Physics for Scientists and Engineers

Chapter 9

Impulse and Momentum

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Linear Momentum

- *Linear momentum* of an object of mass m moving with a velocity $\mathbf{v}$ is defined to be

$$\mathbf{p} = m\mathbf{v}$$

  - "Momentum" and "linear momentum" will be used interchangeably
  - Momentum is a vector quantity
  - SI units of momentum are kg·m/s
- Momentum can be expressed in component form: $p_x = mv_x$, $p_y = mv_y$, $p_z = mv_z$

Newton's 2nd Law & Momentum

- Newton's 2nd law as Newton presented it:

$$\sum \mathbf{F} = \frac{d\mathbf{p}}{dt} = \frac{d}{dt}(m\mathbf{v})$$

  - This is a more general form than the one that we used so far
  - Newton called *m* **v** the *quantity of motion*

- For constant mass (i.e. *dm*/ *dt* = 0), Newton's 2nd law becomes

$$\sum \mathbf{F} = \frac{d}{dt}(m\mathbf{v}) = m\frac{d\mathbf{v}}{dt} = m\mathbf{a}$$

Conservation of Linear Momentum

- When two or more particles in an *isolated* system (i.e. no external forces present) interact, the total momentum of the system remains constant
  - The momentum of the *system* is conserved, not the momenta of *individual* particles
- Can be expressed mathematically in various ways
  - $\mathbf{p}_{total} = \mathbf{p}_1 + \mathbf{p}_2 =$ constant
  - $\mathbf{p}_{1i} + \mathbf{p}_{\mathbf{2i}} = \mathbf{p}_{1f} + \mathbf{p}_{2f}$
- Conservation of momentum can be applied to systems with any number of particles

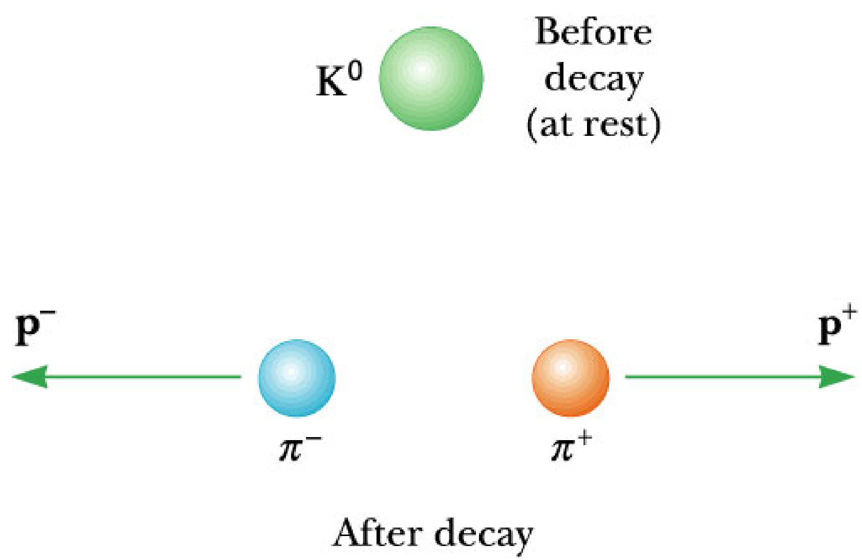
Archer and Arrow

- An archer is standing on a frictionless surface
  - Let the *system* be the archer with bow (particle 1) and the arrow (particle 2)
  - There are no external forces in the *x*-direction, so it is *isolated* in the *x*-direction
- Total momentum before releasing the arrow is 0
  $\Rightarrow$ Total momentum after releasing the arrow is $p_{1f} + p_{2f} = 0$
  - The archer will move in the opposite direction of the arrow
  - Agrees with Newton's 3rd law
  - Because the archer is much more massive than the arrow, his acceleration and velocity will be much smaller than those of the arrow

Kaon Decay

- The kaon decays into a positive π and a negative π particle
- Total momentum before decay is zero
- Therefore, the total momentum after the decay must equal zero:

$$\mathbf{p}^{+} + \mathbf{p}^{-} = 0, \text{ or } \mathbf{p}^{+} = -\mathbf{p}^{-}$$



Impulse and Momentum

- From Newton's 2nd Law $\mathbf{F} = d\mathbf{p}/dt$, $d\mathbf{p} = \mathbf{F}dt$
- Integrating to find the change in momentum over some time interval

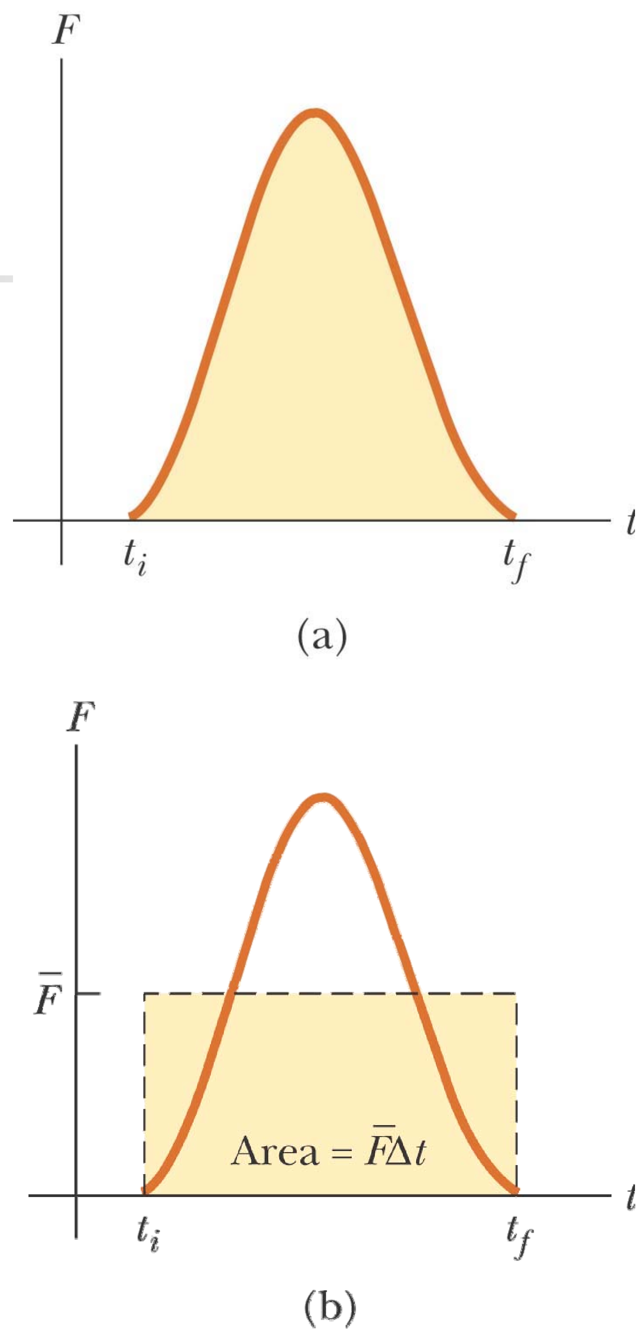
$$\Delta\mathbf{p} = \mathbf{p}_f - \mathbf{p}_i = \int_{t_i}^{t_f} \mathbf{F}dt = \mathbf{I}$$

- The integral is called the *impulse*, $\mathbf{I}$, of the force $\mathbf{F}$ acting on an object over Δt
- This equation expresses the *impulse-momentum theorem*

More About Impulse

- Impulse is a vector quantity
- The magnitude of the impulse is equal to the area under the force-time curve
- Dimensions of impulse are ML/T, with units of N s = kg m s^{-1}
- Impulse can also be found by using the time averaged force:

$$\mathbf{I} = \overline{\mathbf{F}}\Delta t$$



Example 1: Deformed Ball

A hard rubber ball, released at chest height, falls to the pavement and bounces back to nearly the same height. When it is in contact with the pavement, the lower side of the ball is temporarily flattened, with the maximum dent of 1 cm.

Estimate the acceleration of the ball while it is in contact with the pavement and the time duration it is deformed.

To find the contact time, we use the impulse equation:

$$I = F_{ave}\Delta t = p_f - p_i$$

To find F_{ave}, we need to find a. To find a, first find the velocity of the ball at the instant the bottom hits the ground, assuming the ball falls 1.50 m:

$$v_f^2 = v_i^2 + 2a(y_f - y_i) = 0 + 2(-9.80\,\text{m/s}^2)(-1.50\,\text{m} - 0)$$

$$v_f = -5.42\,\text{m/s}$$

Example 1, cont

We use the same equation to find the acceleration, assumed constant:

$$v_f^2 = v_i^2 + 2a(y_f - y_i) = (-5.42\ \text{m/s})^2 + 2a(0 - 0.01\ \text{m}) = 0$$

$$a = 1470\ \text{m/s}^2$$

This is approximately the average acceleration.

The momentum of the ball goes from $p_i = -m(5.42\ \text{m/s})$ to $p_f = 0$ as it is being deformed. Using the impulse theorem:

$$I = F_{ave}\Delta t = p_f - p_i$$

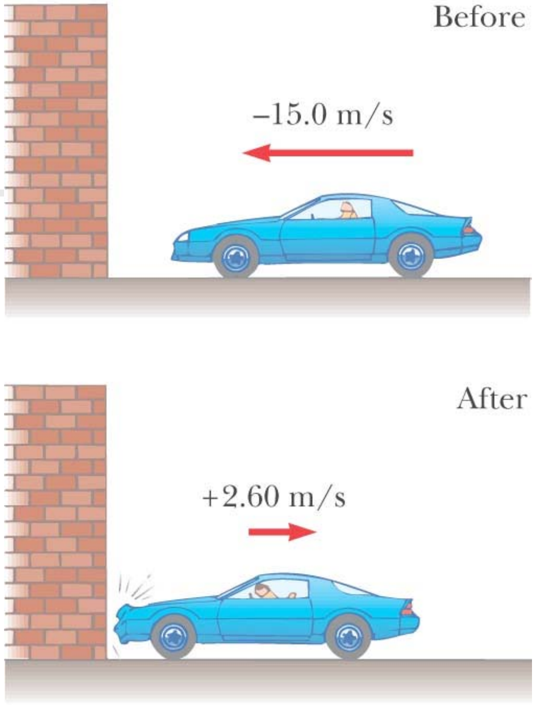
$$\Delta t = \frac{p_f - p_i}{F_{ave}} = \frac{p_f - p_i}{ma} \cong \frac{0 - m(-5.42\ \text{m/s})}{m(1.47\times 10^3\ \text{m/s}^2)} \cong 3.69\times 10^{-3}\ \text{s}$$

Since it bounces back to nearly the same height, the time to deform and un-deform is approximately twice this time.

Collisions

- *Collision* is an event during which two particles come close to each other and interact by means of forces
- The time interval during which the velocity changes from its initial to final values is assumed to be short
- The interaction force is assumed to be much greater than any external forces present
  - This means the impulse approximation can be used

Example 2: Car Crash



In a crash test, a car of mass 1500 kg collides with a wall. The initial and final velocities of the car are v_i = –15.0 m/s and v_f = 2.60 m/s. The collision lasts 0.150 s. Find the impulse and average force on the car.

Assume the force exerted by the wall on the car is greater than any other forces that cause its velocity to change.

$$p_i = (1500 \text{ kg})(-15.0 \text{ m/s}) = -2.25 \times 10^4 \text{ kg m/s}$$

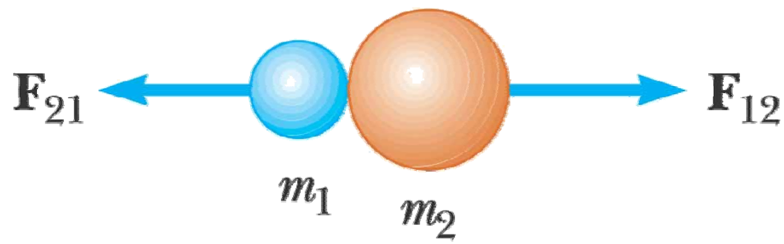
$$P_f = (1500 \text{ kg})(2.60 \text{ m/s}) = 0.39 \times 10^4 \text{ kg m/s}$$

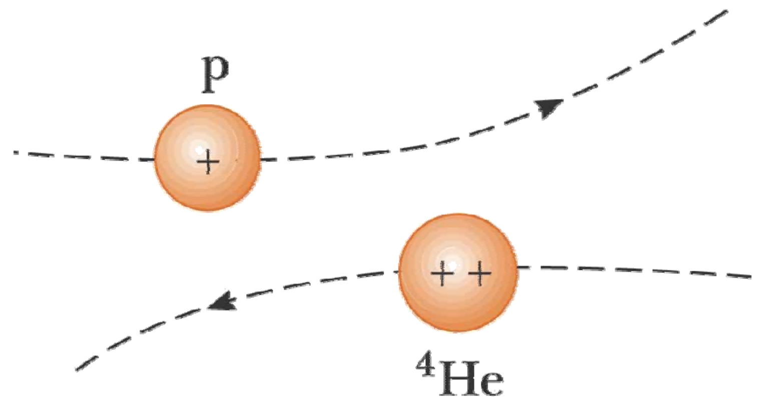
$$I = p_f - p_i = 2.64 \times 10^4 \text{ kg m/s}$$

$$F_{ave} = \Delta p/\Delta t = (2.64 \times 10^4 \text{ kg m/s})/0.150 \text{ s} = 1.50 \times 10^5 \text{ N}$$

Collisions – Examples

- Collisions may be the result of direct contact
  - Impulsive forces may vary in time in complicated ways
- Collision need not include physical contact
  - There are still forces between the particles



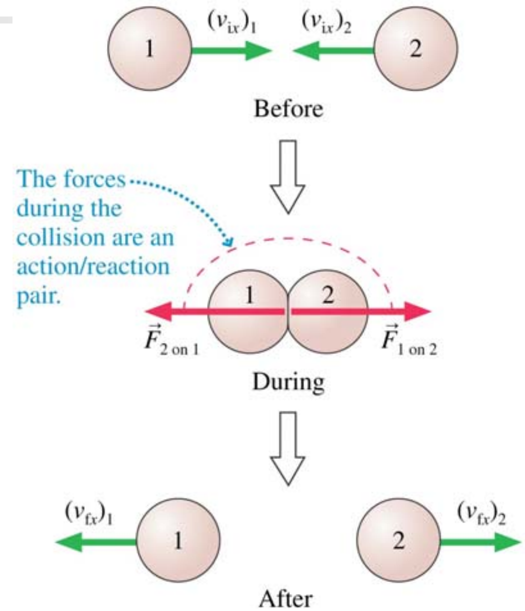


- Total momentum is *always* conserved

Types of Collisions

- In an *elastic* collision, momentum and kinetic energy are conserved
  - Perfectly elastic collisions occur on a microscopic level
  - Macroscopic collisions are only approximately elastic
- In an *inelastic* collision, kinetic energy is not conserved although momentum is conserved
  - If the objects stick together after the collision, it is a *perfectly inelastic* collision
  - Elastic and perfectly inelastic collisions are limiting cases; most actual collisions fall in between these two types

Conservation of Momentum

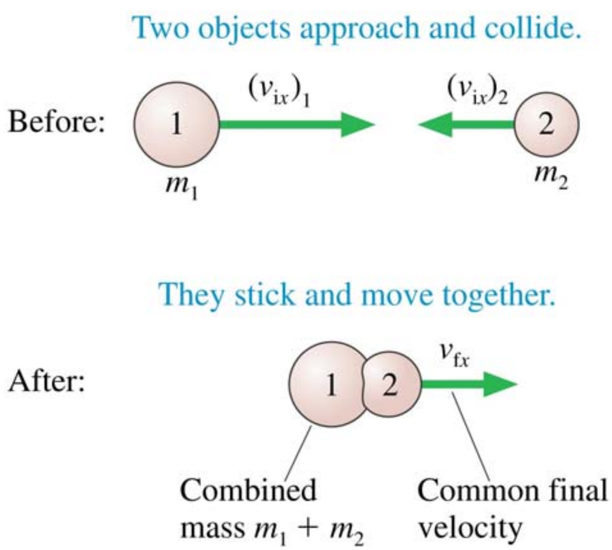


- Since there are no external forces,

$$\mathbf{F}_{\mathbf{ext}}\Delta t = \mathbf{P}_\mathbf{f} - \mathbf{P}_\mathbf{i} = 0, \quad \mathbf{P}_\mathbf{i} = \mathbf{P}_\mathbf{f}$$

- In an *elastic* collision,

$$m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i} = m_1\mathbf{v}_{1f} + m_2\mathbf{v}_{2f}$$



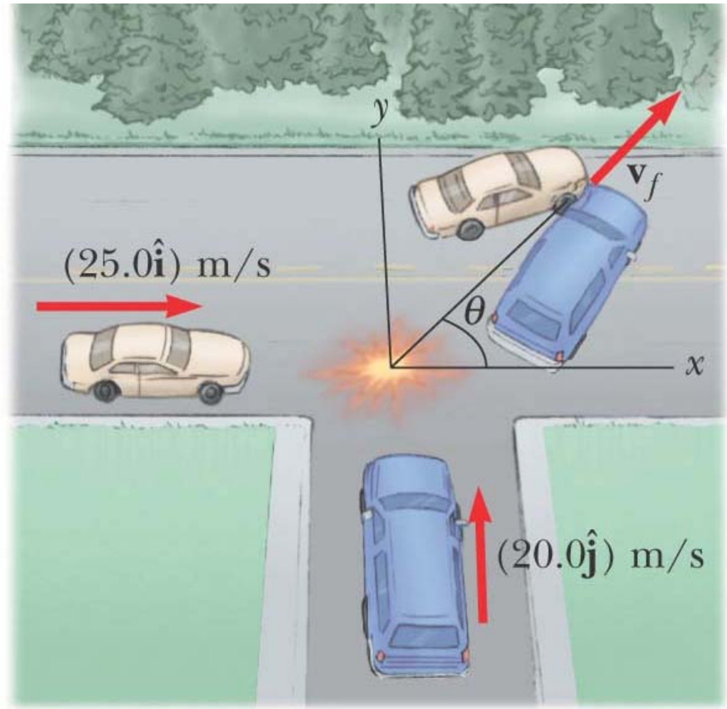
- In a *perfectly inelastic* collision, they share the same velocity after the collision

$$m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i} = (m_1 + m_2)\mathbf{v}_f$$

Example 3: 2D Perf Inel Collision

The car and the van have masses of 1500 kg and 2500 kg, respectively. Before the collision, the car was moving with a speed of 25.0 m/s in the *x*-direction and the van with a speed of 20.0 m/s in the *y*-direction.

The car and the van make a perfectly inelastic collision. What is their velocity after the collision?



Momentum conservation:

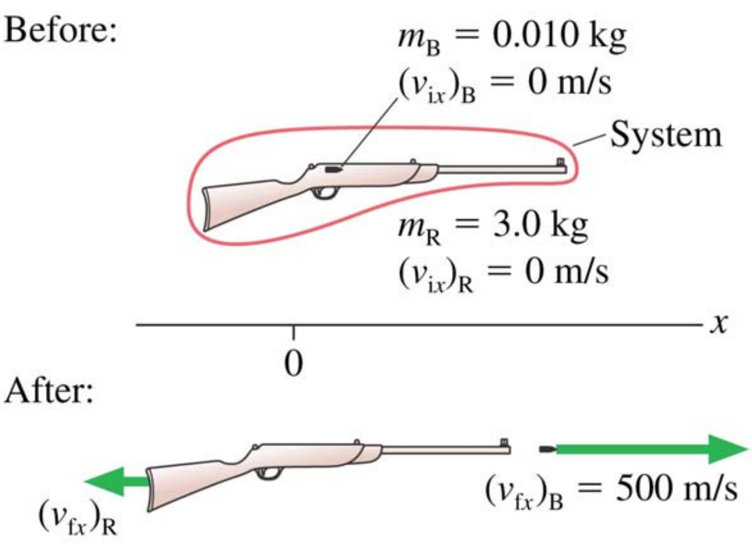
x: $m_1 v_{1ix} = (m_1+m_2) v_f \cos\theta$, $3.75\text{x}10^4$ kg m/s = 4000 $v_f \cos\theta$

y: $m_2 v_{2iy} = (m_1+m_2) v_f \sin\theta$, $5.00\text{x}10^4$ kg m/s = 4000 $v_f \sin\theta$

$\Rightarrow \tan\theta = 5.00/3.75$, $\theta = 53.1°$, $v_f = 5.00\text{x}10^4/(4000 \sin 53.1°) = 15.6$ m/s

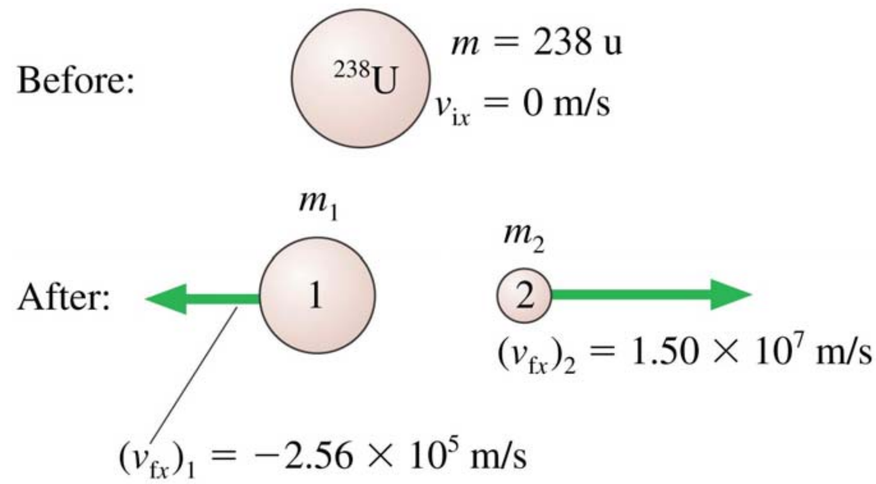
Explosion

- *Explosion* is an event during which particles of the system move apart from each other after a brief interaction
  - Examples: archer and arrow, recoil of rifle, particle decay
- Explosion is the reverse of the collision problem
- Total momentum is conserved



Example 4: Radioactivity

A ^{238}U nucleus spontaneously decays into a small fragment that is ejected with a speed of 1.50 x 10^7 m/s and a "daughter nucleus" that recoils with a speed of 2.56 x 10^5 m/s.



What are the atomic masses of the ejected fragment and the daughter nucleus?

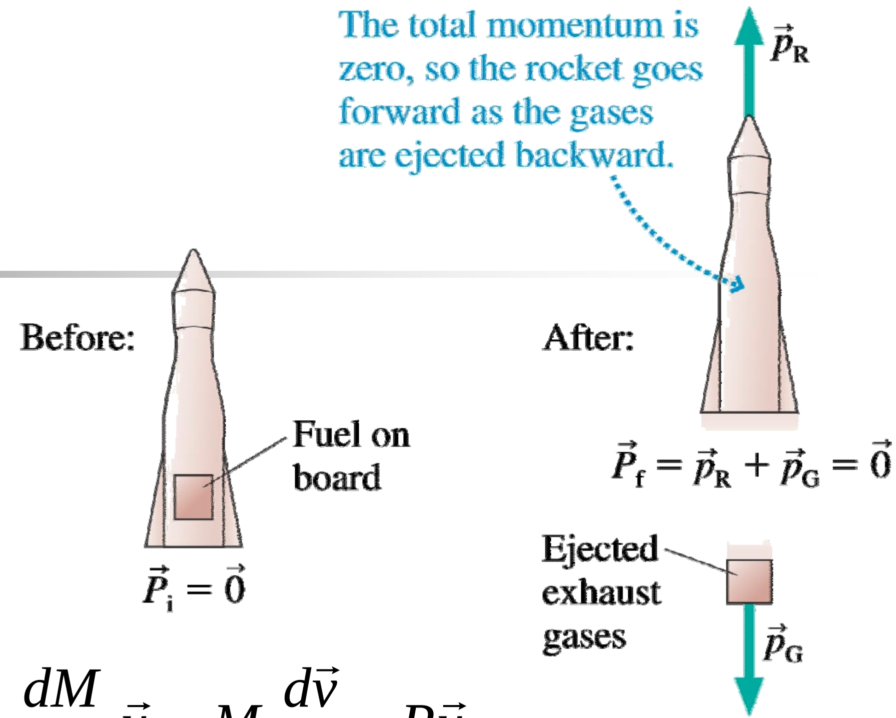
The nucleus was initially at rest so the total momentum remains zero.

$$m_1 v_{1f} + m_2 v_{2f} = (m_1 + m_2) v_i = 0, \ \ m_1 + m_2 = 238\,\mathrm{u}$$

Combining these two conservation laws,

$$m_1 v_{1f} + (238\,\mathrm{u} - m_1) v_{2f} = 0, \ \ m_1 = \frac{v_{2f}}{v_{2f} - v_{1f}} \times 238\,\mathrm{u} = 234\,\mathrm{u}, \ \ m_2 = 4\,\mathrm{u}$$

Rocket Launch



This is an explosion problem where

M continues to decrease : $M = M_0 - Rt$.

Here R is the fuel burn rate.

From Newton's 2nd law,

$$\vec{F}_{ext} = -Mg\hat{j} = \frac{d\vec{P}}{dt} = \frac{d}{dt}\left(M\vec{v}\right) = M\frac{d\vec{v}}{dt} + \frac{dM}{dt}\vec{u} = M\frac{d\vec{v}}{dt} - R\vec{u}$$

Here $\vec{u}$ is the exhaust velocity and the quantity $R\vec{u}$ is the thrust, $\vec{F}_{thrust}$.

For a rocket going straight upward, we have

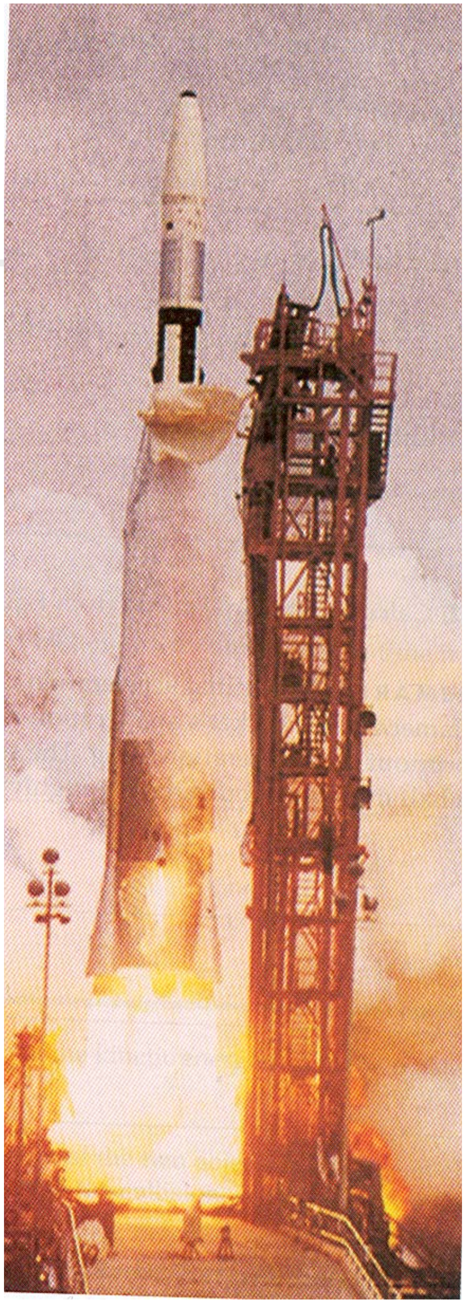
$$-Mg + Ru = M\frac{dv}{dt} \quad \text{or} \quad \frac{dv}{dt} = \frac{Ru}{M} - g = \frac{Ru}{M_0 - Rt} - g$$

Integrating gives the rocket equation : $v = u\ln\left(\frac{M_0}{M_0 - Rt}\right) - gt$

Example 5: Saturn V

Saturn V, the Apollo launch vehicle, has an initial mass of $M_0 = 2.85 \times 10^6$ kg, a payload mass of $M_p = 0.27M_0$, a fuel burn rate of $R = 13.84 \times 10^3$ kg/s, and a thrust of $F_{thrust} = 34.0 \times 10^6$ N.

Find (a) the exhaust speed u, (b) burn time t_b, (c) acceleration at liftoff and burnout, and (d) final speed of the rocket.



Example 5, cont

(a) Exhaust speed: $u = \frac{F_{thrust}}{R} \frac{34.0 \times 10^6 \text{ kg m/s}^2}{13.84 \times 10^3 \text{ kg/s}} = 2.46 \text{ km/s}$

(b) Burn time: $t_b = \frac{M_0 - M_P}{R} = \frac{(1-0.27) 2.85 \times 10^6 \text{ kg}}{13.84 \times 10^3 \text{ kg/s}} = 150 \text{ s}$

(c) Acceleration at liftoff: $\frac{dv}{dt} = \frac{Ru}{M_0} - 9.80 \text{ m/s}^2 = 2.14 \text{ m/s}^2$

Acceleration at burnout: $\frac{dv}{dt} = \frac{Ru}{M_p} - 9.80 \text{ m/s}^2 = 34.3 \text{ m/s}^2$

(d) Final speed at burnout: $v = u \ln\left(\frac{M_0}{0.27 M_0}\right) - 9.80 \text{ m/s}^2 \times 150 \text{ s}$

$$v = (2.46 \times 10^3 \text{ m/s})(1.31) - 1470 \text{ m/s} = 1.75 \text{ km/s}$$