Physics for Scientists and Engineers

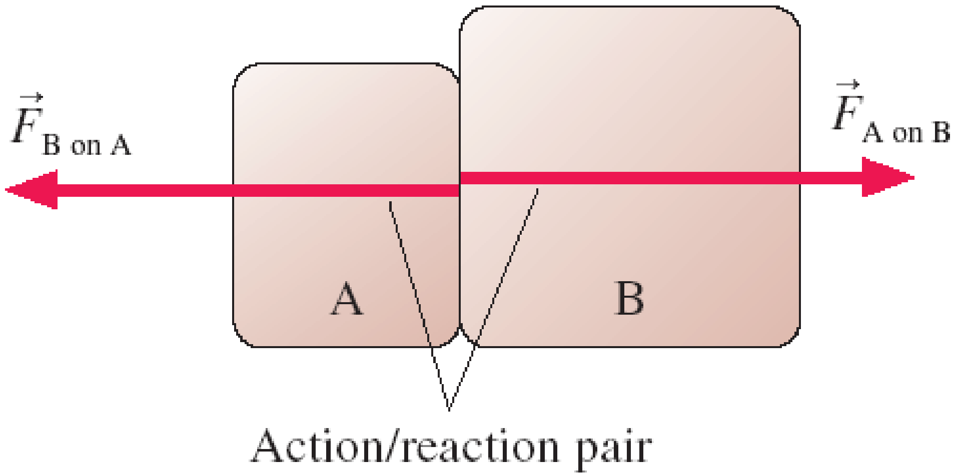
Chapter 7

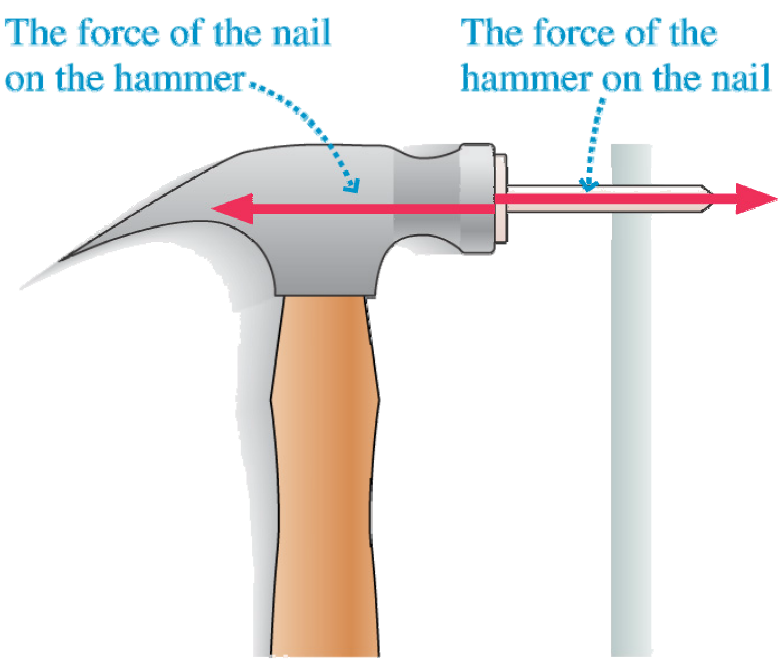
Newton's Third Law

Spring, 2008 Ho Jung Paik

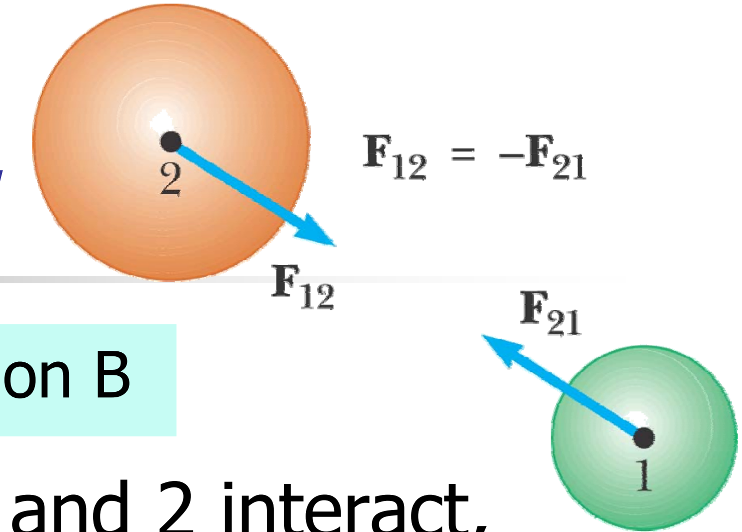
Interacting Objects

- If object A exerts a force on object B, then object B must exert a force on object A.
- The pair of forces, as shown, is called an *action-reaction pair*.





Newton's Third Law



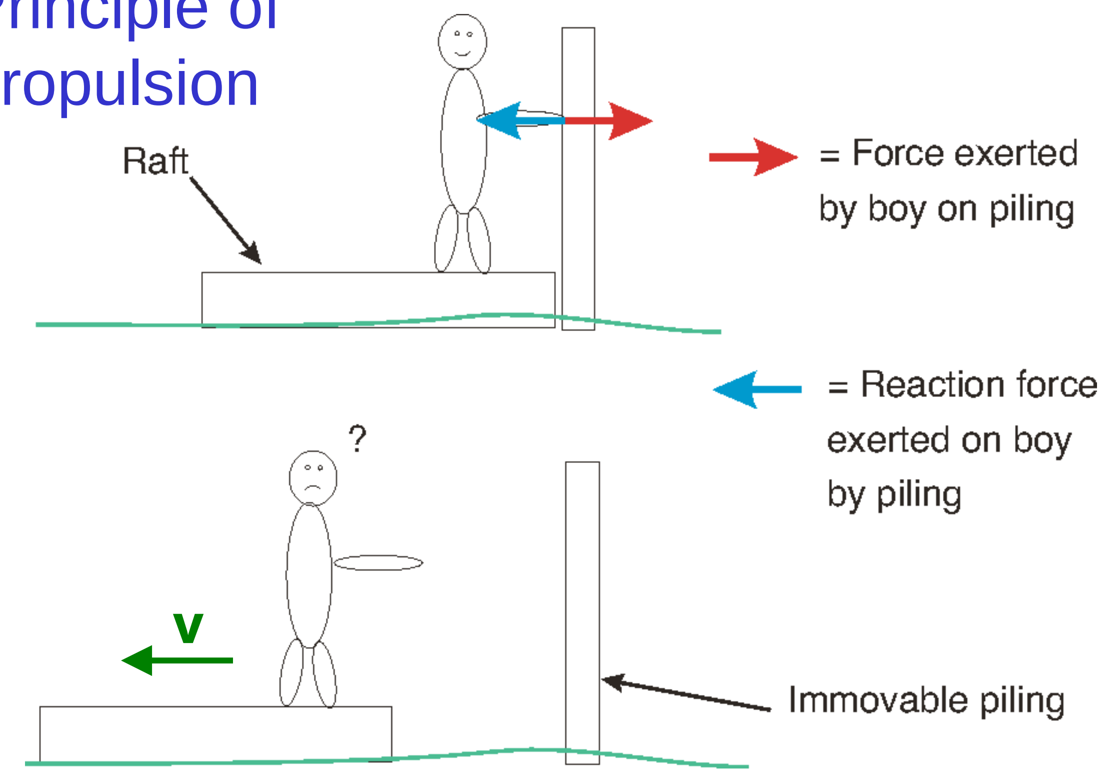
Notation: $\mathbf{F}_{AB}$ is the force exerted by A on B

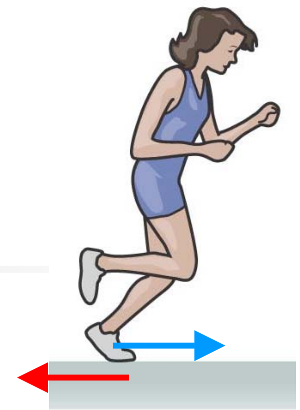
- Newton's 3rd Law: If objects 1 and 2 interact, $\mathbf{F}_{12}$ (action) is equal in magnitude and opposite in direction to $\mathbf{F}_{21}$ (reaction): $\mathbf{F}_{12} = -\mathbf{F}_{21}$
  - It doesn't matter which is considered the action and which the reaction
- Forces *always* occur in pairs
  - A single isolated force cannot exist
  - Action and reaction forces must act on *different* objects

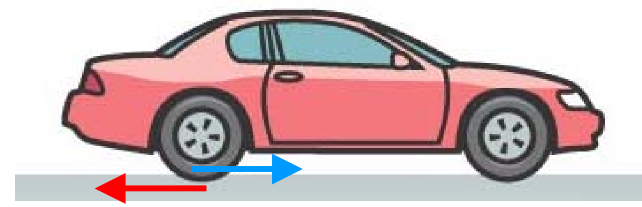
Newton's Third Law, cont

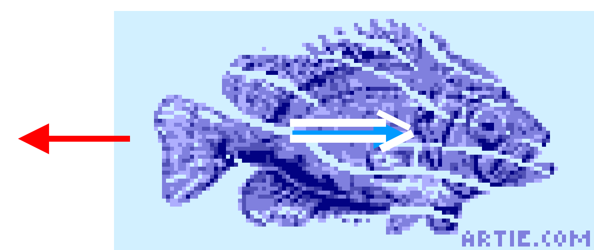
- Without Newton's 3rd Law, no propulsion would be possible

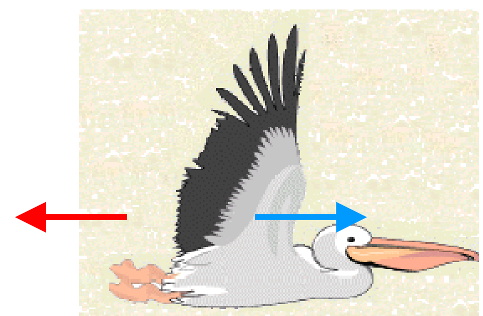
Principle of propulsion

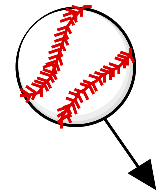




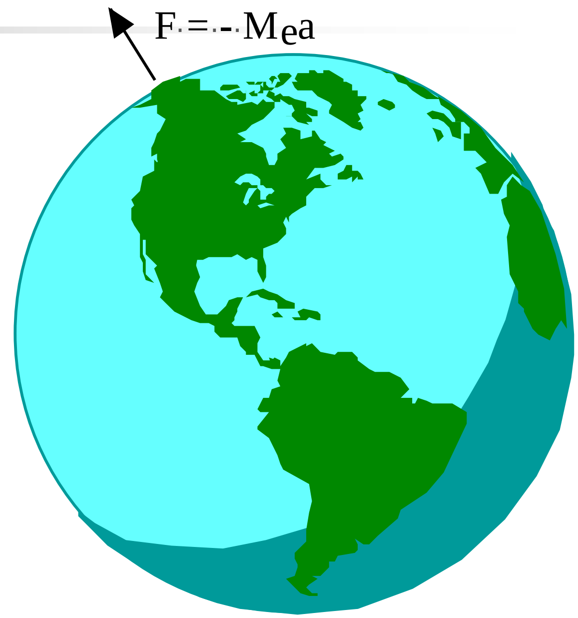








Example 1



If a 1.00-kg ball drops 1.00 m (Δy = –1.00 m), how far does the earth move in that time ?

$$\text{Ball: } \quad \Delta y_b = -\tfrac{1}{2} g t^2, \; t = \sqrt{\frac{2 \times 1.00\ \text{m}}{9.80\ \text{m/s}^2}} = 0.452\ \text{s}$$

$$\text{Earth: } \; \Delta y_e = \tfrac{1}{2} a_e t^2, \; a_e = ?$$

$$F_e = M_e a_e = -F_b = m_b a_b$$

$$a_e = -a_b \frac{m_b}{M_e} = 9.80\ \text{m/s}^2 \times \frac{1.00\ \text{kg}}{5.98 \times 10^{24}\ \text{kg}} = 1.64 \times 10^{-24}\ \text{m/s}^2$$

$$\Delta y_e = \tfrac{1}{2} \times 1.64 \times 10^{-24}\ \text{m/s}^2 \times (0.452\ \text{s})^2 = 1.67 \times 10^{-25}\ \text{m}$$

$M_e = 5.98 \times 10^{24}$ kg

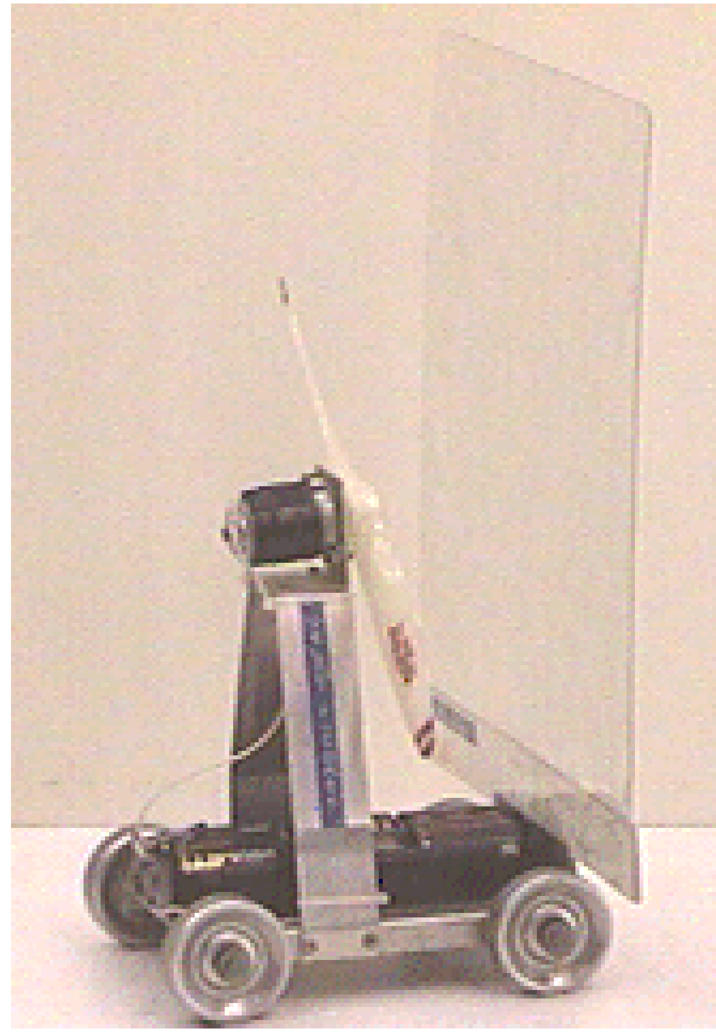
Demo: Fan/Cart

In the following three cases, will the cart move? If so, which way and why?

1) With the sail on and with the fan off the cart

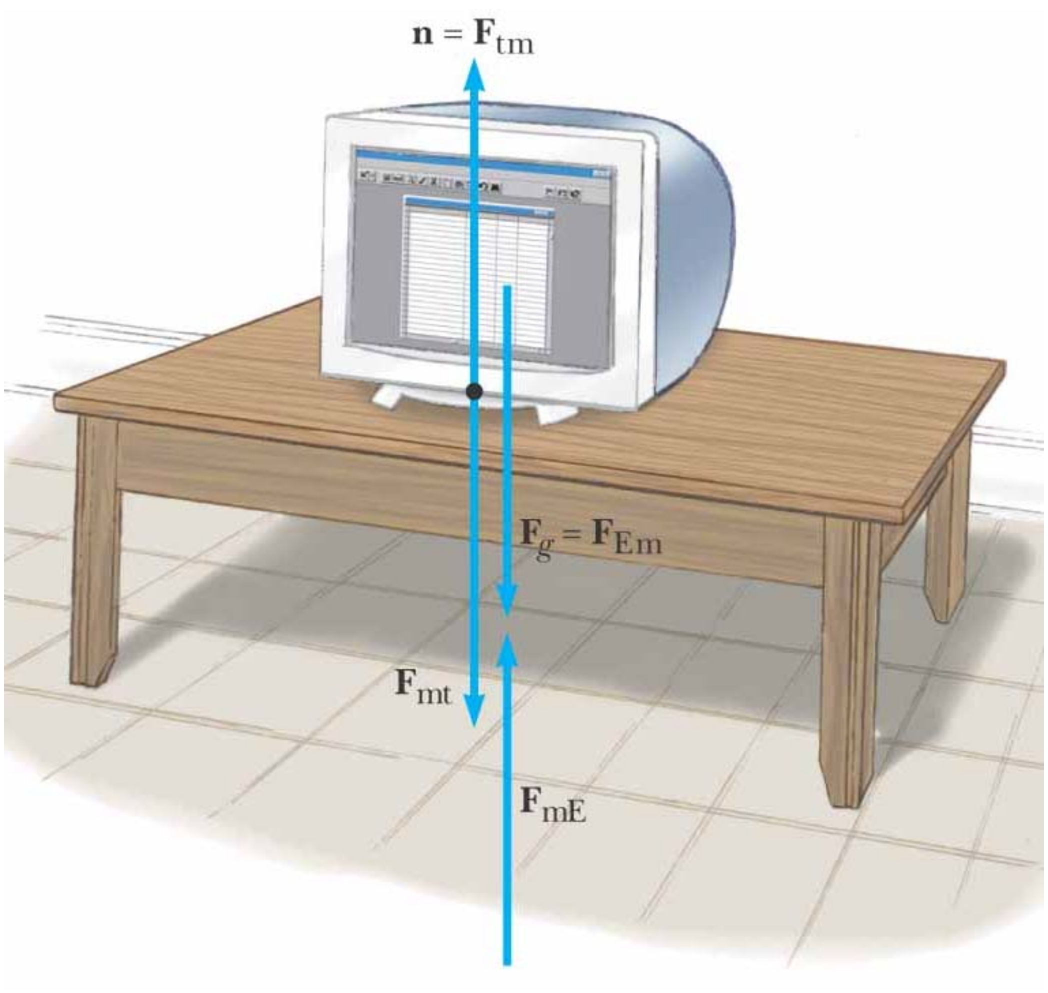
2) With the sail on and with the fan on the cart

3) With the sail off and with the fan on the cart

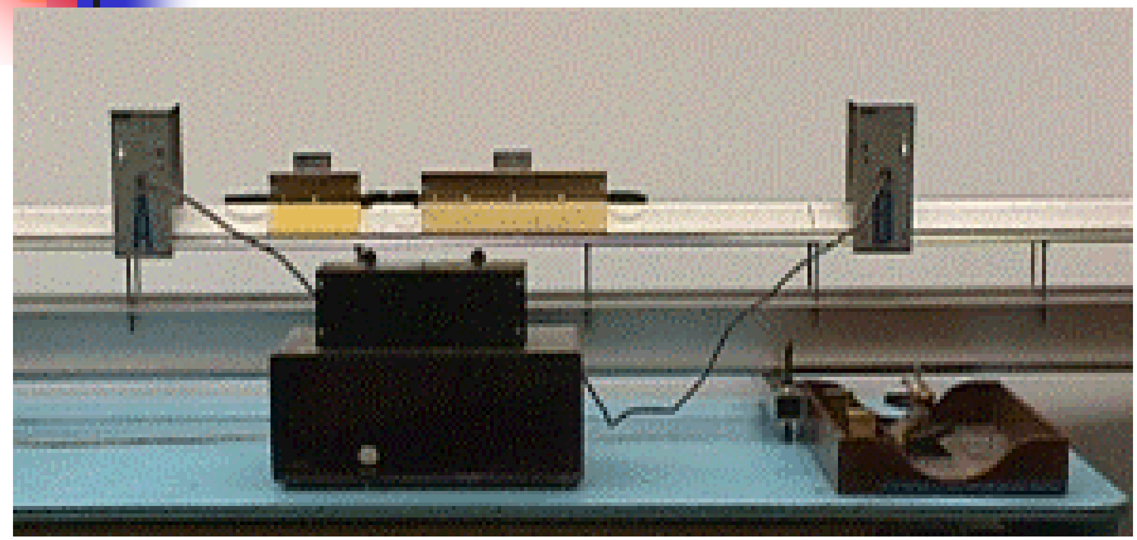


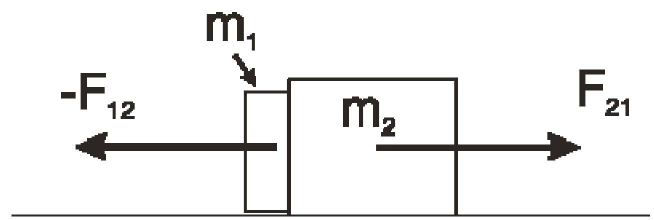
Action-Reaction Example

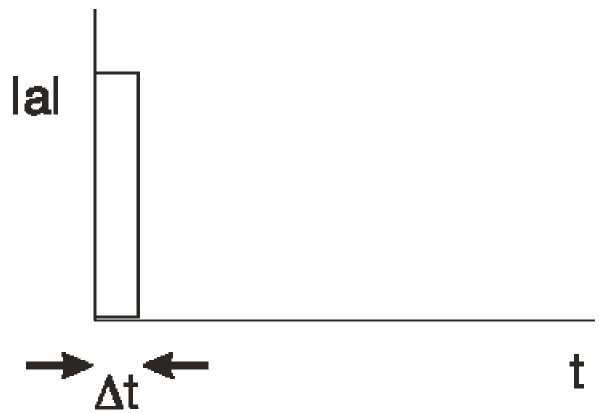
- The normal force **n** (exerted by the table on monitor) is the reaction of the force the monitor exerts on the table, $\mathbf{F}_{mt}$
- The monitor exerts a reaction force $\mathbf{F}_{mE}$, equal in magnitude and opposite in direction to the force the Earth exerts on the monitor, $\mathbf{F}_g$



Demo: Newton's 3rd Law







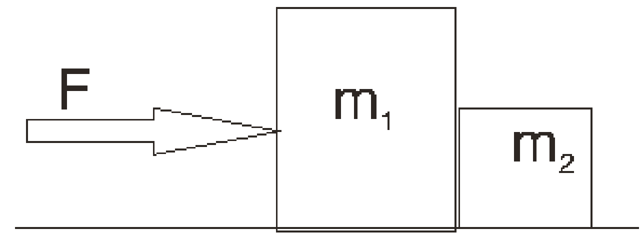
Newton's 2nd law, $a_2 = \dfrac{F_{21}}{m_2}$, $a_1 = \dfrac{F_{12}}{m_1}$

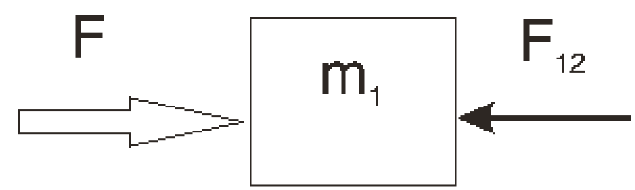
For a short interaction time Δt, $v_2 = a_2 \Delta t$, $v_1 = a_1 \Delta t$

$$\frac{v_2}{v_1} = \frac{a_2}{a_1} = \frac{F_{21}}{m_2}\frac{m_1}{F_{12}} \Rightarrow \frac{F_{21}}{F_{12}} = \frac{v_2}{v_1}\frac{m_2}{m_1}$$

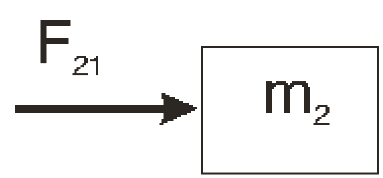
Example 2: Interacting Masses

A 10.0-N force pushes on a 4.00-kg box which is touching a 2.00-kg box.
What force does the 2.00-kg box experience and what is its acceleration?





$$F - F_{12} = m_1 a$$



$$F_{21} = m_2 a$$

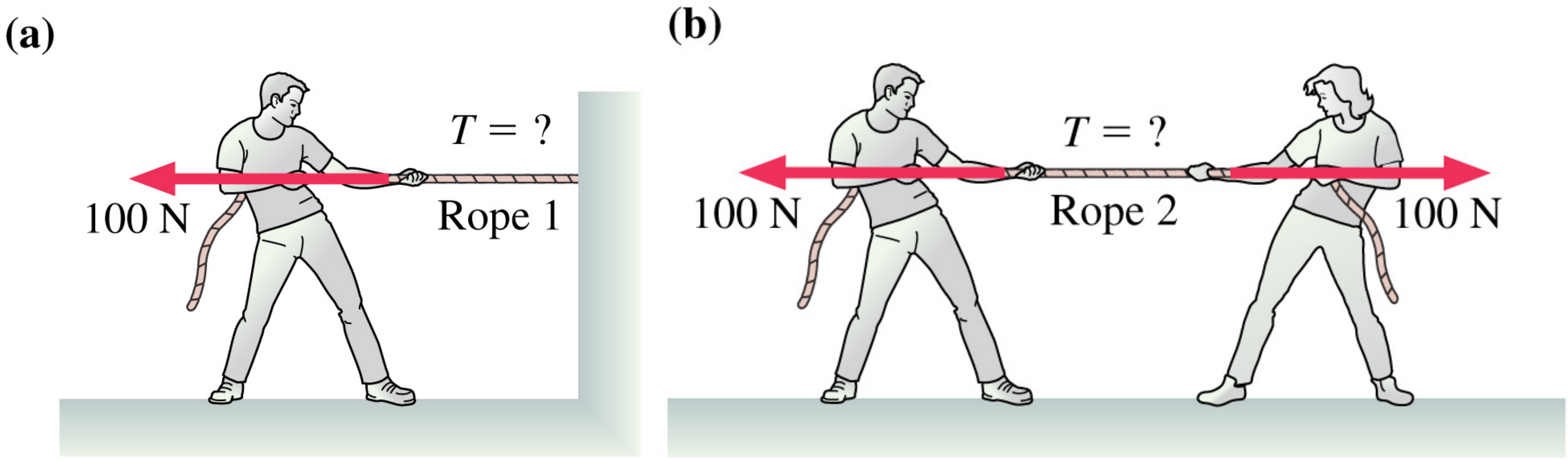
The *a*'s are the same because the boxes are in contact. Add the equations and make use of Newton's 3rd law: $\mathbf{F}_{12} = -\mathbf{F}_{21}$.

$$F = (m_1 + m_2)a \Rightarrow a = \frac{10.0\text{ N}}{6.00\text{ kg}} = 1.67\text{ m/s}^2$$

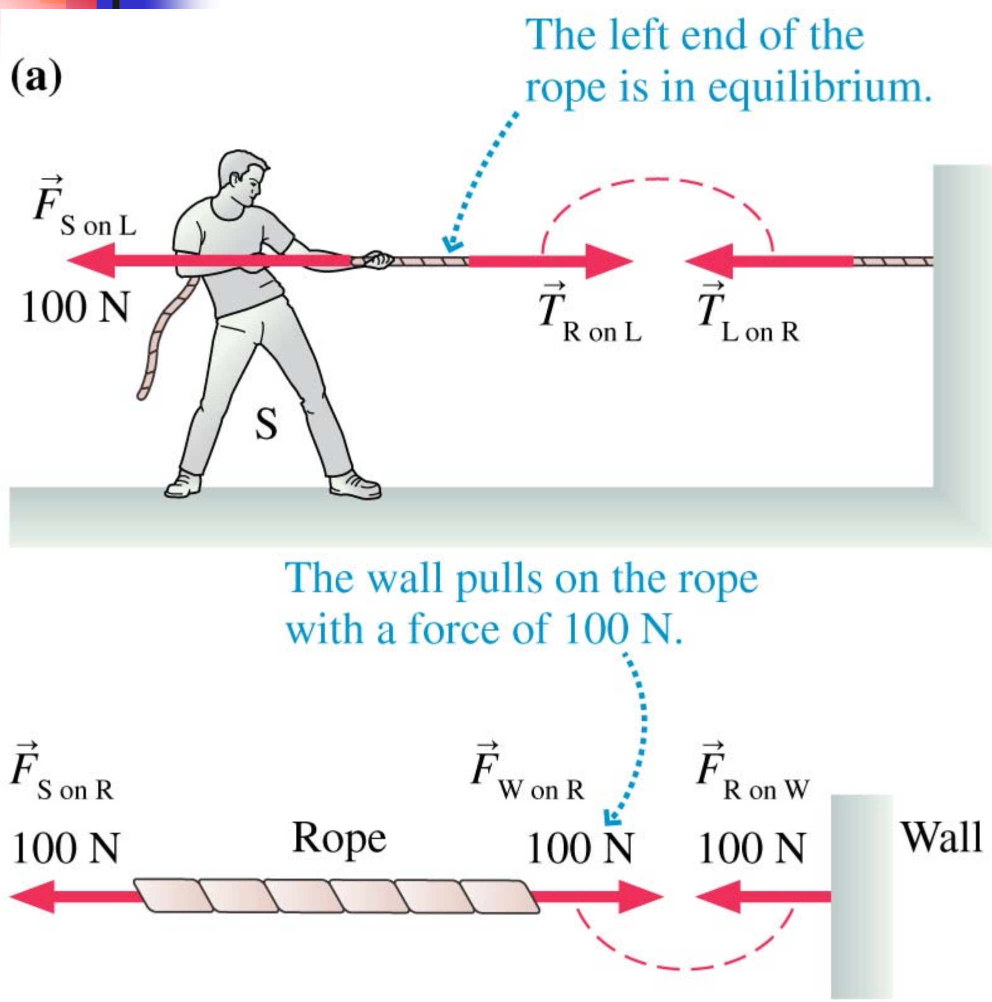
$$F_{21} = m_2 a = 2.00\text{ kg} \times 1.67\text{ m/s}^2 = 3.33\text{ N}$$

Example 3: Pulling the Rope

Does (a) or (b) produce a larger tension in the rope?



3(a): Student-Wall Pull



Imagine cutting the rope in two pieces. There are two forces on the left half of the rope:

$$T_{RL} - 100\text{ N} = 0$$

$$T_{RL} = 100\text{ N}$$

The forces on the broken ends are an action-reaction pair:

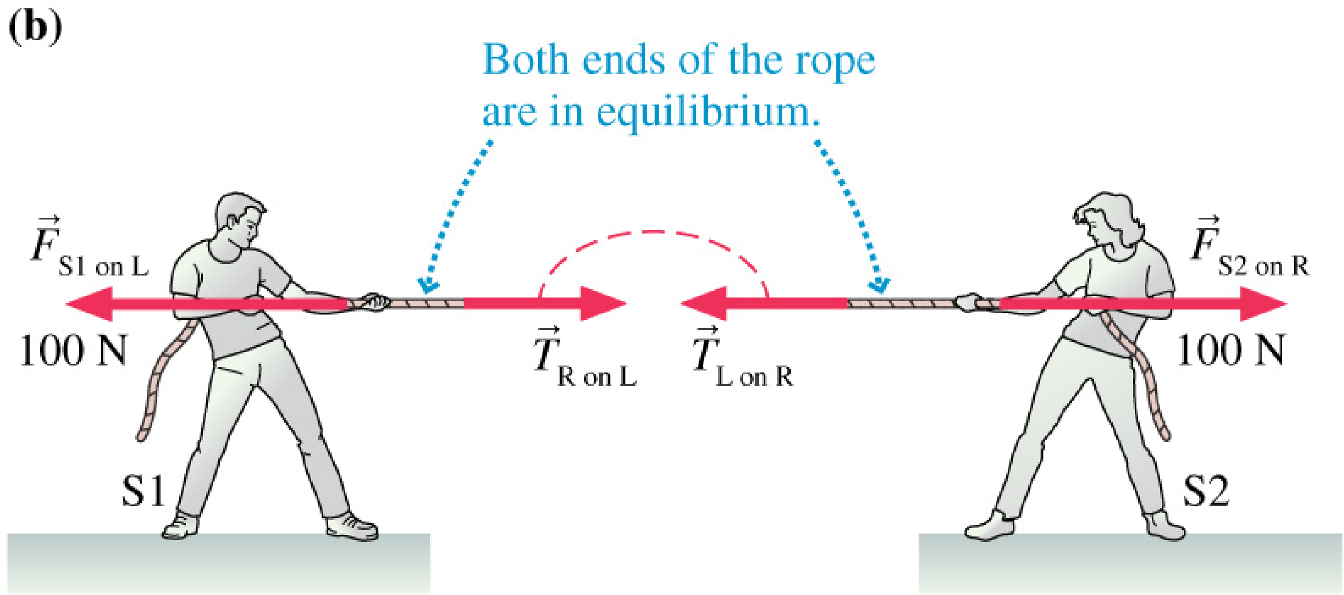
$$T_{LR} = T_{RL} = 100\text{ N}$$

The right half of the rope is not accelerating:

$$F_{WR} - T_{LR} = 0$$

$$F_{WR} = 100\text{ N}$$

3(b): Student-Student Pull

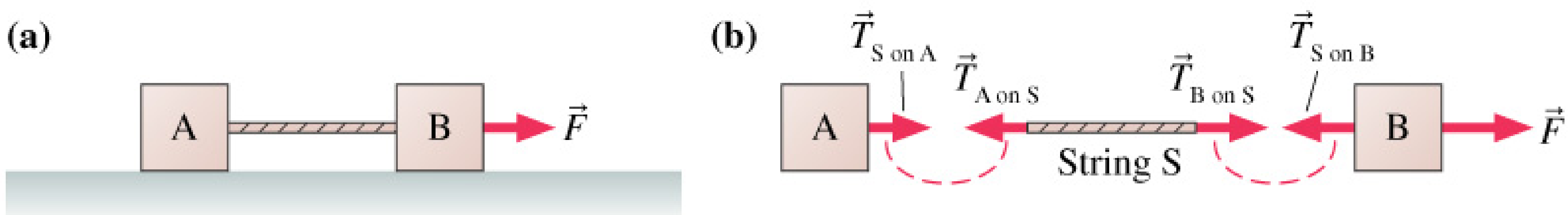


The left half of the rope has the same forces on it as in the wall case: $T_{RL} = 100$ N. There are two forces on the right half of the rope, and the rope doesn't accelerate: $100\text{ N} - T_{LR} = 0$, $T_{LR} = 100$ N.

As a result, the ends of the rope have 100-N forces acting in opposite directions just as in the wall case.

Example 4: Accelerating Rope

In the last example, the rope was not accelerating and the tension was constant throughout the rope. Is the tension the same at both ends if the rope is accelerating?



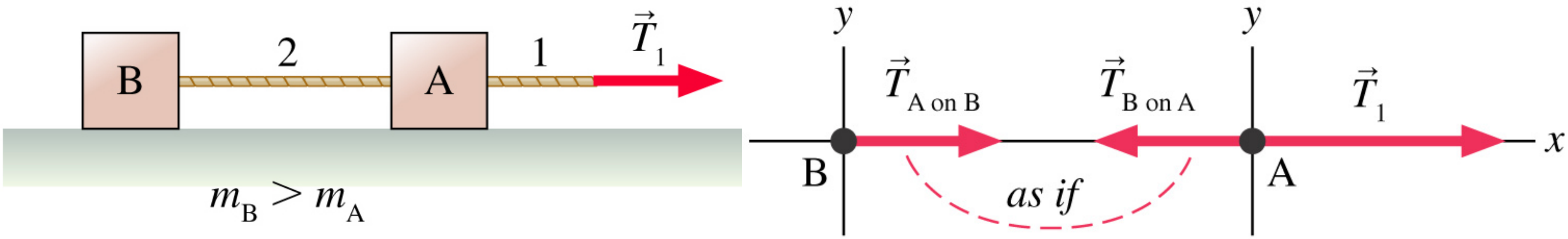
Newton's 2nd law for the rope: $\sum F = T_{BS} - T_{AS} = m_{st}a.$

So if the string is accelerating, the tensions on the ends of a rope *with mass* cannot be the same $T_{BS} = T_{AS} + m_{st}a$.

If the rope has *zero mass*, the tensions will be the same at each end. Often we make this approximation.

Example 4, cont

Take T_1 = 10.0 N, m_B = 4.00 kg and m_A = 2.00 kg. What is the acceleration and the forces on the blocks? Assume massless non-stretch ropes.



$$m_A : \sum F_A = T_1 - T_{BA} = m_A a\,,\ \ m_B : \sum F_B = T_{AB} = m_B a,$$

$$\text{Rope}: \sum F_R = T_{BA} - T_{AB} = 0 \Rightarrow T_{BA} = T_{AB} \Rightarrow T_1 = (m_b + m_A)a$$

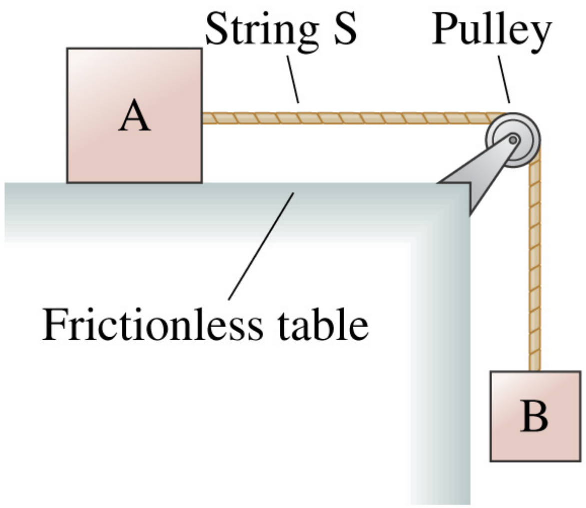
$$\text{From these, we find}: \ \ a = 10.0\ \text{N}/(2.00\ \text{kg} + 4.00\ \text{kg}) = 1.67\ \text{m/s}^2$$

$$\sum F_B = 4.00\ \text{kg} \times 1.67\ \text{m/s}^2 = 6.67\ \text{N},\ \sum F_A = 10\ \text{N} - 6.67\ \text{N} = 3.33\ \text{N}$$

Example 5: Pulleys and Strings

Block B drags block A across a frictionless table as it falls. Assume that the rope is massless and non-stretching and the pulley is massless and frictionless.

Find the tension in the string and the acceleration of the masses.



A *massless* rope around a *massless* pulley cannot have a net force acting on it. So if it is *frictionless*, the tension forces pulling its two ends must cancel each other *as if* they form an action-reaction pair.

Example 5, cont

From Newton's 3rd law,

$$T_{SA} = T_{AS},\ T_{AS} = T_{BS},\ T_{BS} = T_{SB}$$

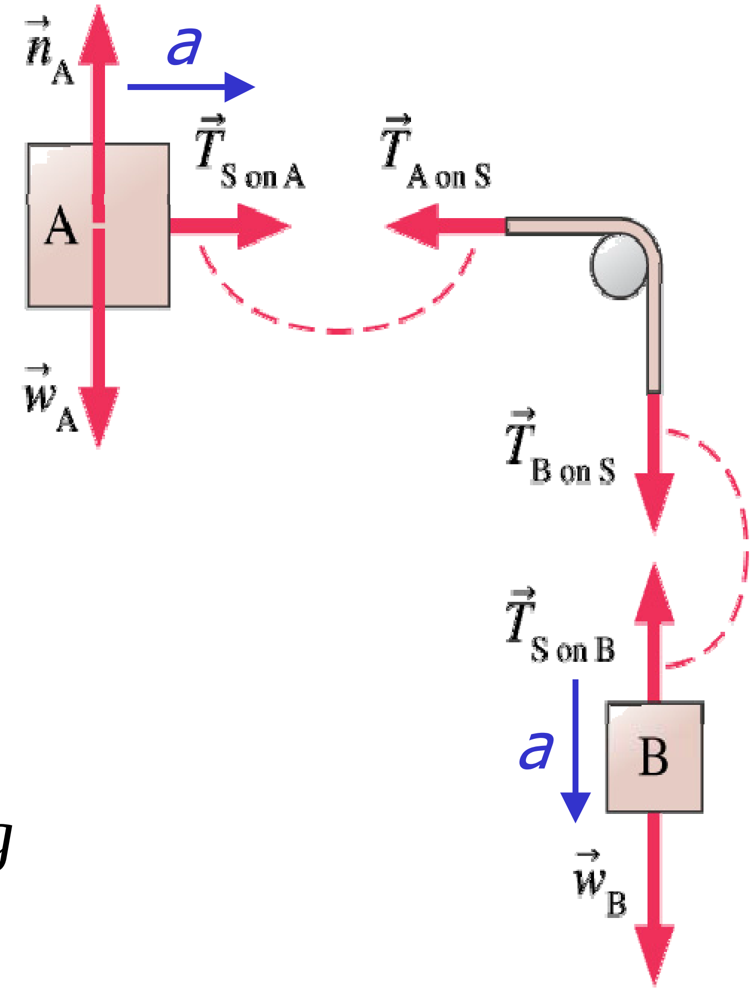
$$\Rightarrow\ T_{SA} = T_{AS} = T_{BS} = T_{SB} = T.$$

Using Newton's 2nd law,

$$\text{A}:\ \sum F_x = T = m_A a$$

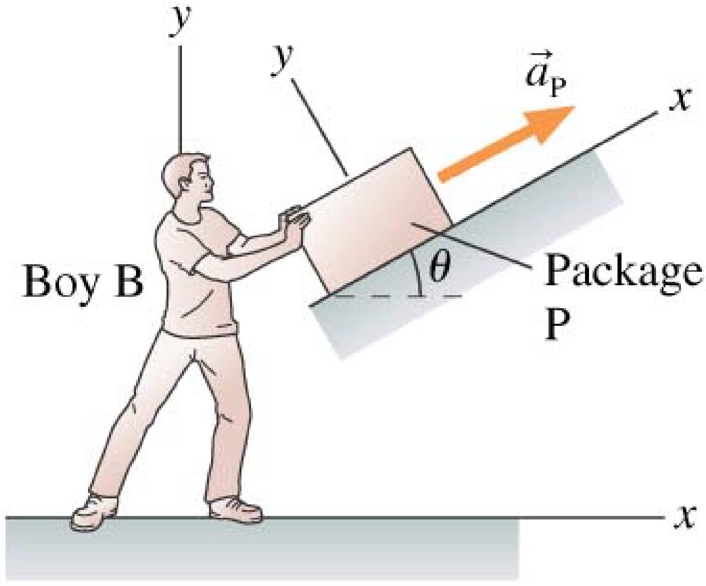
$$\text{B}:\ \sum F_y = T - m_B g = -m_B a$$

$$\Rightarrow\ a = \frac{m_B}{m_A + m_B} g,\quad T = \frac{m_A m_B}{m_A + m_B} g$$



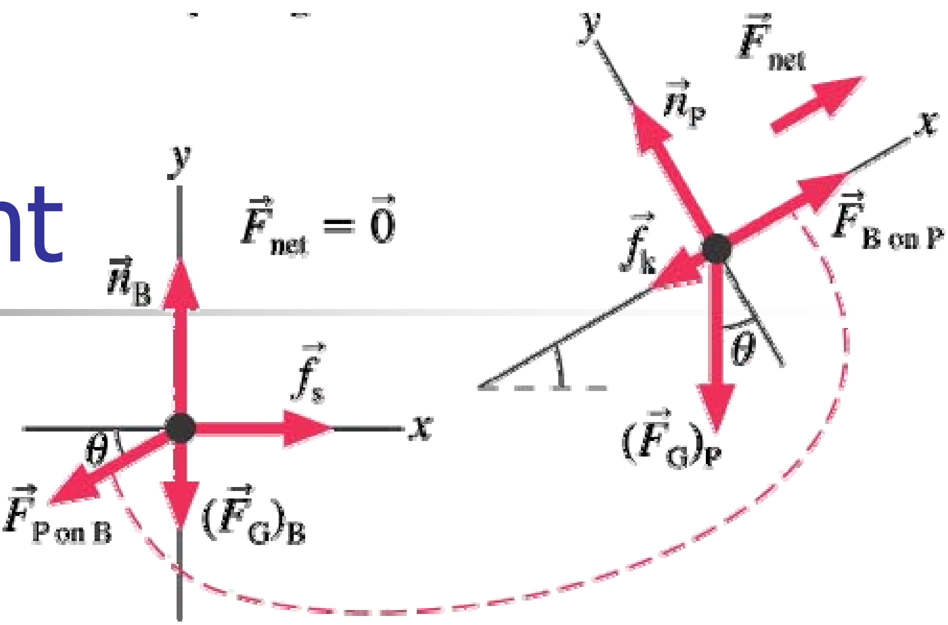
Example 6: Pushing a Package

A 40-kg boy places a 15-kg package on a 30° ramp and shoves it up the ramp into the storeroom. He needs to shove the package with an acceleration of at least 1.0 m/s² in order for the package to make it to the top of the ramp. The coefficient of kinetic friction on the ramp is 0.40. The ground is wet and the coefficient of static friction is only 0.25.



Can he give the package a big enough shove to reach the top of the ramp without his feet slipping?

Example 6, cont



From Newton's 3rd law,

$$F_{PB} = F_{BP} \equiv F$$

Newton's 2nd law for the boy and the package:

$$\sum (F_B)_x = f_s - F\cos\theta = 0, \quad \sum (F_B)_y = n_B - m_B g - F\sin\theta = 0$$

$$\sum (F_P)_x = F - f_k - m_P g\sin\theta = m_P a, \quad \sum (F_P)_y = n_P - m_P g\cos\theta = 0$$

From the package's equations,

$$f_k = \mu_k n_P = \mu_k m_P g\cos\theta, \quad F = m_P(a_x + g\sin\theta + \mu_k g\cos\theta) = 139\ \text{N}$$

Substituting this into the boy's equations,

$$f_{s,\text{req}} = F\cos\theta = 120\ \text{N}, \quad f_{s,\max} = \mu_s n_B = \mu_s(m_B g + F\sin\theta) = 115\ \text{N}.$$

Therefore, the boy *cannot* shove hard enough without slipping.