r}{\sqrt{r^2-x^2}}\, dx$$

$$\vec{r}_{CM} = \frac{1}{m}\cdot\frac{m}{(2+\pi)r}\left[\underbrace{\int_{-r}^{r} x\,\hat{e}_x\, dx}_{0} + \underbrace{\int_{-r}^{r} \frac{x r \hat{e}_x}{\sqrt{r^2-x^2}}\, dx}_{0} + \int_{-r}^{r} r\,\hat{e}_y\, dx \right]$$

integrands are odd function

$$= \frac{1}{(2+\pi)r}\int_{-r}^{r} r\,\hat{e}_y\, dx = \frac{2r}{(2+\pi)}\,\hat{e}_y$$

2.



(a). Volume
$$V = \int_0^h dz \int_0^{2\pi} d\varphi \int_0^{\frac{r(h-z)}{h}} d\rho\, \rho$$
$$= 2\pi \int_0^h dz \int_0^{\frac{r(h-z)}{h}} d\rho\, \rho$$
$$= \pi \int_0^h \frac{r^2(h-z)^2}{h^2}\, dz = \frac{\pi r^2 h}{3}$$

(b)
$$\vec{r}_{CM} = \int \rho \vec{r}\, dv / m = \int \vec{r}\, dv / V$$
$$= \int_0^h dz \int_0^{2\pi} d\varphi \int_0^{\frac{r(h-z)}{h}} d\rho\, \rho \left(\rho\cos\varphi\,\hat{e}_x + \rho\sin\varphi\,\hat{e}_y + z\,\hat{e}_z\right) / V$$
$$= \int_0^h dz \int_0^{\frac{r(h-z)}{h}} d\rho\, \rho\, (0 + 0 + 2\pi z\,\hat{e}_z) / V$$
$$= \int_0^h z\, dz \cdot 2\pi \cdot \frac{1}{2} \cdot \frac{r^2(h-z)^2}{h^2} \Big/ V$$
$$= \frac{\pi r^2}{h^2 V}\int_0^h z(h-z)^2\, dz = \frac{\pi r^2}{h^2 \cdot \frac{\pi r^2 h}{3}} \int_0^h z(h^2+z^2-2hz)\, dz = \frac{h}{4}$$