Physics for Scientists and Engineers

Chapter 11
Power

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System

- A *system* is a small portion of the Universe
  - We will ignore the details of the rest of the Universe
- A critical skill is to identify the system
  - A valid system may be a single object or particle, a collection of objects or particles, or a region of space
- Does the problem require the system approach, or can it be solved by the particle approach?
  - What is the particular system and what is its nature?

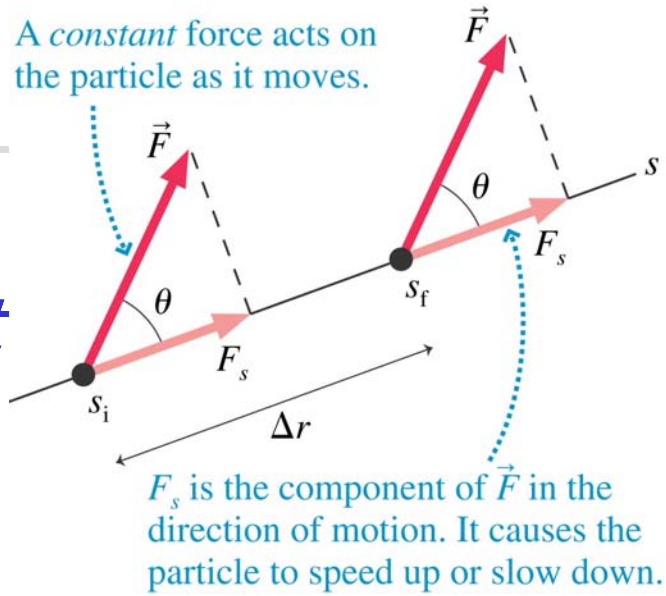
Environment

- There is a *system boundary* around the system
  - The boundary is an imaginary surface
  - It does not necessarily correspond to a physical boundary
- The boundary divides the system from the *environment*
  - The environment is the rest of the Universe

Work

- The work, *W*, done *on* a system *by* an agent exerting a *constant* force on the system is

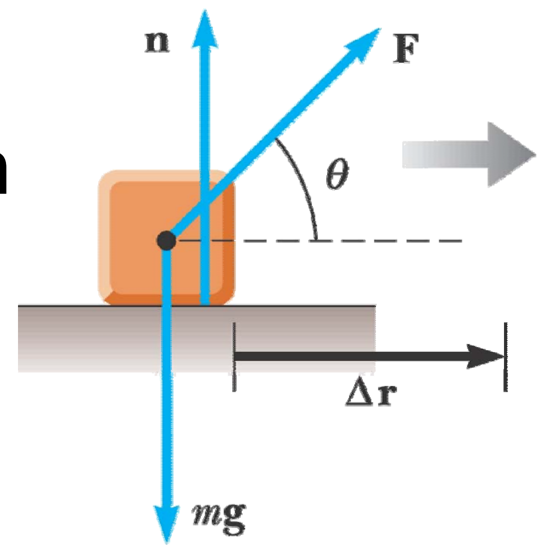
$$W = F\Delta r\cos\theta$$

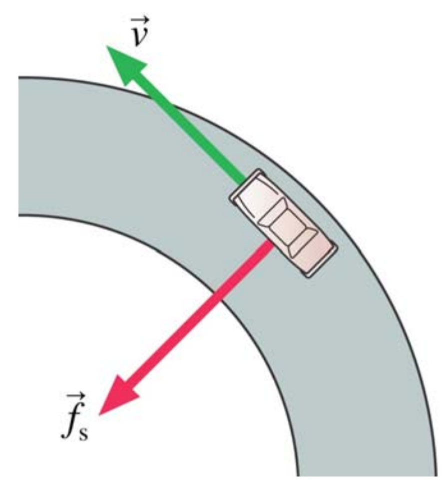


  - Only the component of the applied force *parallel* to Δ**r** does work
  - A force does no work on the object if there is no displacement
  - The work done by a force on a moving object is *zero* when the force is *perpendicular* to the displacement

Examples of Work

- The normal force **n** and the gravitational force $m\mathbf{g}$ do no work on the object moving along the table
  - $\cos\theta = \cos 90° = 0$
- The force **F** does do work on the object equal to $F\Delta r\cos\theta$
- The friction force $\mathbf{f}_s$ on a curve does no work since it is perpendicular to the instantaneous displacement



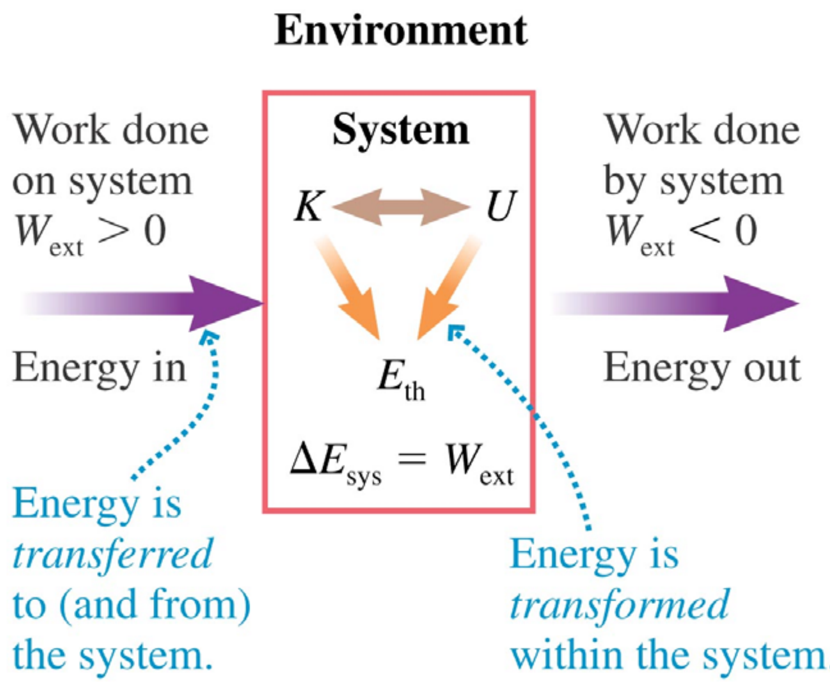


More About Work

- The system and the environment must be determined when dealing with work
  - Work *by the environment on the system* or work done *by the system on the environment* ?
- The sign of the work depends on the direction of **F** relative to Δ**r**
- Work is a scalar quantity
- The unit of work is the joule (J), the same as energy: 1 J = 1 N·m

Work Is an Energy Transfer

- If a system interacts with its environment, this interaction can be described as a *transfer of energy across* the system boundary
  - If the work is done on a system and it is *positive*, energy is transferred *to* the system
  - If the work done on the system is *negative*, energy is transferred *from* the system

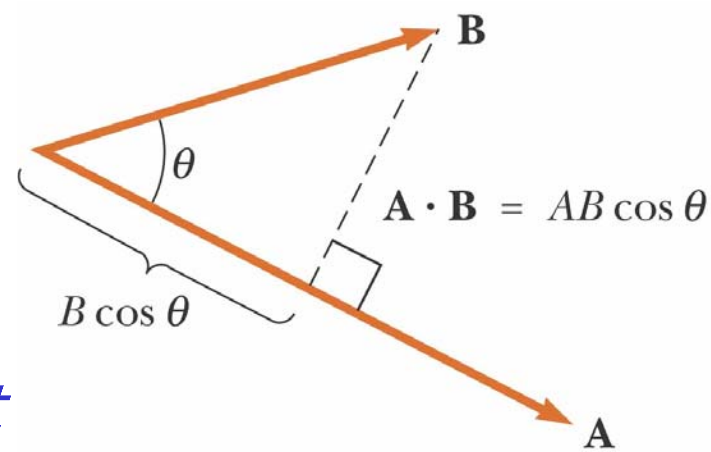


Scalar Product of Two Vectors

- The *scalar product* of two vectors is defined by

$$\mathbf{A}\cdot\mathbf{B} = A\,B\cos\,\theta$$

  - It is also called the *dot product*
  - Work $= \mathbf{F}\cdot\Delta\mathbf{r}$



- The scalar product is *commutative*: $\mathbf{A}\cdot\mathbf{B} = \mathbf{B}\cdot\mathbf{A}$
- The scalar product obeys the *distributive* law of multiplication: $\mathbf{A}\cdot(\mathbf{B}+\mathbf{C}) = \mathbf{A}\cdot\mathbf{B} + \mathbf{A}\cdot\mathbf{C}$

More on Scalar Products

- Dot products of unit vectors are:

$$\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1,\quad \hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$$

- Using component form with **A** and **B**:

$$\mathbf{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k},\quad \mathbf{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

$$\mathbf{A}\cdot\mathbf{B} = A_xB_x + A_yB_y + A_zB_z$$

- The dot product with itself is the magnitude of the vector squared

$$\mathbf{A}\cdot\mathbf{A} = A_x^2 + A_y^2 + A_z^2,\quad \left|\mathbf{A}\right| = \sqrt{A_x^2 + A_y^2 + A_z^2} = \sqrt{\mathbf{A}\cdot\mathbf{A}}$$

Work Done by a Varying Force

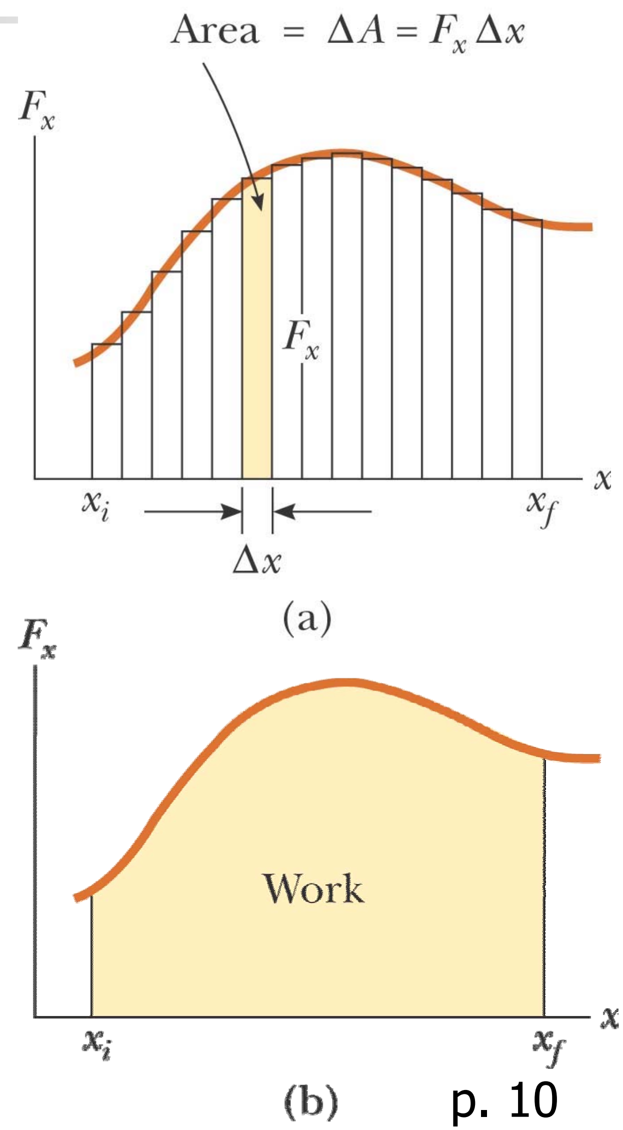
- For a very small displacement Δx, F_x is constant. So the work done for that displacement is:

$$\Delta W = F_x \Delta x = \vec{F} \cdot \Delta \vec{x}$$

- For all of the intervals,

$$W = \lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F_x \Delta x = \int_{x_i}^{x_f} \vec{F} \cdot d\vec{x}$$

- The work done is equal to the area under the F-x curve



Work Done By Multiple Forces

- If more than one force acts on a system, the *total* work done on the system is the work done by the *net* force

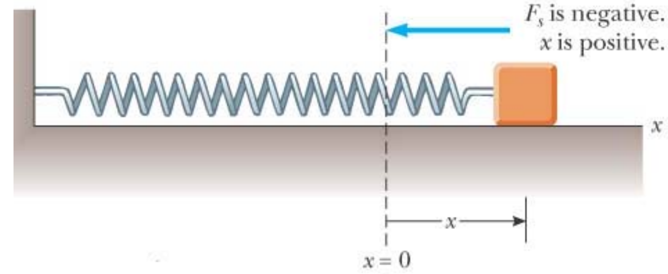
$$\sum W = W_{net} = \int_{x_i}^{x_f} \left(\sum F_x\right) dx$$

- If the system cannot be modeled as a particle, then the total work is equal to the *algebraic sum* of the work done by the individual forces on different parts of the system

$$W_{\text{net}} = \sum W_{\text{by individual forces}}$$

Work Done by a Spring

- Identify the block as the system

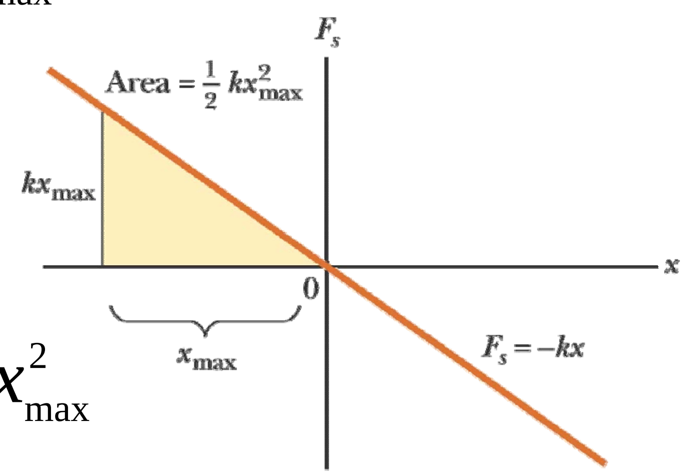


- Calculate the work as the block moves from $x_i = -x_{max}$ to $x_f = 0$

$$W_s = \int_{x_i}^{x_f} F_s dx = \int_{-x_{max}}^{0} (-kx) dx = \frac{1}{2} kx_{max}^2$$

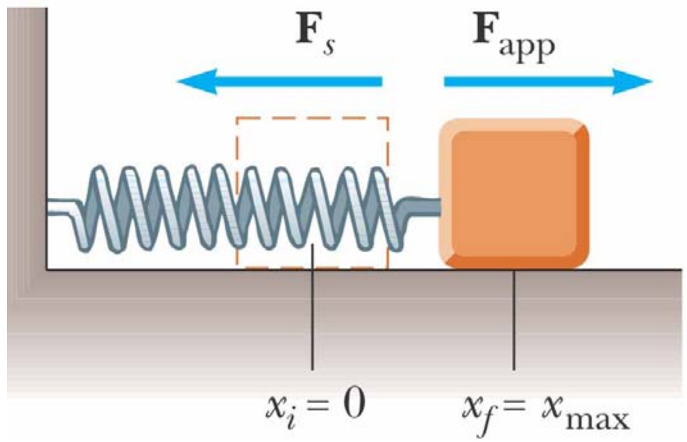
- Work done as the block moves from $x_i = 0$ to $x_f = +x_{max}$ is

$$W_s = \int_{x_i}^{x_f} F_s dx = \int_{0}^{x_{max}} (-kx) dx = -\frac{1}{2} kx_{max}^2$$



Work Done by an Applied Force

- Suppose an *external* agent, F_{app}, slowly stretches the spring so that $a \sim 0$
- The applied force is equal and opposite to the spring force:

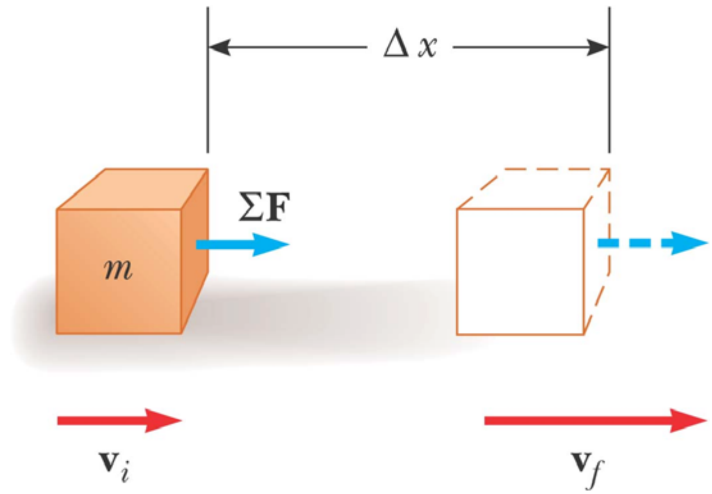


$$F_{app} = -F_s = -(-kx) = kx$$

- Work done by F_{app} is equal to $\frac{1}{2}kx^2_{max}$

Work and Kinetic Energy

- A change in kinetic energy is the result of doing work to transfer energy into a system



- Calculating the work:

$$\sum W = \int_{x_i}^{x_f} \sum F dx = \int_{x_i}^{x_f} m \frac{dv}{dt} dx = \int_{x_i}^{x_f} m \frac{dv}{dx}\frac{dx}{dt} dx = \int_{v_i}^{v_f} mv dv$$

$$\sum W = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

Work-Kinetic Energy Theorem

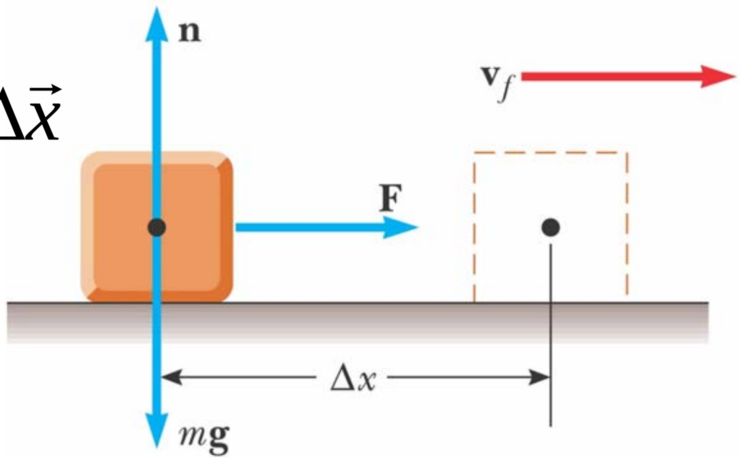
- In the case work is done on a system by one or more forces, the *net work done equals the change in kinetic energy* of the system:

$$\sum W = \Delta K = K_f - K_i$$

- Example:

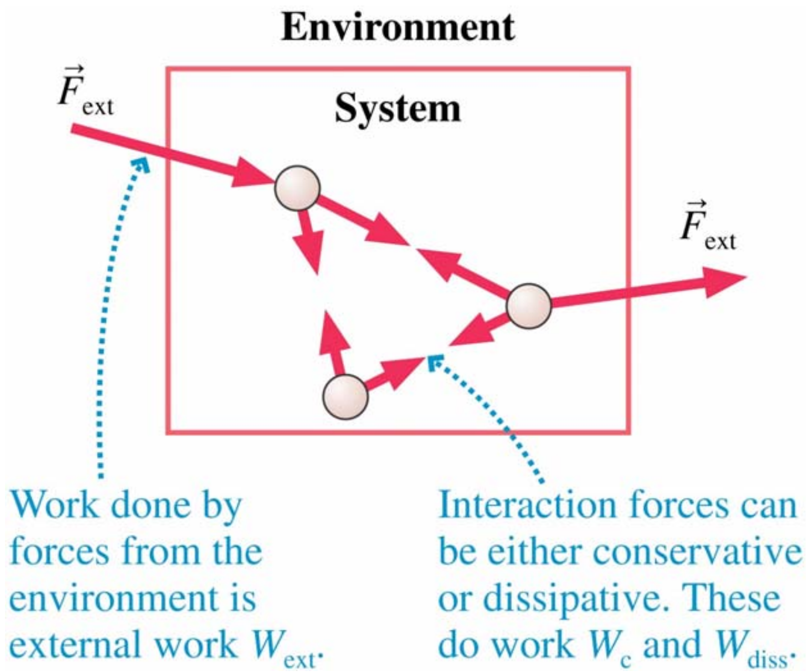
$$\sum W = \sum \vec{F} \cdot \Delta\vec{x} = \left(\vec{n} + m\vec{g} + \vec{F}\right) \cdot \Delta\vec{x}$$

$$= \frac{1}{2} m v_f^2 - 0 = \Delta K$$



Non-isolated System

- A *non-isolated system* is one that interacts with or is influenced by its environment, such as gravity
  - An *isolated system* would not interact with its environment

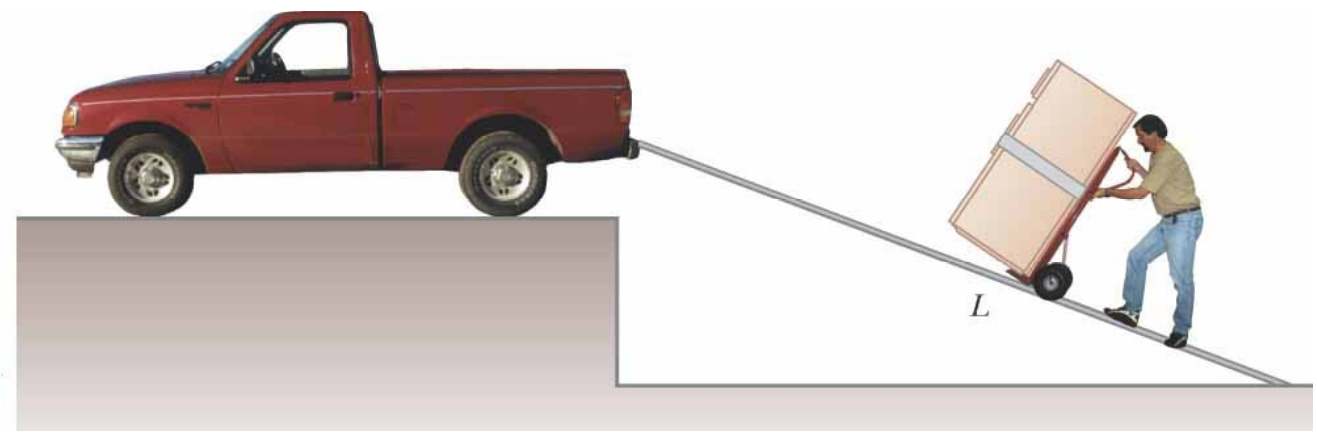


- The Work-Kinetic Energy Theorem can be applied to non-isolated systems

Example 1: Man and Gravity

A man is sliding a box up the ramp. The man and box are a system. Gravity is opposing the man's effort.

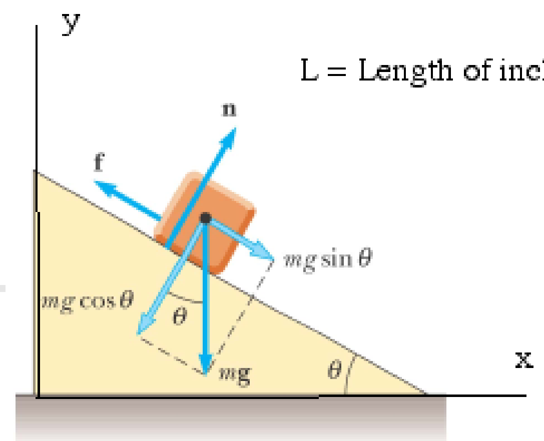
Find the work done by the man and by gravity.



According to the Work - Kinetic Energy Theorem,

$$\sum W = W_{man} + W_{gravity} = \frac{1}{2}Mv_f^2 - \frac{1}{2}Mv_i^2$$

Example 1, cont



Work done by the man:

$$\vec{f} = -f\cos\theta\,\hat{i} + f\sin\theta\,\hat{j},\ \vec{L} = -L\cos\theta\,\hat{i} + L\sin\theta\,\hat{j}$$

$$W_m = \vec{f}\cdot\vec{L} = (-f\cos\theta\,\hat{i} + f\sin\theta\,\hat{j})\cdot(-L\cos\theta\,\hat{i} + L\sin\theta\,\hat{j})$$

$$= fL(\cos^2\theta + \sin^2\theta) = fL$$

Work done by gravity:

$$W_g = \vec{F}_g\cdot\vec{L} = (-mg\,\hat{j})\cdot(-L\cos\theta\,\hat{i} + L\sin\theta\,\hat{j}) = -mgL\sin\theta$$

$$\sum W = W_m + W_g = fL - mgL\sin\theta = \frac{1}{2}Mv_f^2 - \frac{1}{2}Mv_i^2$$

If the man does the minimum work with $f = mg\sin\theta$, then $W_m = -W_g$ and the net work is zero, and hence $v_f = v_i$.

If $f > mg\sin\theta$, then $v_f > v_i$. If $f < mg\sin\theta$, then $v_f < v_i$.

Example 1, cont

Now consider that the man lifts the box *straight up* the same distance as before, $\Delta\mathbf{r} = L \sin\theta\, \mathbf{j}$, without using the incline.

If the man does the minimum work just to overcome the work done by gravity, how much work does he do?

$$W_m = \vec{f} \cdot \Delta\vec{r} = f\,\hat{j} \cdot (L \sin\theta\,\hat{j}) = fL\sin\theta$$

$$W_g = (-mg\,\hat{j}) \cdot (L\sin\theta\,\hat{j}) = -mgL\sin\theta$$

$$W_m + W_g = 0, \quad fL\sin\theta = mgL\sin\theta$$

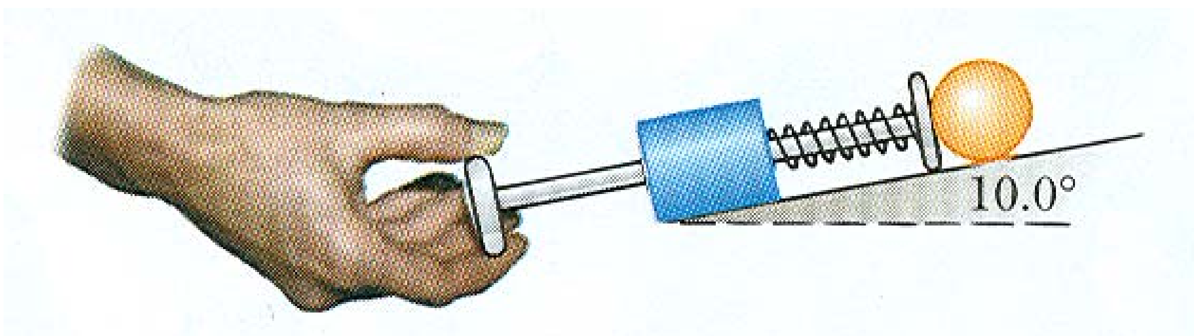
$$f = mg$$

He does the same amount of work but exerts more force than when pushing the box up the incline, i.e. $mg > mg\sin\theta$.

Example 2: Pin Ball Problem

The ball launcher in a pinball machine has a spring that has a force constant of 120 N/m. The surface on which the ball moves is inclined 10.0° with respect to the horizontal.

If the spring is initially compressed 5.00 mm, find the launching speed of a 0.100-kg ball when the plunger is released. Friction and the mass of the plunger are negligible.



Note that the spring force is variable, i.e. it depends on position. The force of gravity is constant.

Example 2, cont

$$s_i = -0.005\ \text{m},\ s_f = 0.000\ \text{m},\ y_i = 0.000\ \text{m},\ y_f = 0.005\ \text{m}\sin 10°$$

$$\sum W = W_{sp} + W_g = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

$$W_{sp} = -\int_{s_i}^{s_f} ks\,ds = \tfrac{1}{2}ks_i^2 - \tfrac{1}{2}ks_f^2 = \tfrac{1}{2}k(x_i^2 + y_i^2)$$

$$W_g = -mg(y_f - y_i) = -mgy_f$$

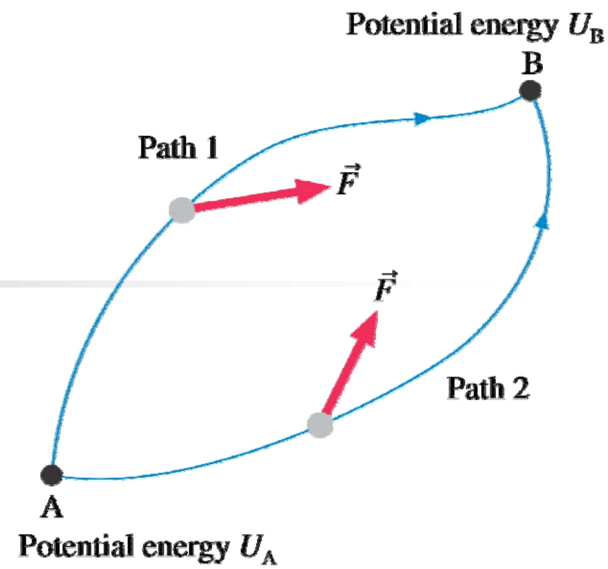
$$\tfrac{1}{2}k(x_i^2 + y_i^2) - mgy_f = \tfrac{1}{2}mv_f^2$$

$$v_f^2 = \frac{k}{m}x_i^2 - 2gy_f = -\frac{120\ \text{N/m}}{0.100\ \text{kg}}(-0.005\ \text{m})^2 - 2(9.80\ \text{m/s}^2)(0.005\ \text{m}\sin 10°)$$

$$= 0.0300\ (\text{m/s})^2 - 0.0170\ (\text{m/s})^2 = 0.0130\ (\text{m/s})^2$$

$$v_f = 0.114\ \text{m/s}$$

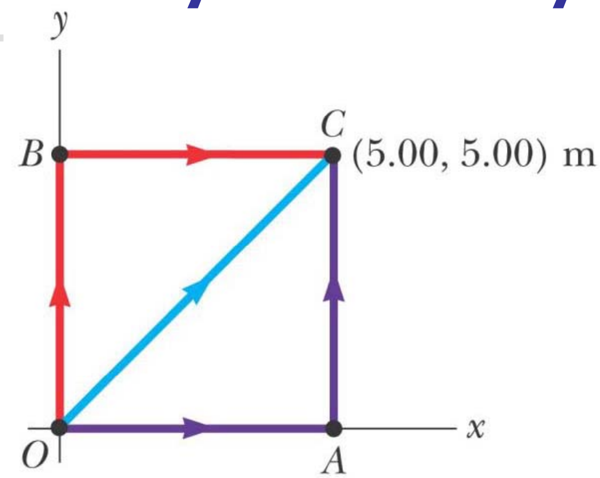
Conservative Forces



- Work done by a *conservative force* on a particle moving between two points is *independent of the path*
  - Gravity and spring forces are conservative forces
  - Work done by a conservative force on a particle moving through any *closed* path is zero
- Work done by *conservative forces* can be related to a *change in potential energy* because the work only depends on the beginning and end points
  - In general, $W_{con} = -\Delta U$

Example 3: Work Done by Gravity

A 4.00-kg particle moves from the origin to position *C*, having coordinates *x* = 5.00 m and *y* = 5.00 m. One force on the particle is the gravitational force acting in the negative *y* direction.



Calculate the work done by the gravitational force in going from *O* to *C* along (a) *OAC*, (b) *OBC*, (c) *OC*.

(a) Work along *OAC*

$$W_{OA} = F_g(x_A - x_O)\cos 90° = 0$$

$$W_{AC} = F_g(y_C - y_A)\cos 180° = -mg(y_C - y_A)$$

$$W_{OAC} = W_{OA} + W_{AC} = 0 - (4.00\,\text{kg})(9.80\,\text{m/s}^2)(5.00\,\text{m}) = -196\,\text{J}$$

Example 3, cont

(b) Work along *OBC*

$$W_{OB} = F_g(y_B - y_O)\cos 180^\circ = -mg(y_B - y_O)$$

$$W_{BC} = F_g(x_C - x_B)\cos 90^\circ = 0$$

$$W_{OBC} = W_{OB} + W_{BC} = -(4.00\text{ kg})(9.80\text{ m/s}^2)(5.00\text{ m}) + 0 = -196\text{ J}$$

(c) Work along *OC*

$$W_{OC} = L_{OC}mg\ \cos 135°,\quad L_{OC} = \sqrt{(5.00\text{ m})^2 + (5.00\text{ m})^2} = 7.07\text{ m}$$

$$W_{OC} = (4.00\text{ kg})(9.80\text{ m/s}^2)(7.07\text{ m})(-0.707) = -196\text{ J}$$

The results are the same for all paths because gravity is a conservative force. Work only depends on the altitudes of the initial and final points. The gravitational potential energies are

$U_i = = mgy_i = 0$ J, $U_f = mgy_f = (4.00\text{ kg})(9.80\text{ m/s}^2)(5.00\text{ m}) = 196$ J

$\Rightarrow W = -(U_f - U_i) = -196$ J

Energy Conservation

- Work-Kinetic Energy Theorem: $\sum W = K_f - K_i$
- For conservative forces, $W = -(U_f - U_i)$
- If *all* the work is due to *conservative* forces,

$$\sum W = -(U_f - U_i)$$

- The Work-Kinetic Energy Theorem can be rewritten as

$$-(U_f - U_i) = K_f - K_i \text{ or } K_i + U_i = K_f + U_f$$

  - Mechanic energy is *conserved*

Force from Potential Energy

- For conservative forces,

$$W = -\vec{F}\cdot\Delta\vec{s} = -\Delta U, \quad \overline{F}_s = -\frac{\Delta U}{\Delta s}$$

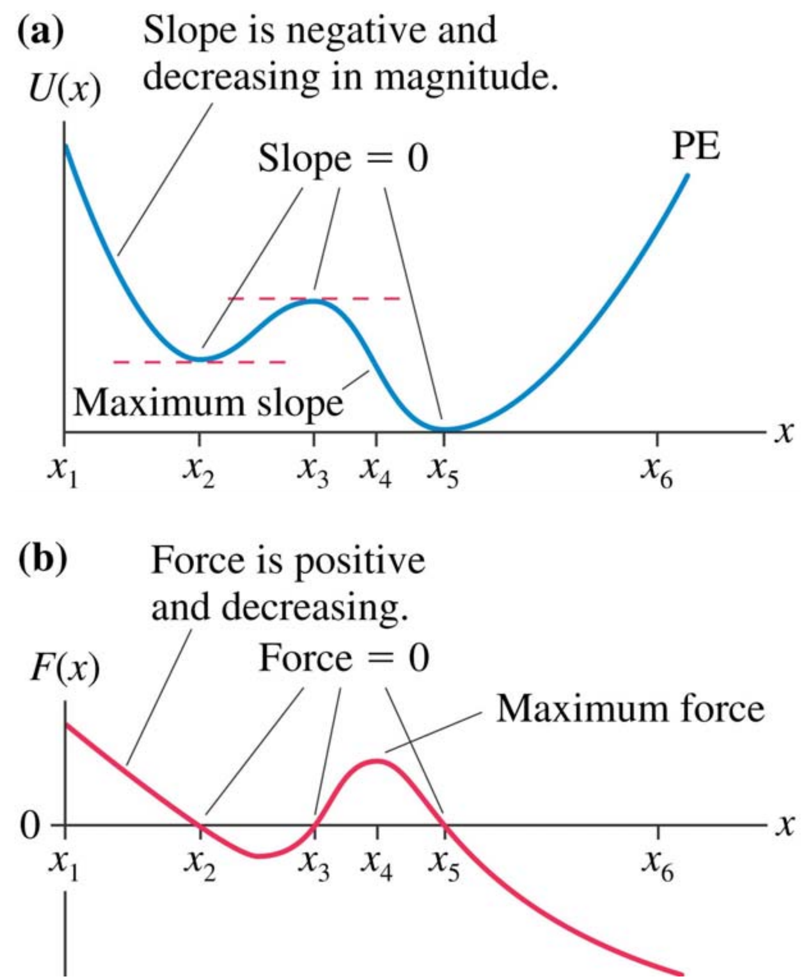
$$F_s = -\lim_{\Delta s\to 0}\frac{\Delta U}{\Delta s} = -\frac{dU}{ds}$$

  - Gravitational force:

$$F_g = -\frac{d}{dy}(mgy) = -mg$$

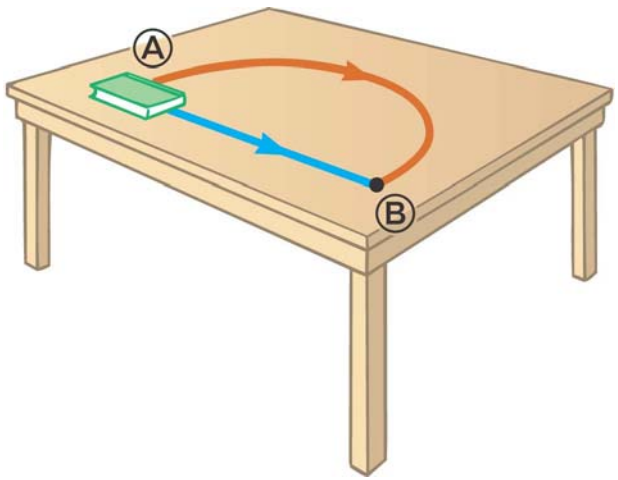
  - Spring force:

$$F_s = -\frac{d}{dx}\left(\frac{1}{2}kx^2\right) = -kx$$



Nonconservative Forces

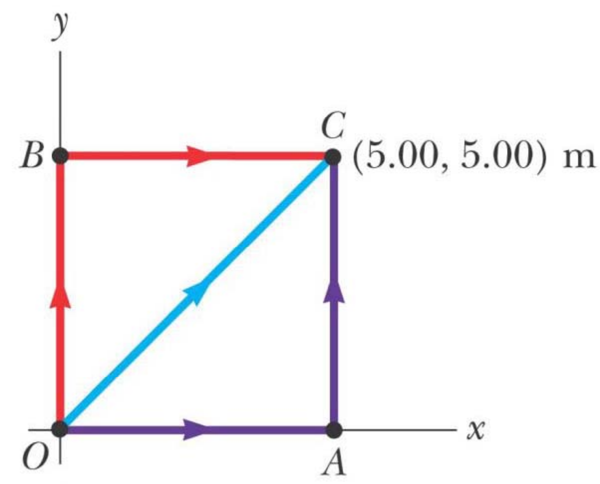
- A *nonconservative force depends on the path* taken, not on the end points alone
- Nonconservative forces acting in a system cause a *change* in the mechanical energy of the system
- Example: The work done against *friction* is greater along the red path than along the blue path
  - Friction is a *nonconservative force*
  - Mechanical energy is *not conserved*



Example 4: Work Done by Friction

A 4.00-kg particle moves from the origin to position *C*, having coordinates $x = 5.00$ m and $y = 5.00$ m, in the horizontal plane. The coefficient of friction is $\mu_k = 0.100$.

Calculate the work done by the friction in going from *O* to *C* along *OAC* and *OC*.



The frictional force is $f = -\mu_k mg = -0.100(4.00\text{ kg})(9.80\text{ m/s}^2) = -3.92$ N.

Work done along *OAC*: $W_{OAC} = W_{OA} + W_{AC} = f(x_A - x_O) + f(y_C - y_A)$
$= (-3.92\text{ N})(10.0\text{ m}) = -39.2\text{ J}$

Work done along OC: $W_{OC} = fL_{OC} = (-3.92\text{ N})(7.07\text{ m}) = -27.2\text{ J}$

⇒ The work done by friction is *path dependent*.

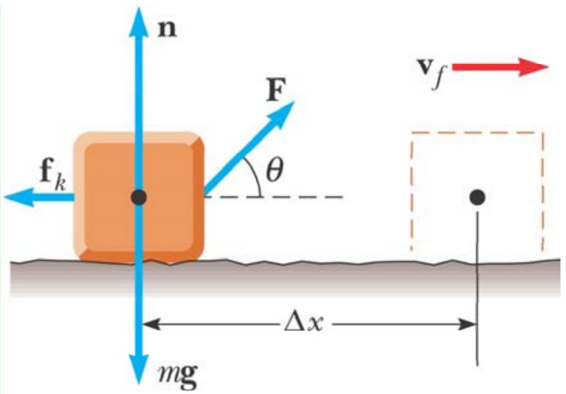
Energy & Nonconservative Forces

- Work done by *nonconservative forces* cannot be related to a *potential energy change*
  - Potential energy depends on the beginning and end points while the work by n.c. force depends on the path
- In general, $\Sigma W = W_c + W_{nc} = \Delta K$
- Substituting $W_c = -\Delta U$, $W_{nc} = \Delta K + \Delta U = \Delta E_{mech}$
  - If friction is zero, this equation becomes the same as Conservation of Mechanical Energy, $\Delta E_{mech} = 0$
  - If friction is present, $W_{nc} = -f\Delta s = \Delta E_{mech} = \Delta K + \Delta U$

Example 5: Work Done w/Friction

A 6.0-kg block is pulled from rest by a force *F* = 12 N over a surface with a coefficient of kinetic friction 0.17 at an angle of 5.0°.

Find the speed of the block after it has been moved 3.0 m.



$$n - mg + F\sin\theta = 0, \quad W_f = f\Delta x\cos 180° = -f\Delta x = -\mu_k n\Delta x$$

$$W_f = -\mu_k(mg - F\sin\theta)\Delta x, \quad W_F = F\cos\theta\,\Delta x, \quad W_f + W_F = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

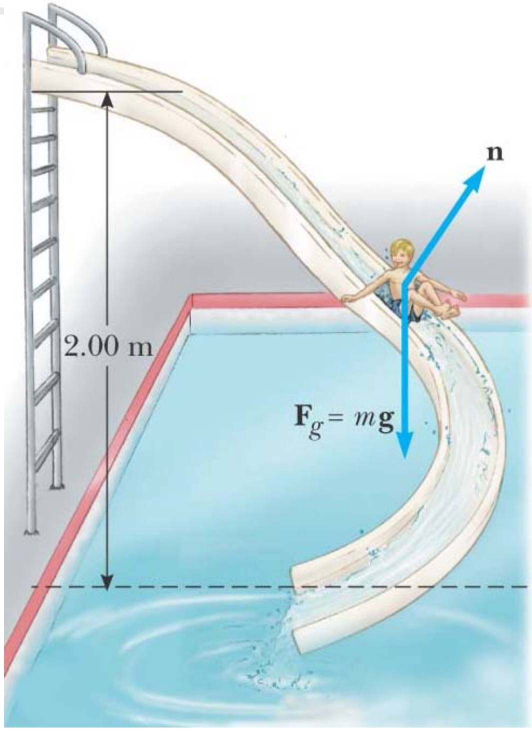
$$v_f = \sqrt{\frac{[-\mu_k(mg - F\sin\theta) + F\cos\theta]\Delta x}{m/2}}$$

$$= \sqrt{\frac{[-0.17(58.8\text{ N} - 12\text{ N}\sin 5.0°) + 12\text{ N}\cos 5.0°]3.0\text{ m}}{3.0\text{ kg}}} = 1.5\text{ m/s}$$

Example 6: Water Slide w/friction

A 20.0-kg child slides down a vertical height of 2.00 m from rest and reaches a final speed of 3.00 m/s.

What is the work done by the friction?



It is not easy to determine the friction *f* because **n** is not always in the same direction. We use the Work-Energy Theorem.

$$W_{nc} = \Delta E_{mech} = \Delta K + \Delta U$$

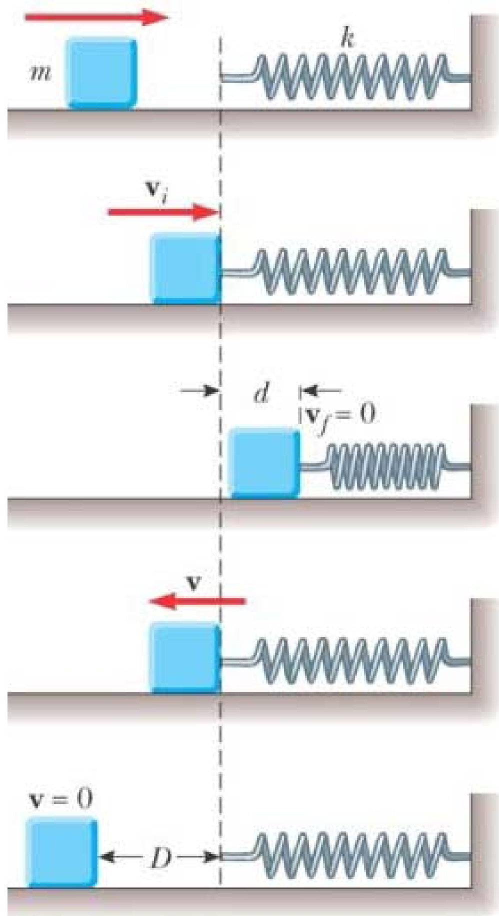
$$\Delta E_{mech} = \left(\frac{1}{2}mv_f^2 - 0\right) + \left(0 - mgh\right) = 90\text{ J} - 392\text{ J} = -302\text{ J}$$

$$W_f = -\int_{s_i}^{s_f} f ds = W_{nc} = \Delta E_{mech} = -302\text{ J}$$

Example 7: Spring Compression

A 1.00-kg object slides to the right on a surface having $\mu_k = 0.250$. The object has a speed of $v_i = 3.00$ m/s when it makes contact with a light spring that has $k = 50.0$ N/m. The object comes to rest after the spring has been compressed a distance *d*. The object is then forced toward the left by the spring and comes to rest a distance *D* to the left of the unstretched spring.

Find (a) the distance of compression *d*, (b) the speed *v* at the unstretched position when the object is moving to the left, and (c) the distance *D* where the object comes to rest.



Example 7, cont

(a) Between the 2nd and 3rd picture, $-fd = \Delta K + \Delta U = \left(0 - \frac{1}{2}mv_i^2\right) + \left(\frac{1}{2}kd^2 - 0\right)$

$$f = -\mu mg = -0.250(1.00\text{ kg})(9.80\text{ m/s}^2) = -2.45\text{ N},$$

$$(2.45\text{ N})d = -\tfrac{1}{2}(1.00\text{ kg})(3.00\text{ m/s})^2 + \tfrac{1}{2}(50.0\text{ N/m})d^2$$

$$d = \frac{-2.45 \pm \sqrt{(2.45)^2 - 4(25.0)(-4.50)}}{50.0} = \frac{(-2.45 \pm 21.35)\text{ N}}{50.0\text{ N/m}} = 0.378\text{ m}$$

(b) Between the 2nd and 4th picture, $-f\,2d = \Delta K + \Delta U = \frac{1}{2}mv^2 - \frac{1}{2}mv_i^2$

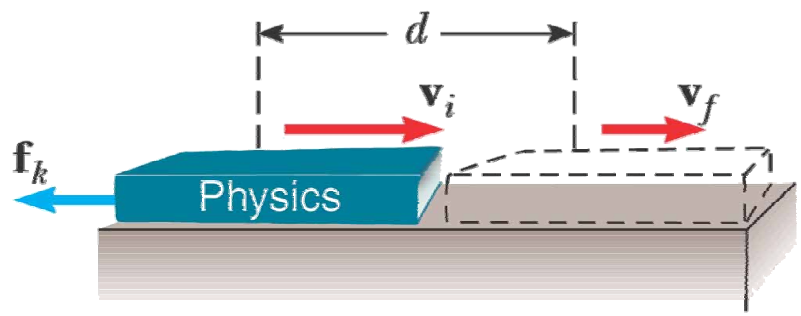
$$v = \sqrt{(3.00\text{ m/s})^2 - \frac{2}{1.00\text{ kg}}(2.45\text{ N})(2 \times 0.378\text{ m})} = 2.30\text{ m/s}$$

(c) Between and 2nd and 5th picture, $-f(D + 2d) = \Delta K + \Delta U = 0 - \frac{1}{2}mv_i^2$

$$D = \frac{\frac{1}{2}(1.00\text{ kg})(3.00\text{ m/s})^2}{2.45\text{ N}} - 2(0.378\text{ m}) = 1.08\text{ m}$$

Internal Energy

- The energy associated with an object's *temperature* is called its *internal energy*, E_{int}
- In this example, the surface of the book is the boundary of the system
  - The friction does work and increases the internal energy of the surfaces



- Where did the mechanical energy go?

Microscopic Picture

- At the microscopic level, atoms move very rapidly
  - The kinetic energy of one vibrating iron atom is ½ (9 x 10^{-26} kg) (500 m/s)^2 ≈ 1 x 10^{-20} J.
  - The atomic mass of iron is 56. 56 g of iron has N_A atoms (N_A = 6.02 x 10^{23}). An iron ball with m = 0.5 kg has 9N_A ≈ 5 x 10^{24} atoms. Therefore, the microscopic internal kinetic energy of the iron ball is ≈ 50,000 J.
  - By comparison, the macroscopic energy of a 0.5-kg iron ball moving at 10 m/s is 25 J.
- This is one form of internal energy

Internal Energy, cont

- The microscopic internal energy of atoms and molecules in matter is called *thermal energy*

$$E_{thermal} = K_{micro} + U_{micro}$$

  - The higher the kinetic energy, the higher the temperature
- There are different types of *internal potential energy*
  - Chemical, nuclear
- The *internal kinetic energy* is called by *temperature*
  - Usually measured empirically in degrees, not joules
- *Total energy is conserved*

Transfer Energy in/out of System

- *Work* – transfer by a force causing a displacement
- *Mechanical waves* – disturbance propagates through a medium
- *Matter transfer* – matter crosses the boundary of the system, carrying energy
- *Heat* – driven by a temperature difference between two regions in space
- *Electrical transmission* – transfer by electric current or EM waves

Conservation of System Energy

- *Energy is conserved*
  - Energy cannot be created or destroyed
  - If the total energy in a system changes, it can only be because energy has crossed the boundary of the system
- Mathematically, $\Delta E_{\text{system}} = \Sigma T = W + Q + \cdots$
  - E_{system} is the total energy of the system
  - T is the energy transferred across the system boundary
- The *Work-Kinetic Energy Theorem* is a special case of Conservation of Energy

Power

- The time rate of energy transfer is called *power*
- The average power is given by $\overline{P} = \Delta W / \Delta t$ when the method of energy transfer is work,
- The *instantaneous power* is the limiting value of the average power as Δt approaches zero

$$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt}$$

  - For constant *F*, $P = \frac{dW}{dt} = \vec{F} \cdot \frac{d\vec{r}}{dt} = \vec{F} \cdot \vec{v}$

Power, cont

- Power can be related to any type of energy transfer
- The SI unit of power is the watt (W):

$$1\ \text{W} = 1\ \text{J/s} = 1\ \text{kg}\cdot\text{m}^2/\text{s}^2$$

  - A unit of power in the US customary system is the horsepower (hp):

$$1\ \text{hp} = 746\ \text{W} = 550\ \text{ft}\cdot\text{lbs/s}$$

- Units of power can also be used to express units of work or energy:

$$1\ \text{kWh} = (1000\ \text{W})(3600\ \text{s}) = 3.6 \times 10^6\ \text{J}$$

Example 8: Power Ski Lift

A skier of mass 70.0 kg is pulled up a slope by a motor-driven cable. Ignore the friction of the slope.

(a) How much work is required to pull him a distance of 60.0 m up a 30.0° slope at a constant speed of 2.00 m/s?

(b) A motor of what power is required to perform this task?

(a) $\sum W = \Delta K = 0$ because he moves at constant speed. $W_{moter} + W_g = 0$

The skier rises a vertical distance of (60.0 m) sin 30°. Thus

$$W_{moter} = (70.0\,\text{kg})(9.80\,\text{m/s}^2)(30.0\,\text{m}) = 2.06\times10^4\ \text{J}$$

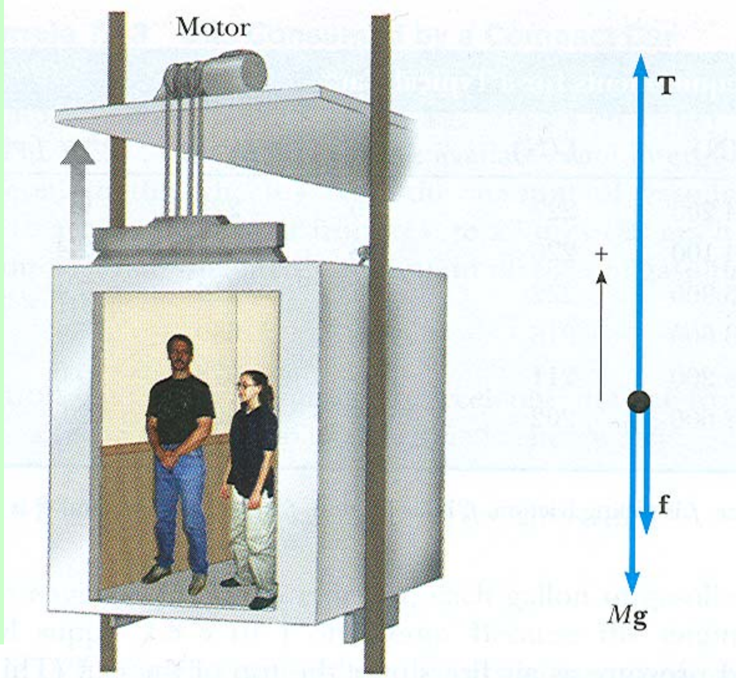
(b) The time to travel 60.0 m at constant speed of 2.0 m/s is 30.0 s. Thus,

$$P_{moter} = \frac{\Delta W_{motor}}{\Delta t} = \frac{2.06\times10^4\ \text{J}}{30.0\,\text{s}} = 686\ \text{W} = \frac{1\,\text{hp}}{746\ \text{W}}\times 686\ \text{W} = 0.920\,\text{hp}$$

Example 9: Elevator Power

An elevator car has a mass of 1600 kg and is carrying passengers having a combined mass of 200 kg. A constant frictional force of 4000 N retards its motion upward.

What power must the motor deliver when the speed is 3.00 m/s, if the elevator is accelerating at 1.00 m/s²?



$$T - f - Mg = Ma, \;\; T = f + M(g + a)$$

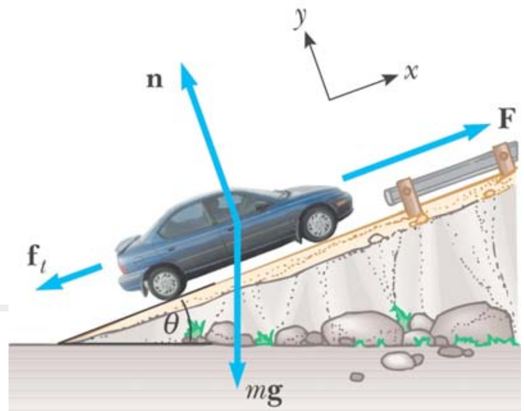
$$T = 4000\ \text{N} + (1800\ \text{kg})(9.80\ \text{m/s}^2 + 1.00\ \text{m/s}^2) = 23.4\times10^3\ \text{N}$$

$$P = Tv = (23.4\times10^3\ \text{N})(3.00\ \text{m/s}) = 70.2\ \text{kW} = 70.2\ \text{kW}\times\frac{1\ \text{hp}}{0.746\ \text{kW}} = 94.1\ \text{hp}$$

Energy and the Automobile

- Analyze automobile fuel consumption
  - 67% of energy available from the fuel is lost in the engine
  - 10% is lost by friction in the transmission, bearings, etc.
  - 6% goes to internal energy and 4% to operate the fuel and oil pumps and accessories
  - This leaves about 13% to actually propel the car
- The magnitude of the total friction force is the sum of the rolling friction and the air drag: $f_t = f_r + f_a$
  - At low speeds, rolling friction dominates: $f_r = \mu_r n$
  - At high speeds, air drag dominates: $f_a = \frac{1}{2} C \rho v^2 A$

Automotive Power



F is the driving force on the wheels from the engine.

$f = 218 + 0.70v^2$ is an empirical rolling and air drag force.

$$\sum F_x = F - f - mg\sin\theta = ma, \quad F = ma + mg\sin\theta + (218 + 0.70v^2)$$

$$P = Fv = mva + mgv\sin\theta + 218v + 0.70v^3$$

Friction Forces and Power Requirements for a Typical Car[a]

v(mi/h)	v(m/s)	n(N)	f_r(N)	f_a(N)	f_t(N)	$\mathcal{P} = f_t v$(kW)
0	0	14 200	227	0	227	0
20	8.9	14 100	226	48	274	2.4
40	17.9	13 900	222	192	414	7.4
60	26.8	13 600	218	431	649	17.4
80	35.8	13 200	211	767	978	35.0
100	44.7	12 600	202	1 199	1 400	62.6

[a] In this table, n is the normal force, f_r is rolling friction, f_a is air friction, f_t is total friction, and $\mathcal{P}$ is the power delivered to the wheels.

Example 10: Gas Consumption

A car has a mass of 1371 kg. Its efficiency is rated at 13%, i.e. 13% of the available fuel energy is delivered to the wheels.

How much gasoline is used to accelerate the car to 27 m/s (60 mph)? The energy equivalent of gasoline is 1.3×10^8 J/gal.

The energy of the car at 27 m/s (60 mph) is

$$K = \tfrac{1}{2} mv^2 = \tfrac{1}{2}(1371\text{ kg})(27\text{ m/s})^2 = 5.0 \times 10^5\text{ J}$$

Since the car is only 13% efficient, each gallon of gas yields

$$0.13\,(1.3 \times 10^8\text{ J/gal}) = 1.7 \times 10^7\text{ J/gal}$$

Hence the number of gallons needed is

$$\text{No. of gallons} = \frac{5.0 \times 10^5\text{ J}}{1.7 \times 10^7\text{ J/gal}} = 0.029\text{ gal}$$

Example 10, cont

Suppose it takes 10 s to accelerate from 0 to 60 mph. The car travels

$$\Delta x = v_{ave}\Delta t = \frac{0+27\text{ m/s}}{2}\times 10\text{ s} = 135\text{ m}\times\frac{1\text{ mi}}{1600\text{ m}} = 0.08\text{ mi}$$

The car consumes 0.029 gallons of gas to go 0.08 mi.

So during the acceleration time, the gas consumption rate is or 2.8 mi/gal.

From the previous table, at v = 60 mph the required power to the wheels is

$$P_{\text{wheels}} = 17.4\text{ kW}$$

With a 13% efficiency, the engine must have $P_{eng} = P_{wheels}/0.13 = 134$ kW

The gasoline consumption rate is $1.34\times10^5\ \frac{\text{J}}{\text{s}}\ \frac{1\text{ gal}}{1.38\times10^8\text{ J}}\times\frac{3600\text{ s}}{1\text{ hr}} = 3.50\ \frac{\text{gal}}{\text{hr}}$.

For a car going 60 mph, we will get $\frac{60\text{ mi/hr}}{3.50\text{ gal/hr}} = 17\text{ mi/gal}$