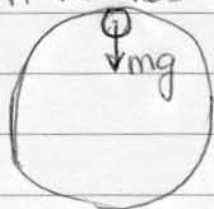


Homework #10

$$\begin{aligned} 1. (a) \quad I &= \sum_i m_i r_i^2 = \int_0^R r^2 dm \\ &= \int_0^R r^2 dr \frac{M}{R} = \frac{M}{R} \int_0^R r^2 dr \\ &= \frac{M}{R} \cdot \frac{R^3}{3} = \frac{MR^2}{3} \end{aligned}$$

$$\begin{aligned} (b) \quad I &= \int_0^R dr \int_0^{2\pi} d\varphi \, r \cdot r^2 \cdot \frac{M}{2R^2} \\ &= \frac{M}{2R^2} \cdot 2\pi \int_0^R r^3 dr = \frac{2M}{R^2} \cdot \frac{R^4}{4} = \frac{MR^2}{2} \end{aligned}$$

2. "In order to make it around the loop without falling off" means when it reaches the top of the circle $N \geq 0$



Suppose it's released at height h :

Energy conservation:

$$mgh = \frac{1}{2}mv^2 + mg \cdot 2R$$

Newton's second law:

$$N + mg = m \frac{v^2}{R}$$

$$\text{and } N \geq 0$$

$$\text{then } h \geq \frac{5}{2}R$$