Physics for Scientists and Engineers

Chapter 2

Kinematics in One Dimension

Spring, 2008

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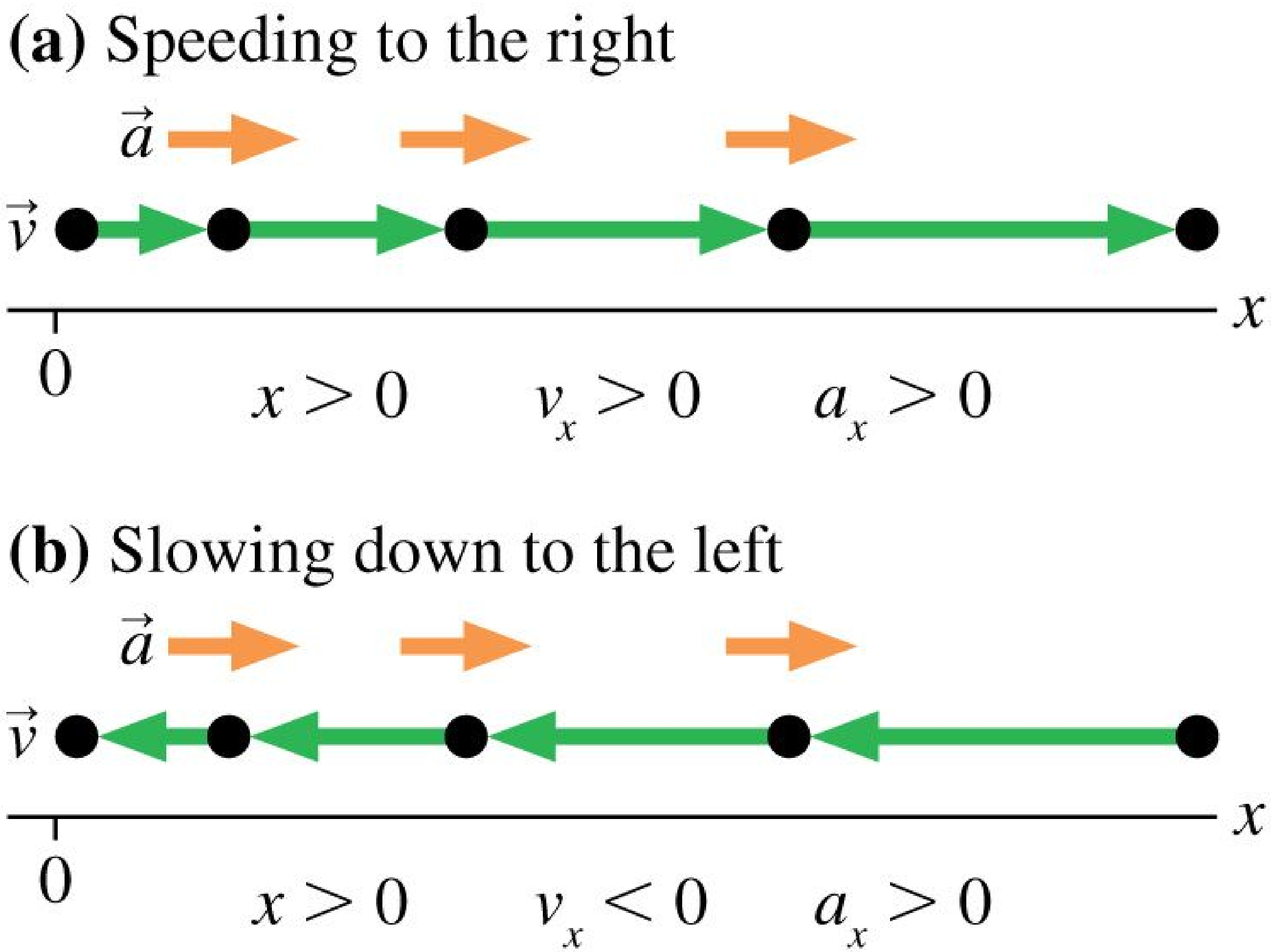
Kinematics

- Describes motion while ignoring the agents (forces) that caused the motion
- For now, will consider motion in one dimension
  - Along a straight line
- Will use the particle model
  - A particle is a point-like object, has mass but infinitesimal size

Acceleration and Velocity



Motion Diagrams

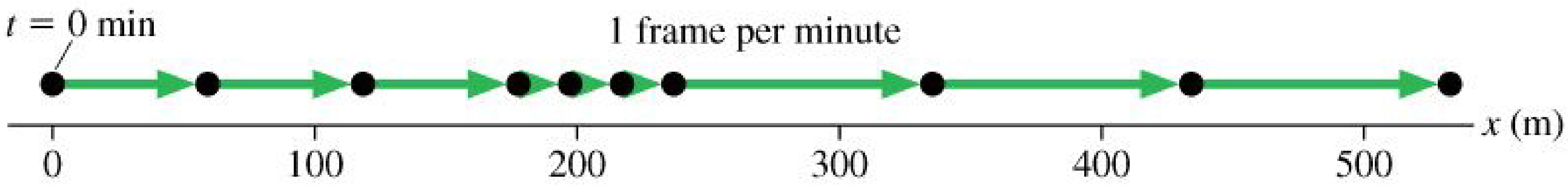
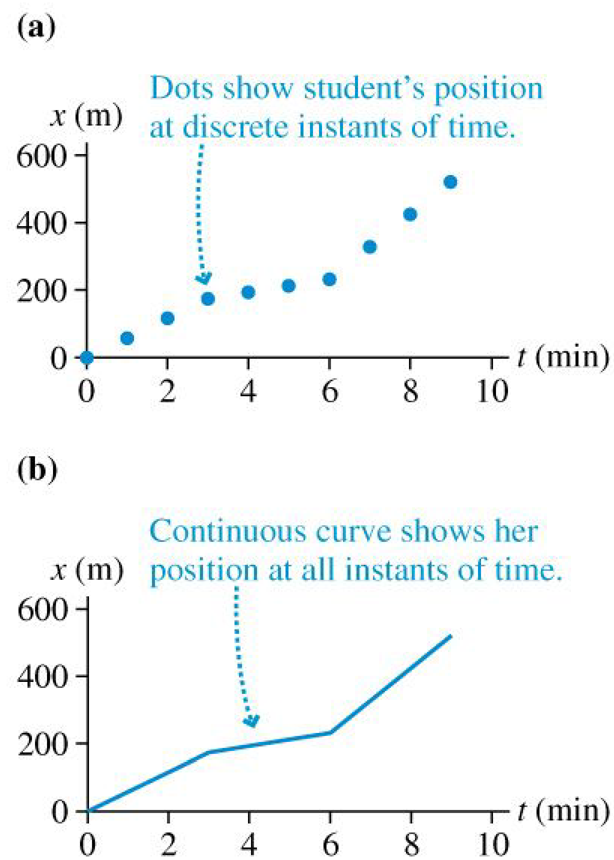


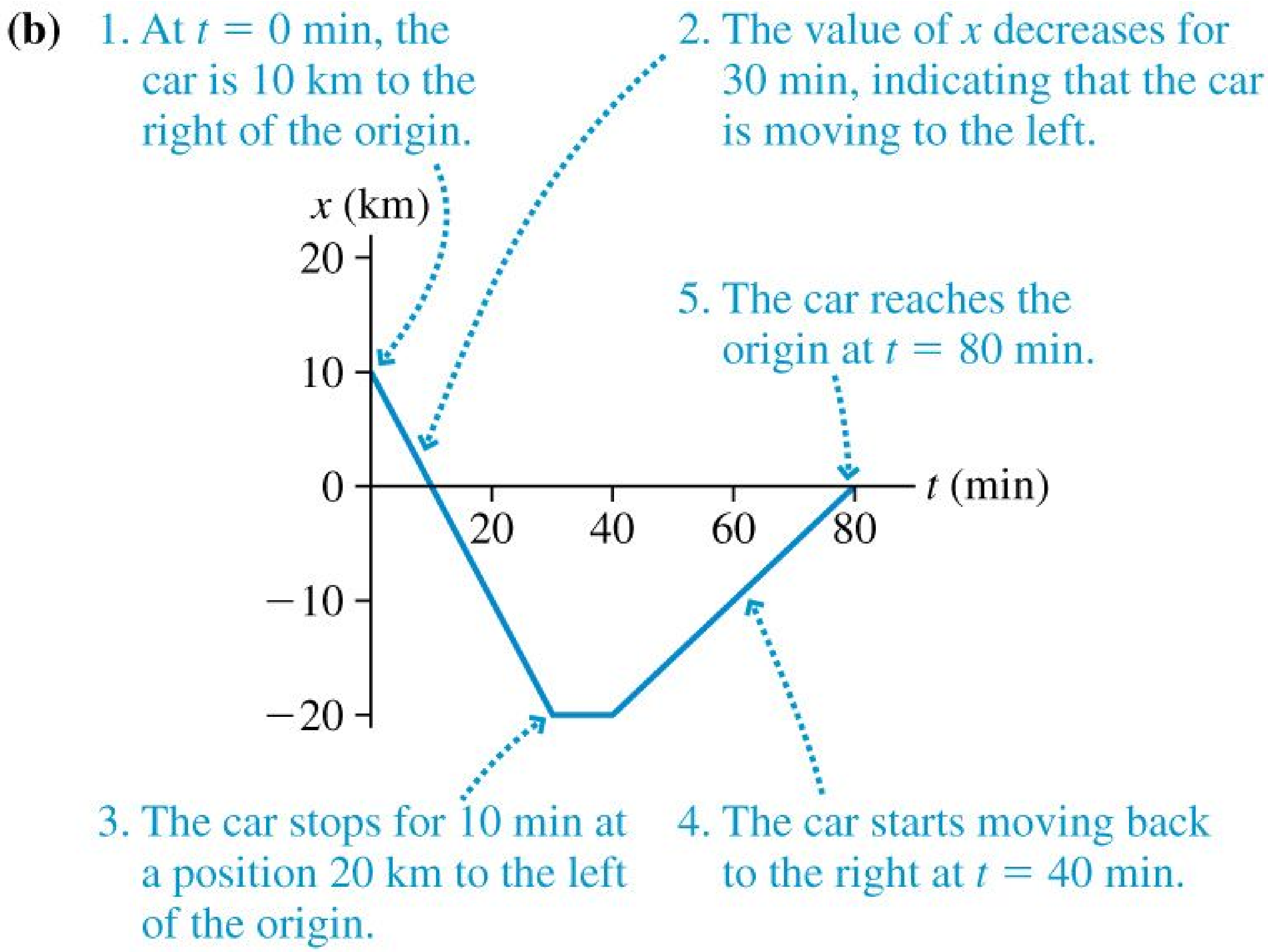
TABLE 2.1 Measured positions of a student walking to school

Time t (min)	Position x (m)
0	0
1	60
2	120
3	180
4	200
5	220
6	240
7	340
8	440
9	540

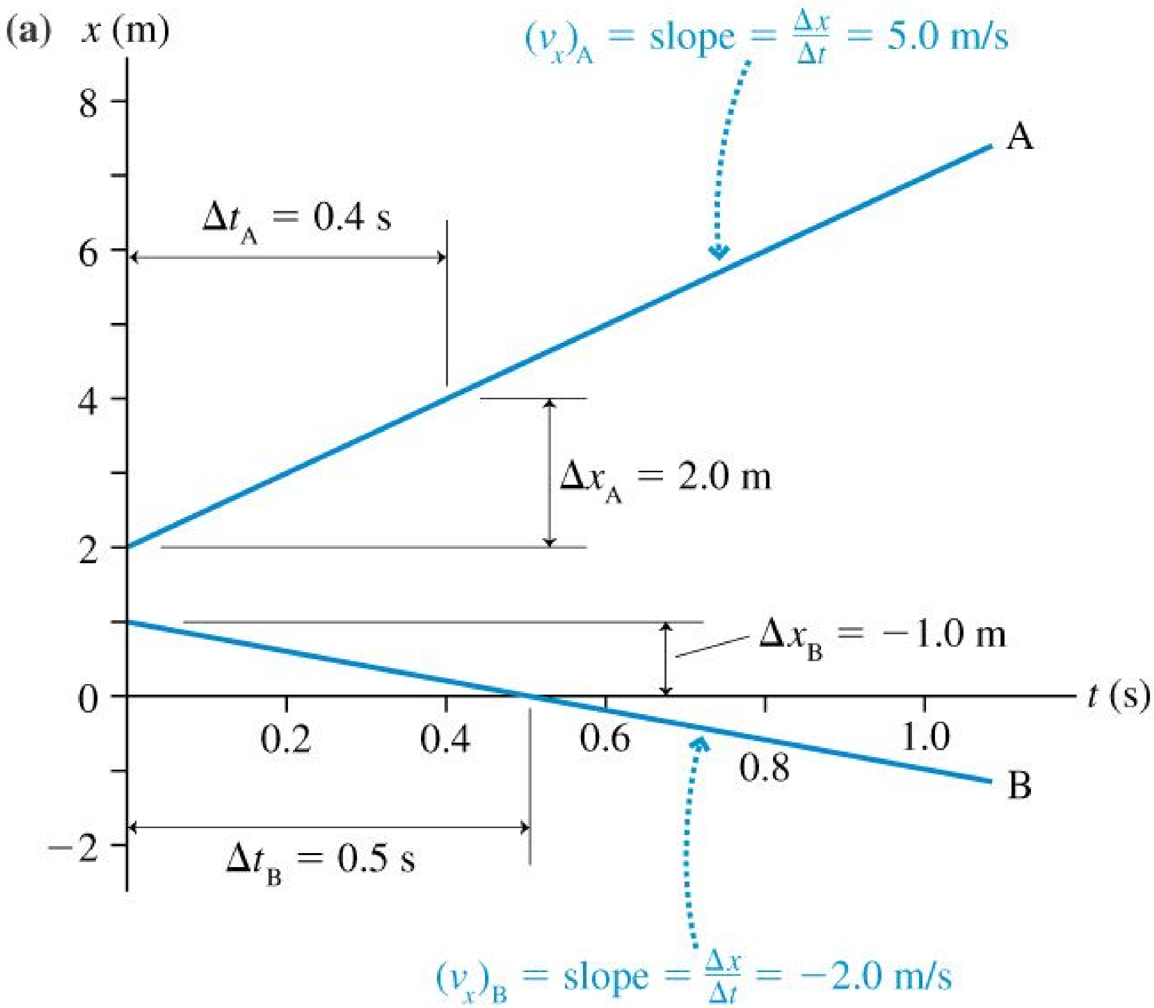
Position graphs ⇒



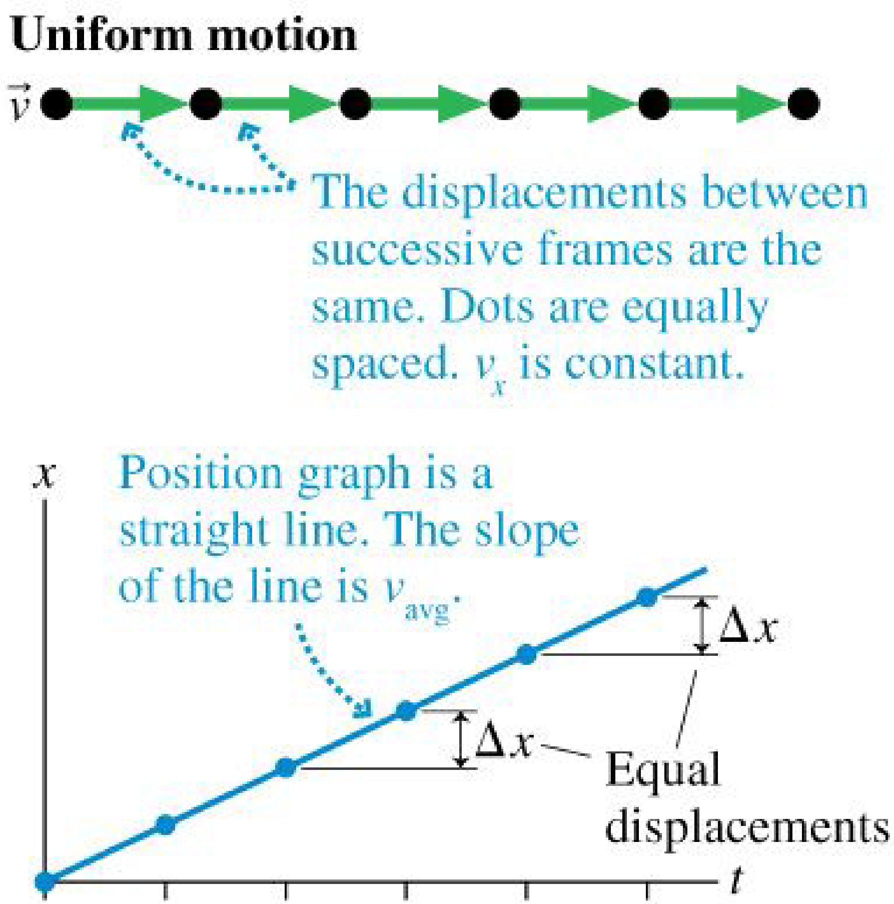
Interpreting a Position Graph

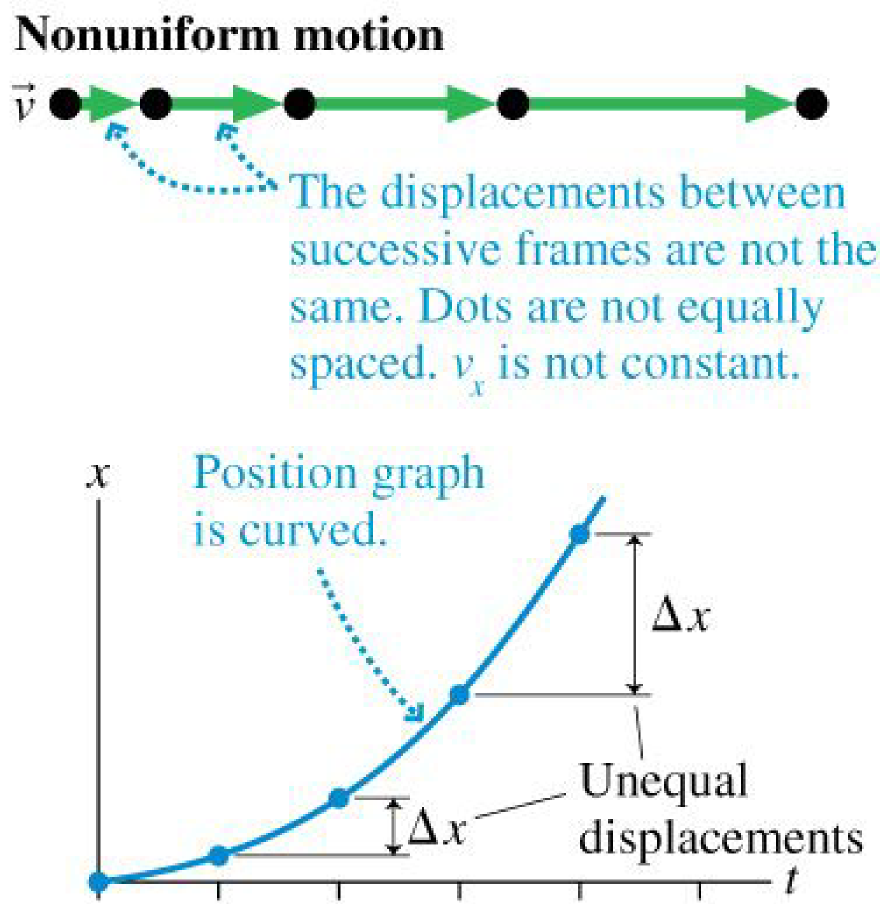


Positive & Negative Slopes



Uniform & Nonuniform Motion





Uniform Motion

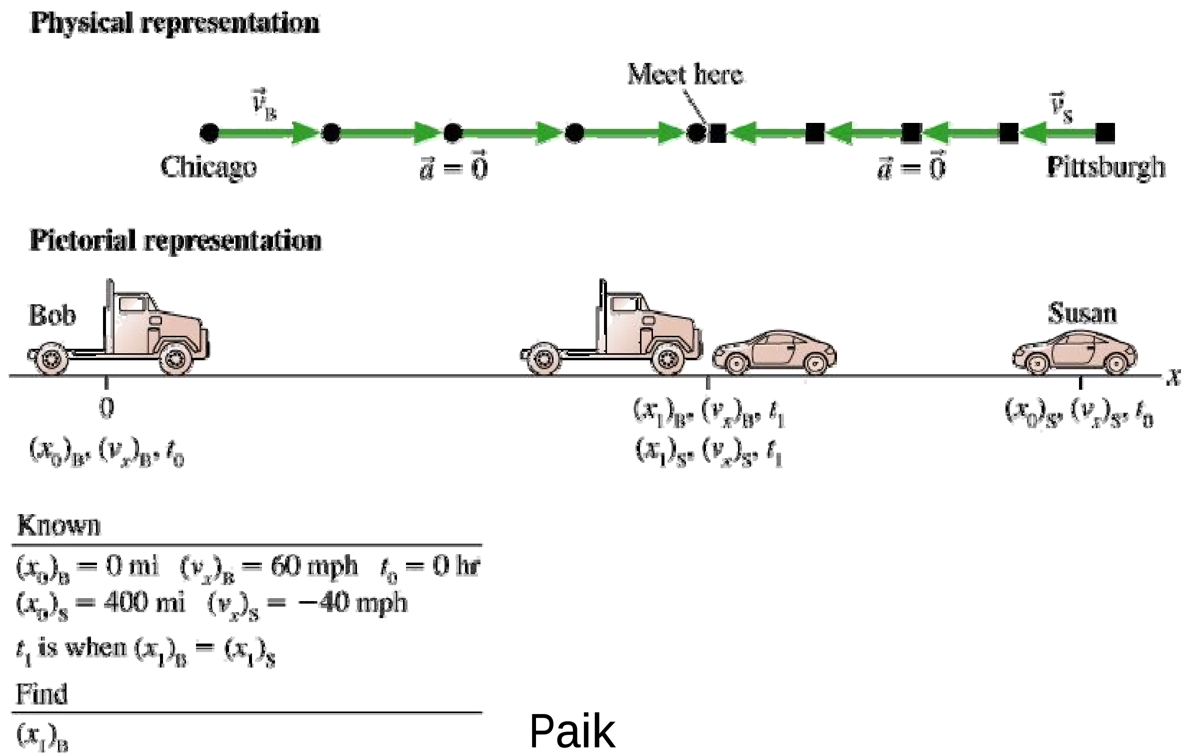
- A motion with $\vec{v} = \text{constant}$
- Consider 1-D motion in x direction

$$v_x \equiv \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t}$$

$$x_f = x_i + v_x \Delta t$$

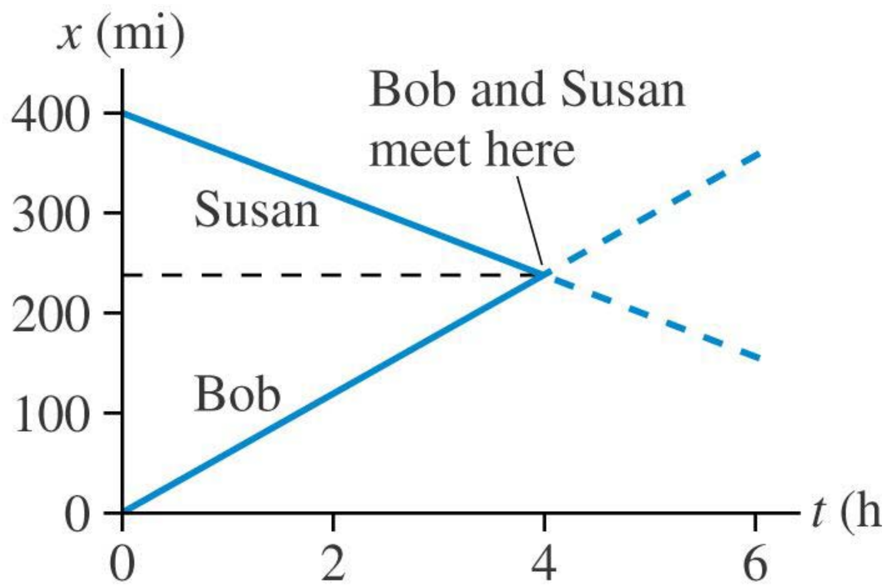
Collision Problem

Bob leaves home in Chicago at 9:00 am and travels east at a steady 60 mph. Susan, 400 miles to the east in Pittsburgh, leaves at the same time and travels west at a steady 40 mph. Where will they meet for lunch?



Position vs Time and Math

$$x_{1B} = x_{0B} + v_B(t_1 - t_0)$$
$$= 0 \text{ mi} + v_B(t_1 - 0 \text{ h}) = v_B t_1$$
$$x_{1S} = x_{0S} + v_S(t_1 - t_0)$$
$$= 400 \text{ mi} + v_S t_1$$



They meet at $t = t_1$ and when $x_{1B} = x_{1S}$.

$$x_{1B} = x_{1S}$$
$$v_B t_1 = 400 \text{ mi} + v_S t_1$$
$$(v_B t_1 - v_S t_1) = 400 \text{ mi}$$
$$t_1 = \frac{400 \text{ mi}}{[60 \text{ mph} - (-40 \text{ mph})]} = 4.0 \text{ h}$$

Using this t_1 in Bob's equation

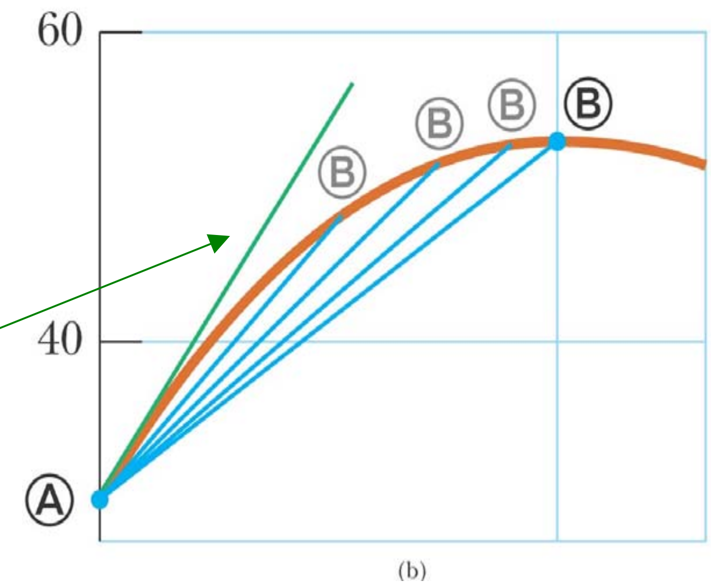
$$x_{1B} = v_{1B} t_1 = 60 \text{ mph} \times 4.0 \text{ h}$$
$$x_{1B} = 240 \text{ mi} = x_{1S}$$

Instantaneous Velocity

- The limit of the average velocity as the time interval becomes infinitesimally short.

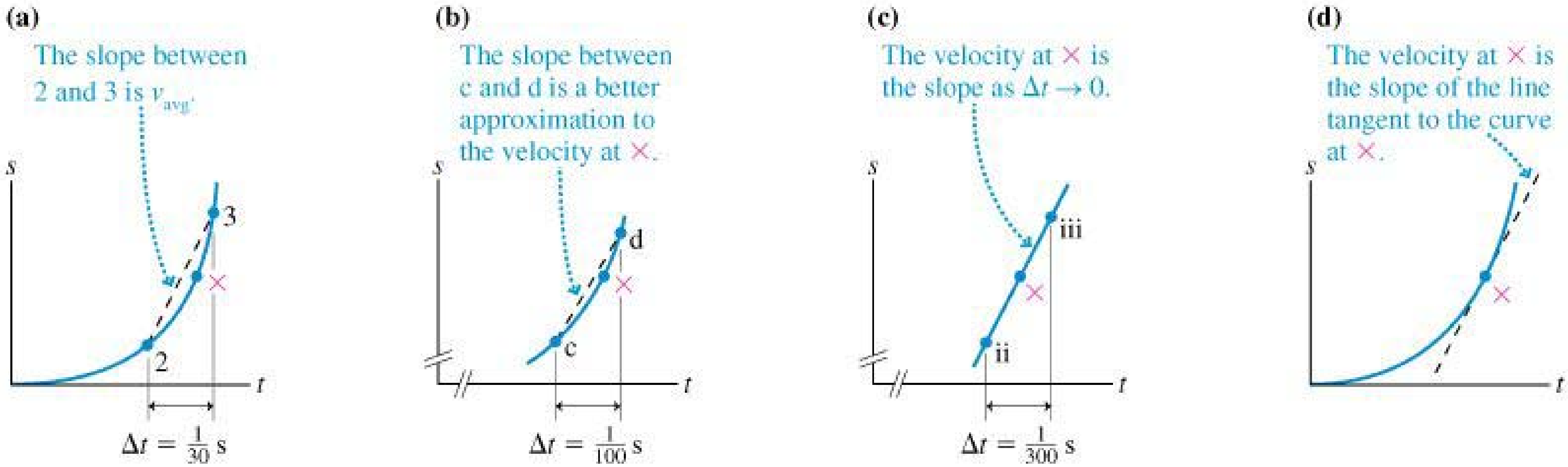
$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Slope of the green line



- The instantaneous velocity indicates what is happening at every point of time.

Instantaneous Velocity, cont



- The instantaneous speed is the magnitude of the instantaneous velocity
  - The average speed is *not* always the magnitude of the average velocity!

Position from Velocity

- Since the instantaneous velocity is

$$v_x = \frac{dx}{dt}$$

the change in position of a moving object is given by

$$dx = v_x dt$$

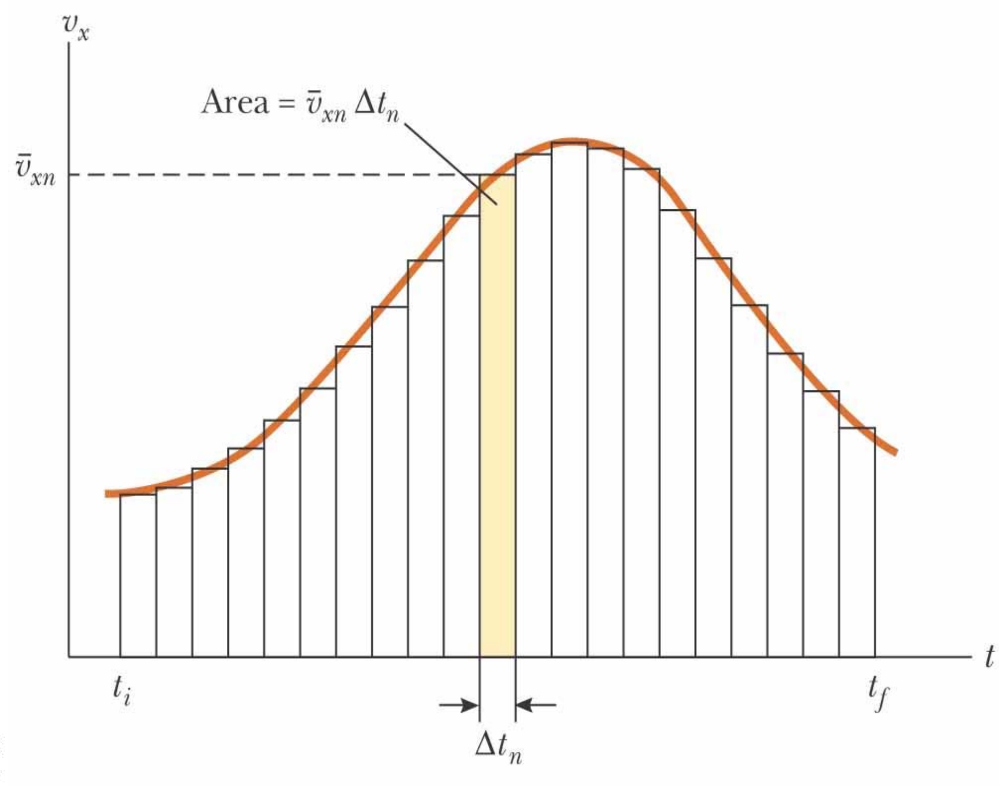
$$\int_{x_0}^{x_1} dx = \int_{t_0}^{t_1} v_x dt$$

$$x_1 - x_0 = \int_{t_0}^{t_1} v_x dt$$

Motion Equation from Calculus

- Displacement equals the area under the velocity–time curve
- The limit of the sum is a definite integral:

$$\lim_{\Delta t_n \to 0} \sum_n v_{xn} \Delta t_n = \int_{t_i}^{t_f} v_x(t)\,dt$$



Instantaneous Acceleration

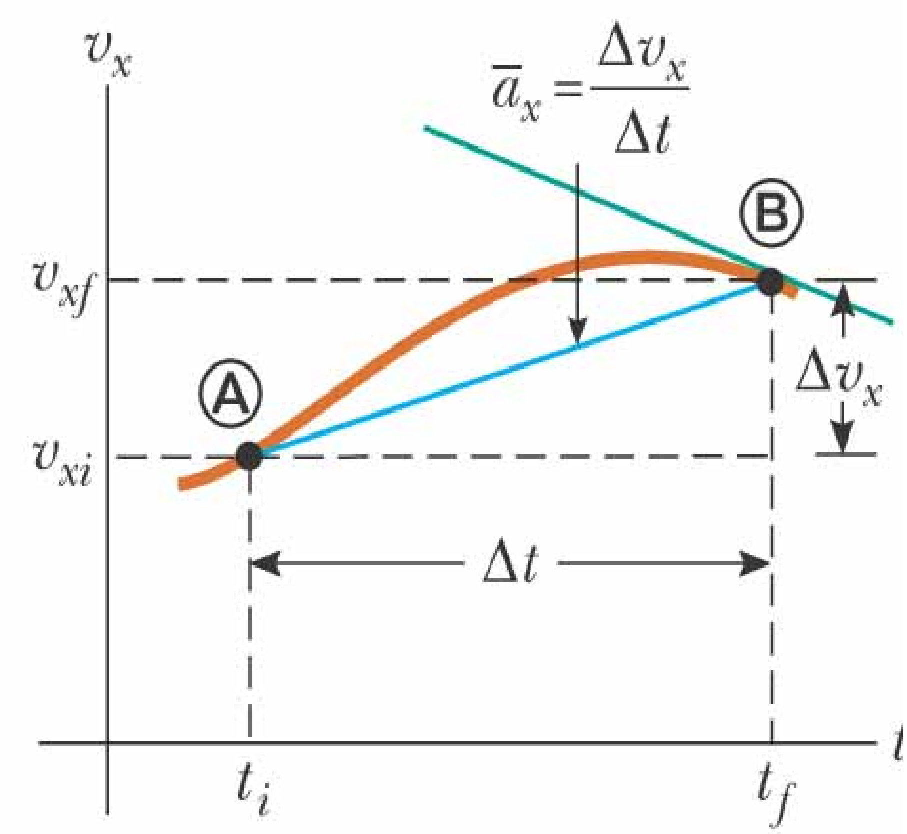
- Instantaneous acceleration is the limit of the average acceleration as Δt approaches 0:

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt}$$

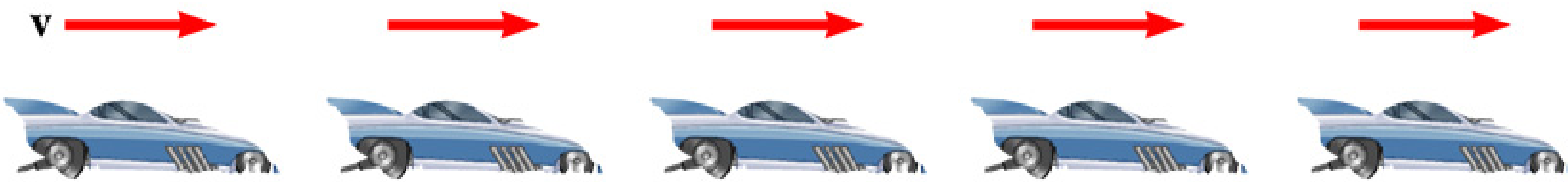
$$\text{Since } v_x = \frac{dx}{dt}, \quad a_x = \frac{d^2x}{dt^2}$$

Instantaneous Acceleration, cont

- The slope of the velocity vs time graph is the acceleration
- The blue line is the average acceleration between t_i and t_f
- The green line represents the instantaneous acceleration at t_f



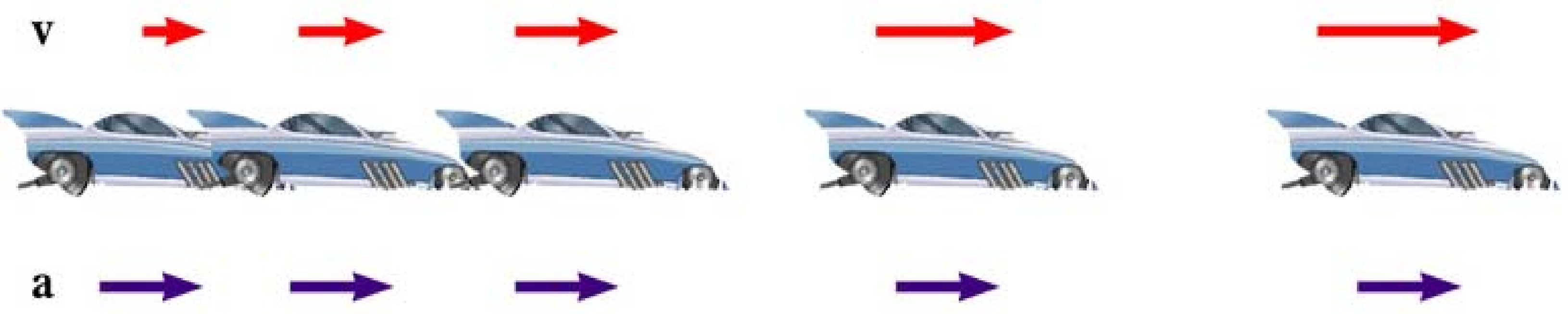
Acceleration and Velocity, 1



Equal time delay snapshots

- The car is moving with constant positive velocity (red arrows maintaining the same size)
- Acceleration equals zero

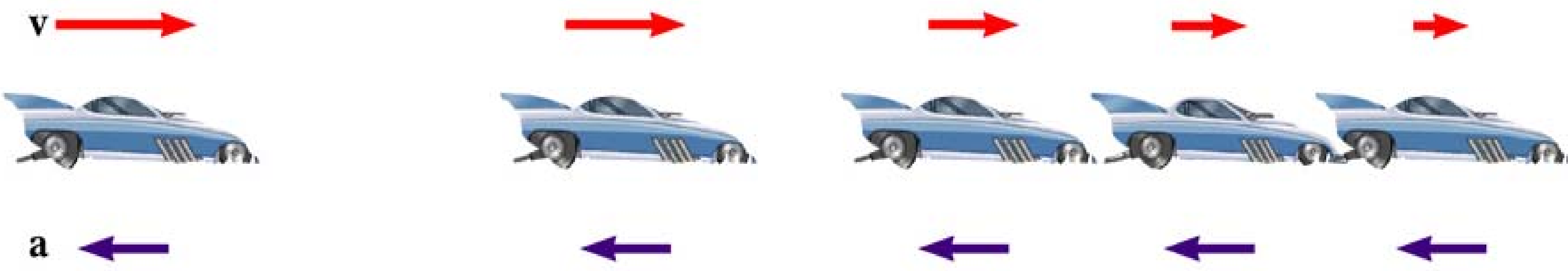
Acceleration and Velocity, 2



Equal time delay snapshots

- Velocity and acceleration are in the same direction
- Acceleration is positive and uniform (blue arrows maintaining the same length)
- Velocity is positive and increasing (red arrows getting longer)

Acceleration and Velocity, 3



Equal time delay snapshots

- Acceleration and velocity are in opposite directions
- Acceleration is negative and uniform (blue arrows maintaining the same length)
- Velocity is positive and decreasing (red arrows getting shorter)

Constant Acceleration

- For constant acceleration, the average velocity can be expressed as the arithmetic mean of the initial and final velocities

$$v_{ave} = \frac{v_0 + v}{2}$$

  - Not true if *a* is not constant

Velocity from Acceleration

- Since the instantaneous acceleration is

$$a = \frac{dv}{dt}$$

the change in velocity is given by

$$\int_{t_0}^{t} dv = v - v_0 = \int_{t_0}^{t} a\,dt$$

- If *a* is a constant, $\int_{t_0}^{t} a\,dt = a(t - t_0)$

$$\boxed{v = v_0 + a(t - t_0)} \qquad \text{Eq. (1)}$$

Displacement from Acceleration

- For constant acceleration,

$$x = x_0 + v_{ave}(t - t_0)$$

- Since $v_{ave} = \frac{1}{2}(v_0 + v)$ and $v = v_0 + a(t - t_0)$

$$v_{ave} = \frac{1}{2}[v_0 + v_0 + a(t - t_0)] = v_0 + \frac{1}{2}a(t - t_0)$$

- Therefore,

$$x = x_0 + v_0(t - t_0) + \frac{1}{2}a(t - t_0)^2 \quad \text{Eq. (2)}$$

With Time Eliminated

- From Eq. (1),

$$t - t_0 = \frac{v - v_0}{a}$$

- Substituting this into Eq. (2),

$$x - x_0 = v_0 \left(\frac{v - v_0}{a} \right) + \tfrac{1}{2} a \left(\frac{v - v_0}{a} \right)^2$$

$$= \frac{1}{2a} \left(2v_0 v - 2v_0^2 + v^2 - 2v_0 v + v_0^2 \right)$$

- Therefore,

$$v^2 = v_0^2 + 2a(x - x_0) \qquad \text{Eq. (3)}$$

1-D Kinematic Equations

- With constant acceleration,

$$(1)\ \ v = v_0 + a(t - t_0)$$

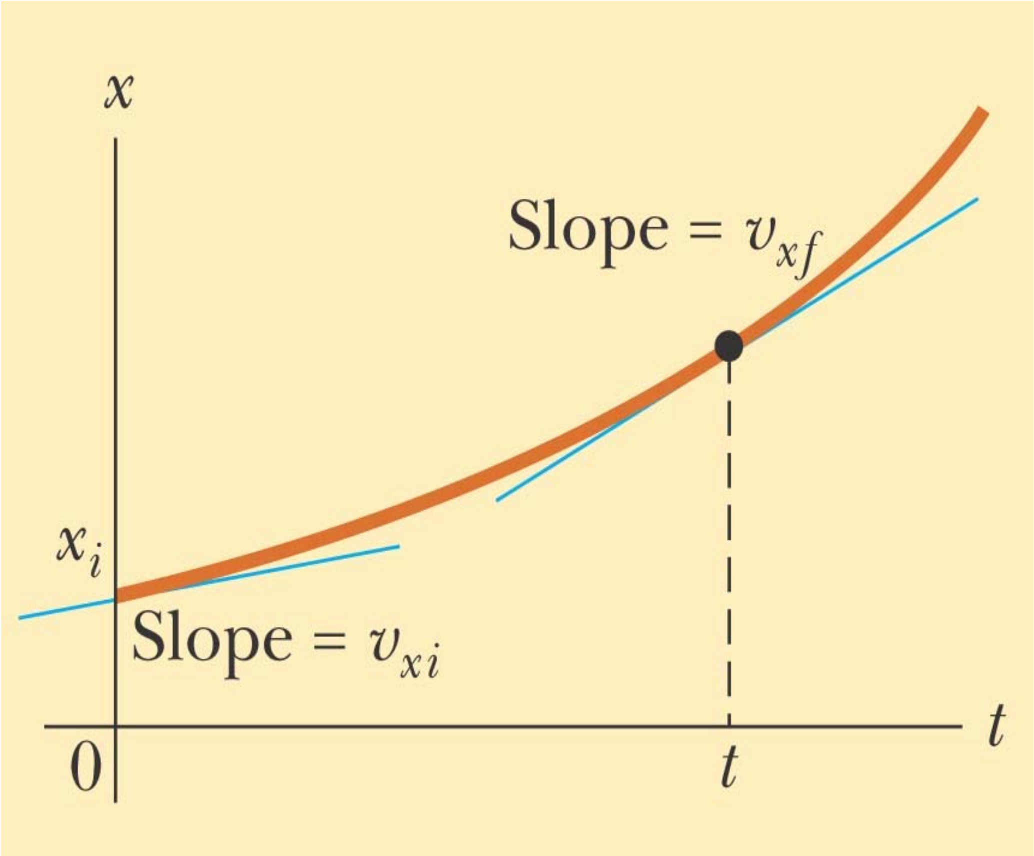
$$(2)\ \ x = x_0 + v_0(t - t_0) + \tfrac{1}{2}a(t - t_0)^2$$

$$(3)\ \ v^2 = v_0^2 + 2a(x - x_0)$$

- You may need to use two of the equations to solve one problem
- Many times there is more than one way to solve a problem

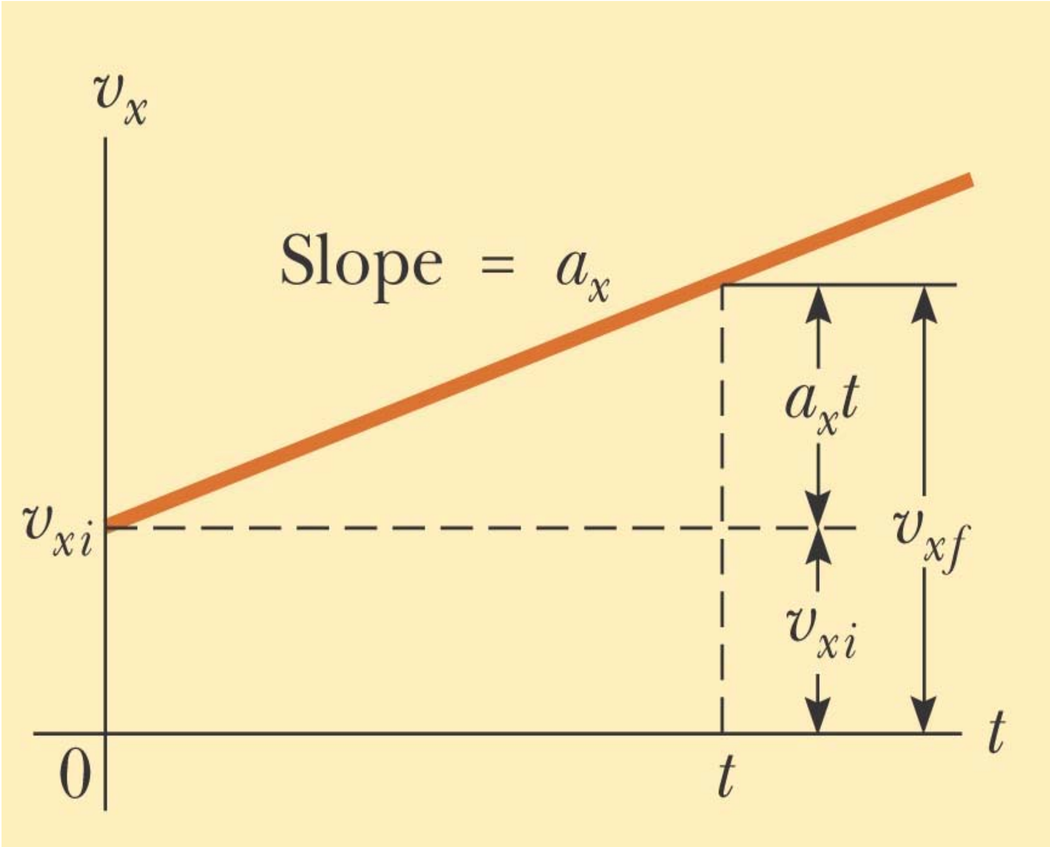
Displacement–Time Curve

- Slope of the curve is the velocity
- Curved line indicates the velocity is changing
  - Therefore, there is an acceleration



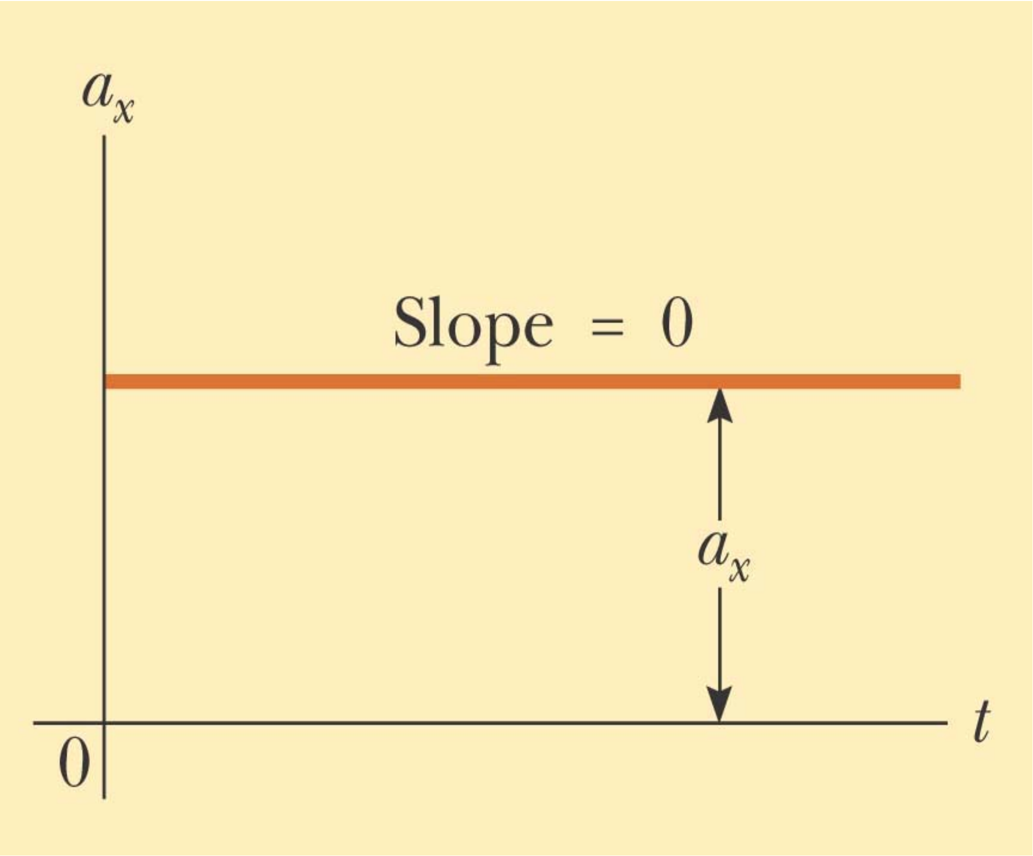
Velocity–Time Curve

- Slope is the acceleration
- Straight line indicates the velocity is changing at a constant rate
  - Therefore, a constant acceleration



Acceleration–Time Curve

- Slope is the rate of change in acceleration
- Zero slope indicates a constant acceleration



Example of $a = 0$

A car is moving at a constant velocity of 60 mph. The driver suddenly sees an animal crossing the road ahead. If the driver's reaction time is 0.20 s, show how far the car will go before the driver pushes on the brake pedal.

- We know

  v_0 = 60 mph x (0.447 m/s / 1 mph) = 27 m/s,

  a = 0 m/s^2, $t - t_0 = t_{react}$ = 0.20 s

- Use the equation:

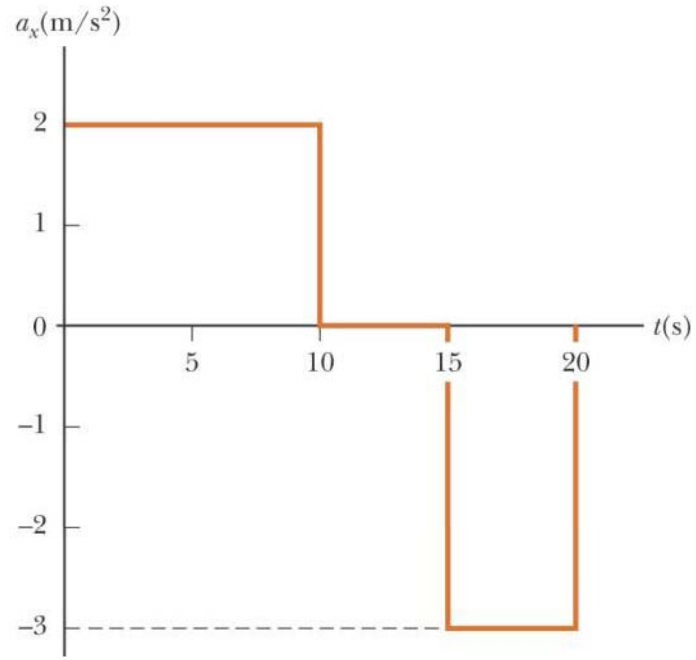
$$x = x_0 + v_0(t - t_0) + \tfrac{1}{2}a(t - t_0)^2$$

- The displacement of the car before putting on the brakes:

  $x - x_0$ = 27 m/s x 0.20 s + 0 m = 5.4 m

Example of $a \neq 0$

A particle starts from rest and accelerates as shown in the figure. Determine (a) the particle's speed at t = 10.0 s and 20.0 s, and (b) the distance traveled in the first 20.0 s.



$$\text{(a)}\ v_{10} = v_0 + a_1\Delta t_1 = 0 + 2.00\times 10.0 = 20.0\ \mathrm{m/s}$$

$$v_{15} = v_{10} + a_2\Delta t_2 = 20.0 + 0\times 5.0 = 20.0\ \mathrm{m/s}$$

$$v_{20} = v_{15} + a_3\Delta t_3 = 20.0 + (-3.00)\times 5.0 = 5.0\ \mathrm{m/s}$$

$$\text{(b)}\ x_{10} = x_0 + v_0\Delta t_1 + \tfrac{1}{2}a_1(\Delta t_1)^2 = 0 + 0\times 10.0 + \tfrac{1}{2}\times 2.00\times 10.0^2 = 100\ \mathrm{m}$$

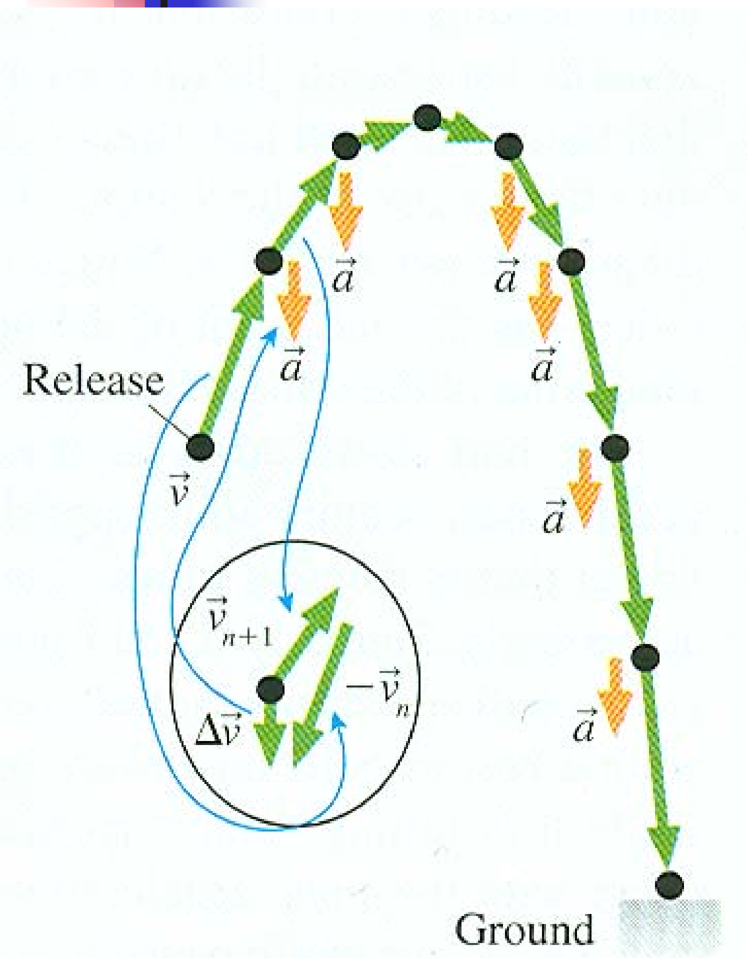
$$x_{15} = x_{10} + v_{10}\Delta t_2 + \tfrac{1}{2}a_2(\Delta t_2)^2 = 100 + 20.0\times 5.0 + \tfrac{1}{2}\times 0\times 5.0^2 = 200\ \mathrm{m}$$

$$x_{20} = x_{15} + v_{15}\Delta t_3 + \tfrac{1}{2}a_3(\Delta t_3)^2 = 200 + 20.0\times 5.0 + \tfrac{1}{2}(-3.00)\times 5.0^2 = 263\ \mathrm{m}$$

Freely Falling Objects

- A *freely falling* object is any object moving freely under the influence of gravity alone, i.e., with negligible air drag
- Object has a constant acceleration due to gravity
- Demonstration 1: Free fall in vacuum
- Demonstration 2: Measure *a*

Acceleration of Free Fall



- Acceleration of an object in free fall is downward, regardless of the initial motion
- The magnitude of free fall acceleration is usually taken as $g = 9.80\ m/s^2$, however
  - g decreases with altitude
  - g varies with latitude
  - $9.80\ m/s^2$ is the average at the Earth's surface

Free Fall Example

At A, initial velocity is upward (+20 m/s) and acceleration is $-g$ (-9.8 m/s^2).

At B, velocity is 0 and acceleration is $-g$.

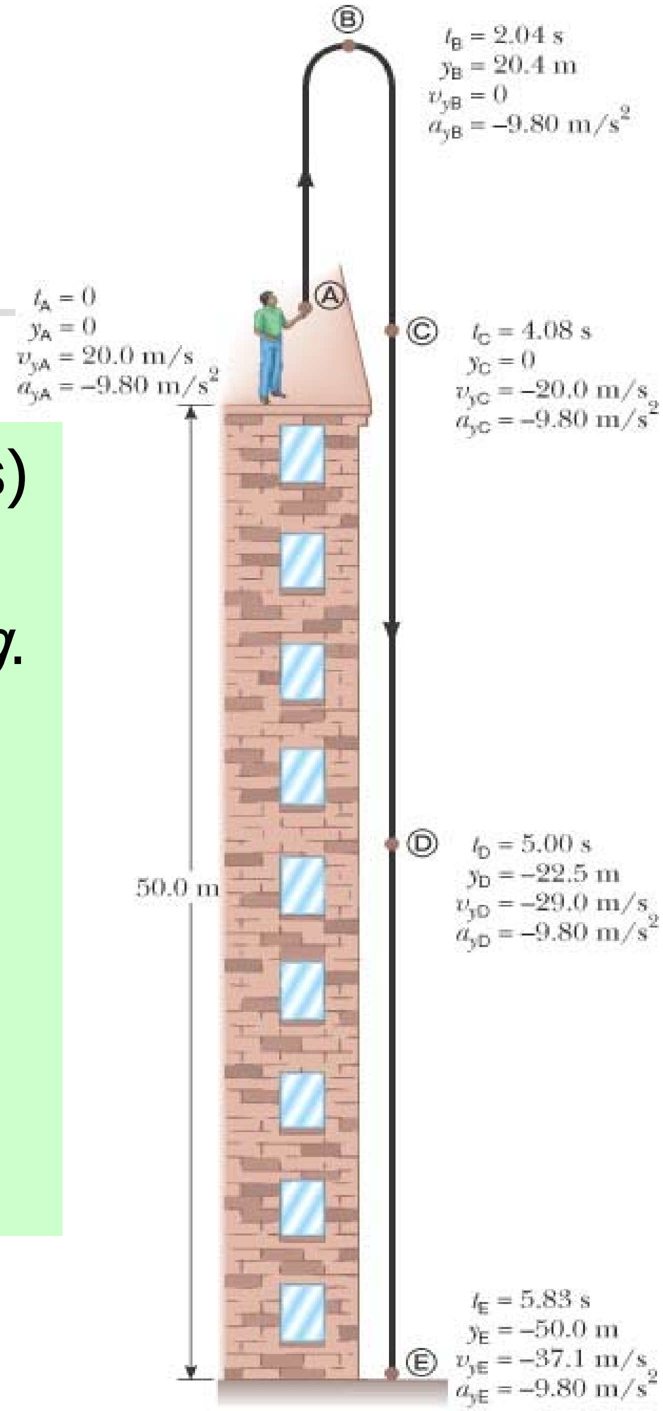
At C, velocity has the same magnitude as at A, but is in the opposite direction.

The final displacement is -50.0 m.

(a) Find the distance from A to B.

(b) Find the velocity at C.

(c) Find the velocity at $y_E = -50.0$ m.



Free Fall Example, cont

$$\text{(a) } v_B^2 = v_A^2 + 2a(y_B - y_A)$$

$$0 = (20.0\,\text{m/s})^2 + 2(-9.80\,\text{m/s}^2)(y_B - 0\,\text{m})$$

$$y_B = \frac{(20.0\,\text{m/s})^2}{2(9.80\,\text{m/s}^2)} = \frac{400(\text{m/s})^2}{18.60\,\text{m/s}^2} = 20.4\,\text{m}$$

$$\text{(b) } v_C^2 = v_B^2 + 2a(y_C - y_B)$$

$$= 0 + 2(-9.80\,\text{m/s}^2)(0\,\text{m} - 20.4\,\text{m}) = 399.8(\text{m/s})^2$$

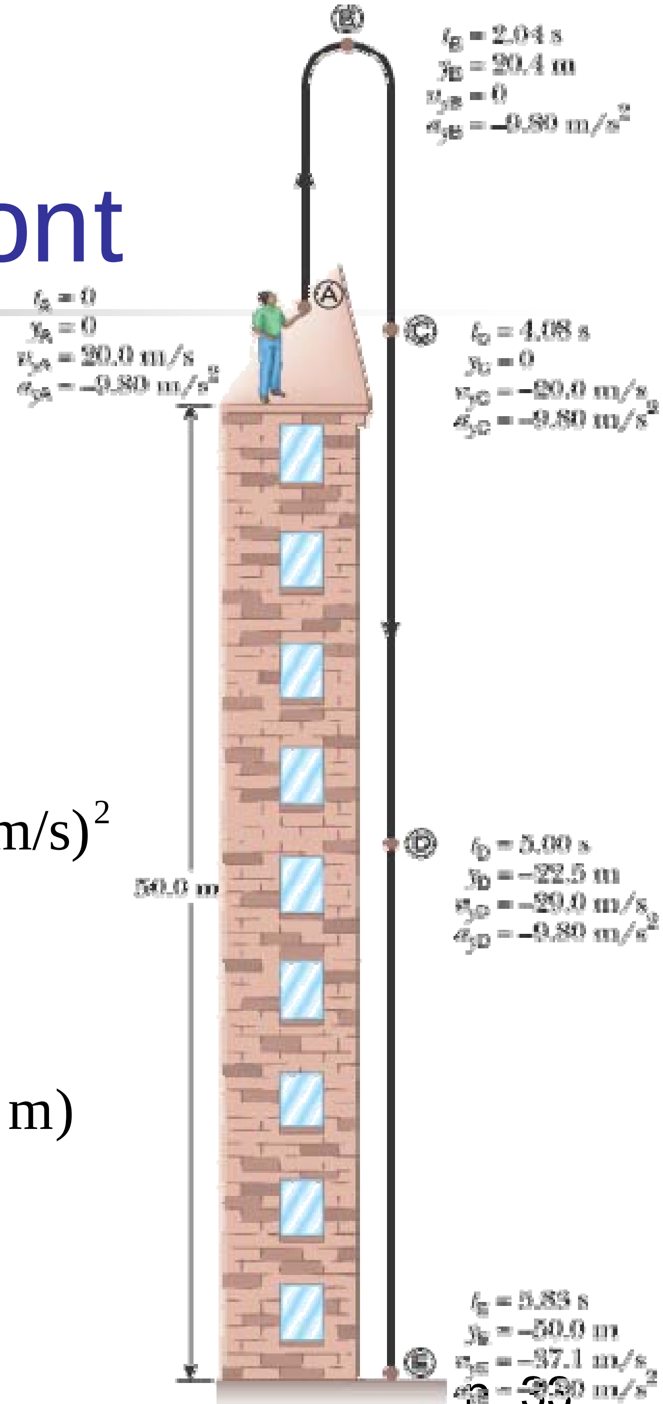
$$v_C = \pm 20.0\,\text{m/s (choose minus sign, as down)}$$

$$\text{(c) } v_E^2 = v_C^2 + 2a(y_E - y_C)$$

$$v_E^2 = (-20.0\,\text{m/s})^2 + 2(-9.80\,\text{m/s}^2)(-50.0\,\text{m} - 0\,\text{m})$$

$$= 400(\text{m/s})^2 + 980(\text{m/s})^2 = 1380(\text{m/s})^2$$

$$v_E = \pm 37.1\,\text{m/s} \quad \text{(choose the minus sign)}$$



Problem Solving – Conceptualize

- Think about and understand the situation
  - Make a quick drawing of the situation
- Gather the numerical information
  - Write down the "givens"
- Focus on the expected result
  - What are you asked to find?
- Think about what a reasonable answer should be, e.g., sign of quantities, units

Problem Solving – Categorize

- Simplify the problem
  - Can you ignore air resistance?
  - Model objects as particles
- Try to identify similar problems you have already solved

Problem Solving – Analyze

- Select the relevant equation(s) to apply
- Solve for the unknown variable
  - Substitute appropriate numbers
- Calculate the results
  - Include units
  - Round the result to the appropriate number of significant figures

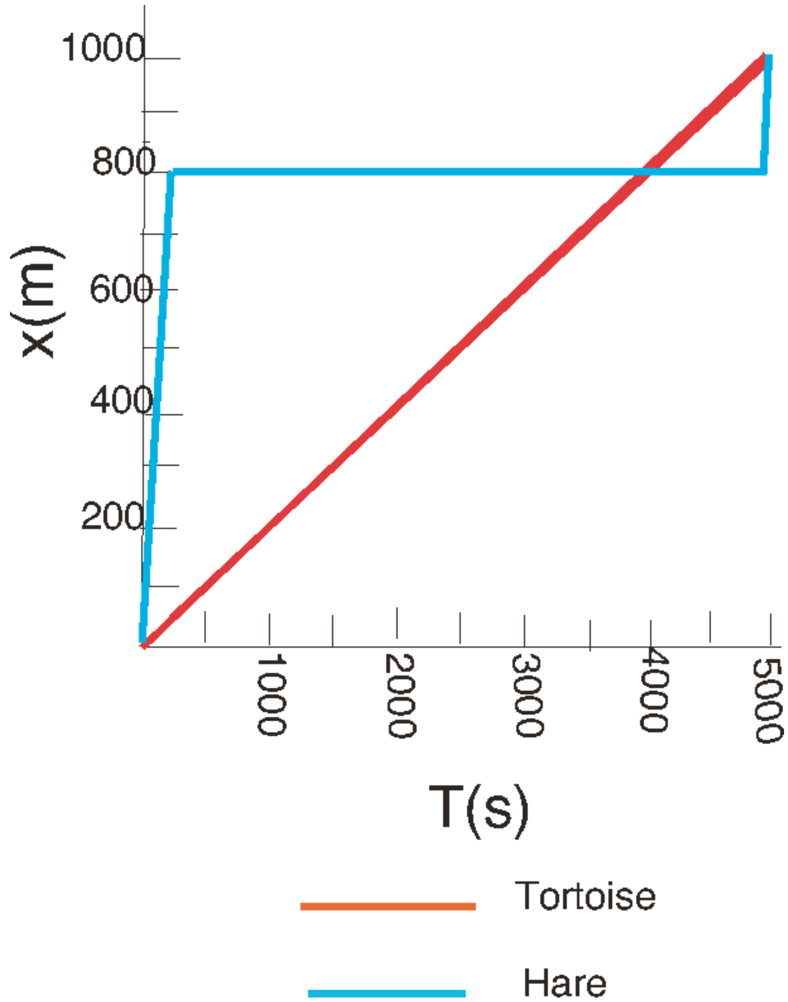
Problem Solving – Finalize

- Check your result
  - Does it have the correct units?
  - Does it agree with your conceptualized ideas? Does the answer have the correct signs?
- Look at limiting situations to be sure the results are reasonable
  - e.g., the correct limit if $a = 0$?
- Compare the result with those of similar problems

Example 1

A hare and a tortoise compete in a 1.00 km race. The tortoise crawls steadily at its maximum speed of 0.200 m/s toward the finish line. The hare runs at its maximum speed of 8.00 m/s toward the goal for 0.800 km and then stops to tease the tortoise.

How close to the goal can the hare let the tortoise approach before resuming the race?



Example 1, cont

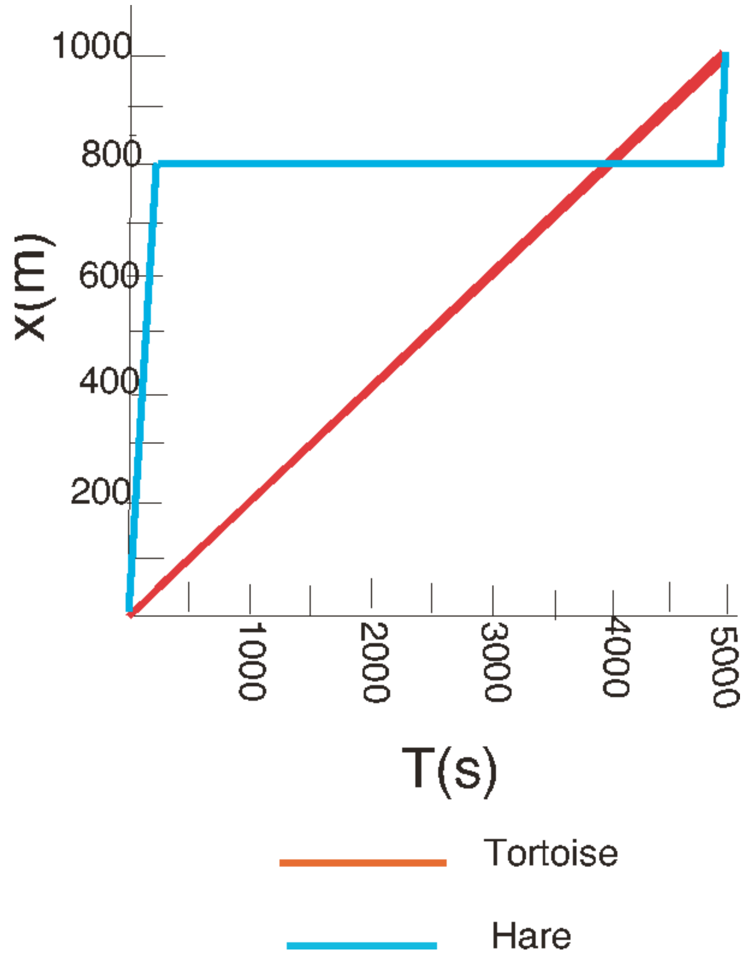
The hare sits at 800 m waiting for the tortoise. To go the last 200 m, the hare will take

Δt = 200 m/(8.00 m/s) = 25.0 s

In 25.0 s, the tortoise can go

Δx = 0.200 m/s x 25.0 s = 5.00 m

In order for the hare to win, he must restart before the tortoise gets within 5.00 m of the finish line.



Example 2

Jules Verne in 1865 suggested sending people to the Moon by firing a space capsule from a 220-m long cannon with a final velocity of 10.97 km/s.

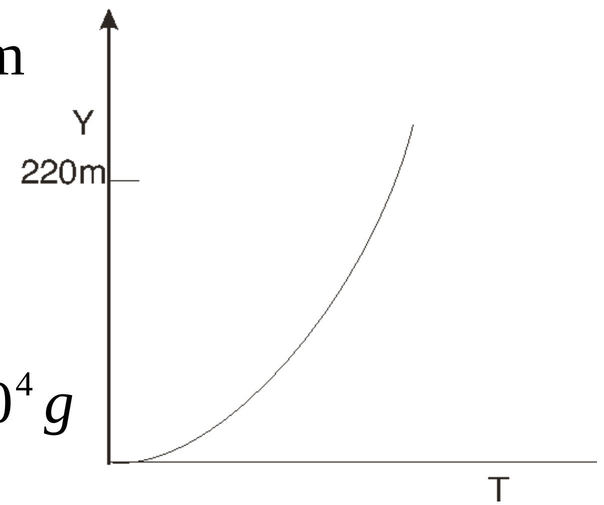
What would have been the acceleration experienced by the space travelers during launch? Compare your answer with the free-fall acceleration 9.80 m/s^2.

$$v_i = 0\text{ m/s}, v_f = 10.97\times10^3\text{ m/s}, y_i = 0\text{ m}, y_f = 220\text{ m}$$

$$v_f^2 = v_i^2 + 2a(y_f - y_i)$$

$$(10.97\times10^3\text{ m/s})^2 = (0\text{ m/s})^2 + 2a(220\text{ m})$$

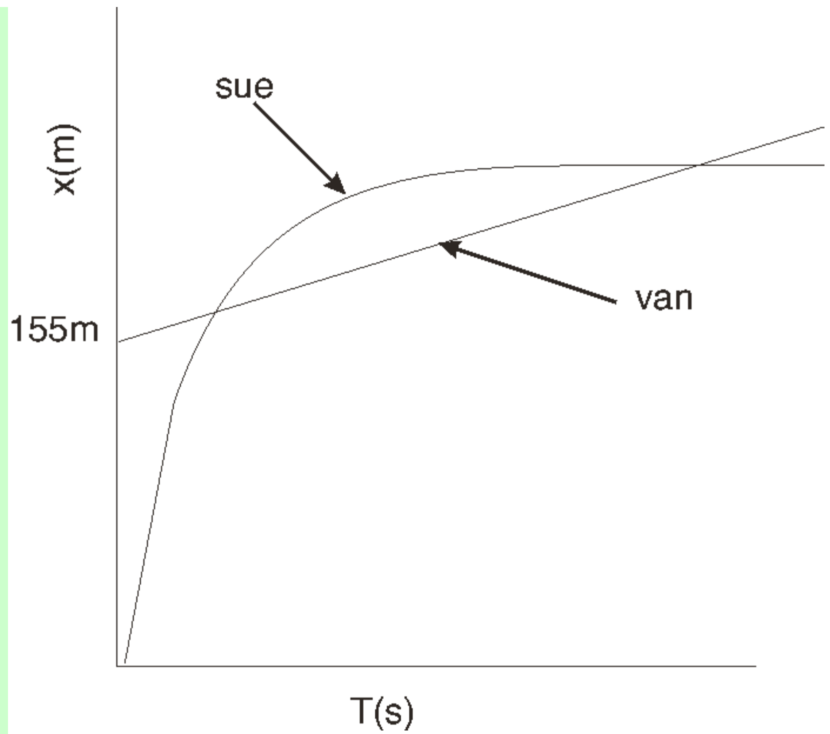
$$a = \frac{120.3\times10^6(\text{m/s})^2}{440\text{ m}} = 2.735\times10^5\text{ m/s}^2 = 2.79\times10^4 g$$



Example 3

Speedy Sue, driving at 30.0 m/s, enters a one-lane tunnel. She then observes a slow-moving van 155 m ahead traveling at 5.00 m/s. Sue applies her brakes but can accelerate only at –2.00 m/s^2 because the road is wet.

Will there be a collision? If yes, determine how far into the tunnel and at what time the collision occurs.



Example 3, cont

Sue's position :

$$x_S(t) = x_{0S} + v_{0S}t + \tfrac{1}{2}a_S t^2 = 0\text{ m} + (30.0\text{ m/s})t + \tfrac{1}{2}(-2.00\text{ m/s}^2)t^2$$

Van's position :

$$x_V(t) = x_{0V} + v_{0V}t + \tfrac{1}{2}a_V t^2 = 155\text{ m} + (5.00\text{ m/s})t + \tfrac{1}{2}(0\text{ m/s}^2)t^2$$

Collision occurs if there is a solution t_C to $x_S(t_C) = x_V(t_C)$:

$$30.0t_C - t_C^2 = 155 + 5.00t_C \quad \text{or} \quad t_C^2 - 25.0t_C + 155 = 0$$

$$t_C = \frac{25.0 \pm \sqrt{25.0^2 - 4\times 155}}{2} = 13.6\text{ s} \quad \text{or} \quad 11.4\text{ s}$$

The smaller one is the collision time. The wreck happens at position :

$$x_V(t_C) = 155\text{ m} + 5.00\text{ m/s} \times 11.4\text{ s} = 212\text{ m}$$

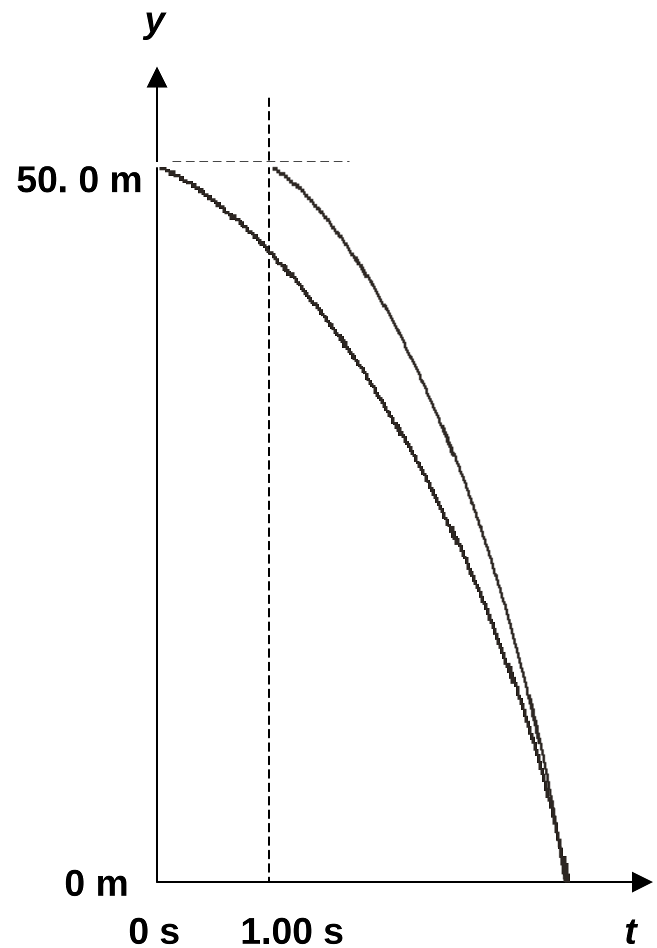
Example 4

A student climbs a 50.0-m cliff that overhangs a calm pool of water. He throws two stones vertically downward, 1.00 s apart, and observes that they cause a single splash. The first stone has an initial speed of 2.00 m/s.

(a) How long after release of the first stone do the two stones hit the water?

(b) What initial velocity must the second stone have?

(c) What is the speed of each stone at the instant the two hit the water?



Example 4, cont

(a) First stone: $y_1 = y_{01} + v_{01}(t - t_{01}) + \frac{1}{2}a(t - t_{01})^2$

$$0\text{ m} = 50.0\text{ m} + (-2.00\text{ m/s})(t - 0\text{ s}) + \tfrac{1}{2}(-9.8\text{ m/s}^2)(t - 0\text{ s})^2$$

$$t = \frac{-2.00 \pm \sqrt{2.00^2 - 4(4.90)(-50.0)}}{2(4.90)} = 3.00\text{ s} \text{ or } -3.40\text{ s} \text{ (Choose +)}$$

(b) The second stone: $y_2 = y_{20} + v_{20}(t - t_{20}) + \frac{1}{2}a(t - t_{20})^2$

$$-50.0\text{ m} = 0\text{ m} + v_{20}(3.00\text{ s} - 1.00\text{ s}) + \tfrac{1}{2}(-9.80\text{ m/s}^2)(3.00\text{ s} - 1.00\text{ s})^2$$

$$v_{20} = \frac{-50.0 + 19.6}{2.00} = -15.2\text{ m/s}$$

(c) $v_1 = v_{10} + a(t - t_{10})$

$$= -2.00\text{ m/s} + (-9.80\text{ m/s}^2)(3.00\text{ s} - 0\text{ s}) = -31.4\text{ m/s}$$

$$v_2 = \text{v}_{20} + a(t - t_{20})$$

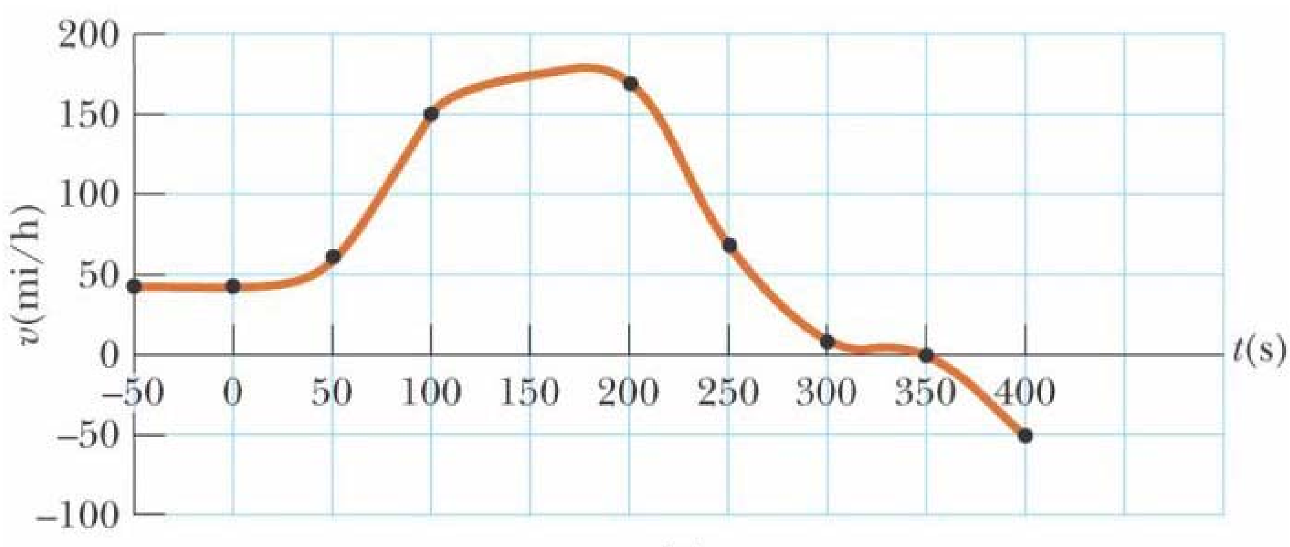
$$= -15.3\text{ m/s} + (-9.80\text{ m/s}^2)(3.00\text{ s} - 1.00\text{ s}) = -34.9\text{ m/s}$$

Example 5

The Acela is the Porsche of American trains. It can carry 304 passengers at 170 mi/h. A velocity-time graph for the Acela is shown in the figure.

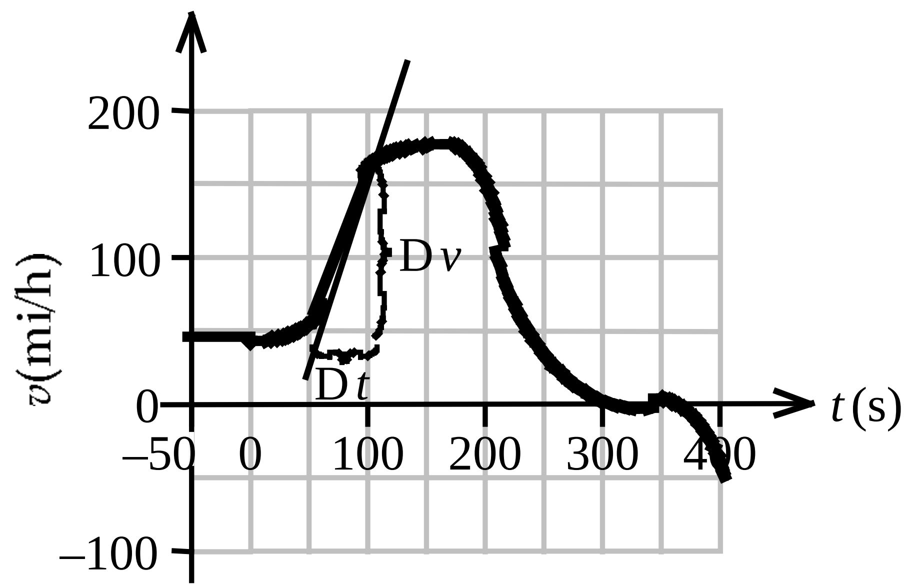
(a) Find the peak positive acceleration of the train.

(b) Find the train's displacement between t = 0 and t = 200 s.



Example 5, cont

(a) Peak acceleration is given by the slope of the steepest tangent to the v-t curve.



From the tangent line shown, we find

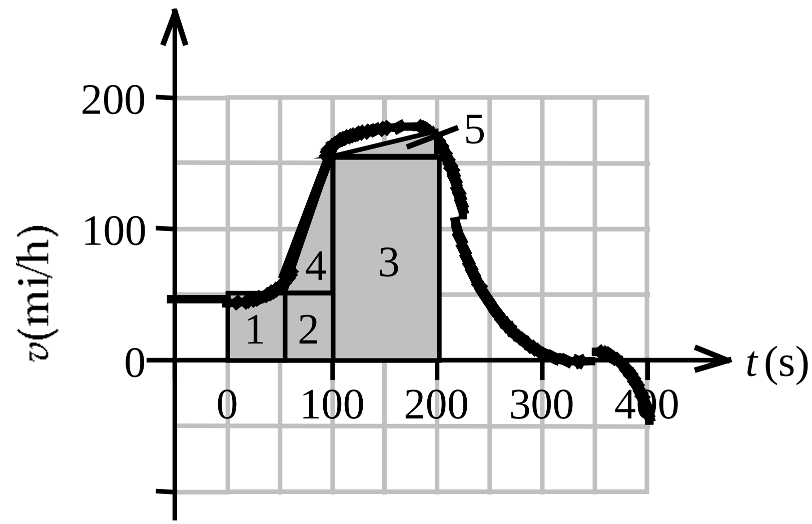
$$a = \frac{\Delta v}{\Delta t} = \frac{(155 - 45)\ \text{mi/h}}{(100 - 50)\ \text{s}} = 2.2\ (\text{mi/h})/\text{s}$$

$$= 2.2 \times \frac{1600\ \text{m}}{3600\ \text{s}} \times \frac{1}{\text{s}} = 0.98\ \text{m/s}^2$$

Example 5, cont

(b) Area under the v - t curve equals the displacement.

We approximate the area with a series of triangles and rectangles.



$$\Delta x = \text{area } 1 + \text{area } 2 + \text{area } 3 + \text{area } 4 + \text{area } 5$$

$$= 50 \text{ mi/h} \times 50 \text{ s} + 50 \text{ mi/h} \times 50 \text{ s} + 160 \text{ mi/h} \times 100 \text{ s}$$

$$= \tfrac{1}{2} \times 100 \text{ mi/h} \times 50 \text{ s} + \tfrac{1}{2} \times (170 - 160) \text{ mi/h} \times 100 \text{ s}$$

$$= 24000 \text{ mi/h} \times \text{s} = \frac{24000 \text{ mi}}{3600 \text{ s}} \times \text{s} = 6.7 \text{ mi}$$