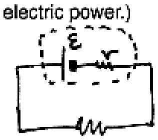
PHYS 142, EXAM 2 NAME: ______________________ SECTION: ________

1. (10 pts) A 100W audio amplifier with an internal impedance ("resistance") of 8Ω is connected to a speaker with 8Ω of resistance. (a)(6 pts) How much power is actually delivered to the speaker? (b)(4 pts) Show that 8Ω is the optimal resistance of the "load" speaker for getting maximum power out of this amp.
(Hint: You can treat the amp as a battery with internal resistance. You can say that this "battery" produces 100W of electric power.)

(6) a)



$R = r = 8\Omega$

$P_A = 100W$ – power produced by amp

$P_s = ?$ power used up by speaker

(2) picture

(3) equations

(1) 50W (or $\frac{1}{2}$)

$P_A = I\mathcal{E}$ ← how power is produced

$P_A = I^2 r + I^2 R$ ← how power is used up in the circuit

$\Rightarrow P_A = I^2(r+R) = I^2(2R) = 2I^2R = 2P_s$

$P_A = \frac{V^2}{2R}$

$\Rightarrow \boxed{P_s = \frac{1}{2}P_A}$ $P_s = 50W$

(4) b)

$P_s(R) = I^2 R$ ← power on the speaker as a function of speaker's resistance

$\Rightarrow$ find where this function is maximum

$\Rightarrow \frac{\partial P_s(R)}{\partial R} = 0$

$\mathcal{E} - Ir - IR = 0$ ← from Kirchhoff's loop rule

$\mathcal{E} = I(R+r) \Rightarrow I = \frac{\mathcal{E}}{(R+r)}$

(1) for each equation or something like that (max = ?)

(2) $R = r$

$\Rightarrow P_s(R) = \frac{\mathcal{E}^2 R}{(R+r)^2}$

$\frac{\partial P_s(R)}{\partial R} = \frac{\mathcal{E}^2(R+r)^2 - 2(R+r)\mathcal{E}^2 R}{(R+r)^4} = 0$

$\Rightarrow \cancel{\mathcal{E}^2}(R+r)^{\cancel{2}} - 2\cancel{(R+r)}\cancel{\mathcal{E}^2}R = 0$

$\Rightarrow R + r - 2R = 0$

$\Rightarrow R = r = 8\Omega$ ← value of R for which P_s is maximal!

$\varepsilon_0 = 8.85\text{x}10^{-12}\ C^2/(Nm^2)$ $e = 1.6\text{x}10^{-19}\ C$ $m_e = 9.1\text{x}10^{-31}\ kg$ $m_p = 1.67\text{x}10^{-27}\ kg$ $\mu_0 = 4\pi\text{x}10^{-7}\ Tm/A$

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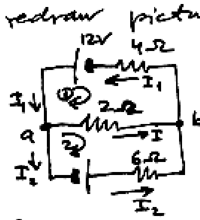
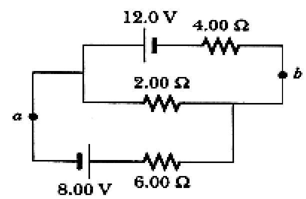
2. (10 pts) In the circuit shown, calculate (a)(7 pts) the current flowing through the 2Ω resistor and (b)(3 pts) the potential difference between points *a* and *b*. Make sure you have the correct signs.

can redraw picture :

(1) labeling currents





a) 2 loop rules
+ 1 junction rule at (a)

$I = ?$

(3) equations

$$(1)\quad -12V + 4\Omega \cdot I_1 + 2\Omega \cdot I = 0 \quad \Rightarrow 4I_1 = 12 - 2I \quad \Rightarrow I_1 = 3 - \tfrac{1}{2}I$$

$$(2)\quad -8V - 2\Omega \cdot I + 6\Omega I_2 = 0 \quad \Rightarrow 6I_2 = 8 + 2I \quad \Rightarrow I_2 = \tfrac{4}{3} + \tfrac{1}{3}I$$

$$(3)\quad I_1 = I_2 + I \quad \Rightarrow \quad I = I_1 - I_2$$

plug (1), (2) into (3) :

$$I = 3 - \tfrac{1}{2}I - \left(\tfrac{4}{3} + \tfrac{1}{3}I\right)$$

$$I = 3 - \tfrac{4}{3} - \tfrac{1}{2}I - \tfrac{1}{3}I$$

(2) calculation

$$I\left(1 + \tfrac{1}{2} + \tfrac{1}{3}\right) = 3 - \tfrac{4}{3}$$

$$I\left(\tfrac{6}{6} + \tfrac{3}{6} + \tfrac{2}{6}\right) = \tfrac{9}{3} - \tfrac{4}{3}$$

$$I\left(\tfrac{11}{6}\right) = \tfrac{5}{3}$$

$$I = \frac{\frac{5}{3}}{\frac{11}{6}} = \frac{30}{33} = \frac{10}{11}$$

$$\boxed{I = 0.909\,A}$$

(1) correct value

(3) b) $\Delta V_{ab} = V_a - V_b = I \cdot \underset{}{2\Omega}^{\leftarrow R_2}$

$$\boxed{\Delta V_{ab} = 1.82V}$$

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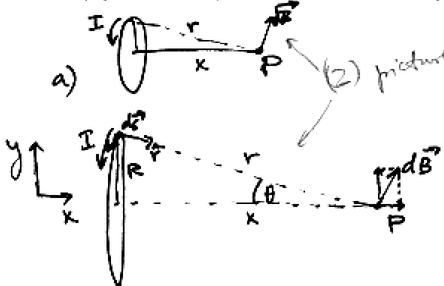
3. (10 pts) Consider a closed circular current loop of radius R and current I. (a)(8 pts) Show that the magnitude of the magnetic field at a point P along the loop's axis a distance x from the loop, is given by:

$$B = \frac{\mu_o IR^2}{2(x^2+R^2)^{3/2}}$$

(b)(2 pts) Now, if I put this loop on the table and the current is flowing in the counter-clockwise direction, what direction will the magnetic field have?
(Hint: Draw a picture, maybe even two different views, set up the loop such that its axis is along the x-axis, so that point P is on the x-axis. Look at the symmetry of the problem.)
(1 pt extra credit) What is the form of the equation if point P is very far from the loop?

a)



(2) pictures

- because of symmetry, all the little $d\vec{B}$ vectors in the $\hat{y}$ direction will cancel out, and the final $\vec{B}$ will be in $+\hat{x}$ direction

(- so let's only worry about dB_x for magnitude)

$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{s}\times\hat{r}}{r^2}$$ ← start there!

$$|d\vec{s}\times\hat{r}| = ds$$

$$r^2 = R^2 + x^2$$

$$\Rightarrow dB_x = |dB|\cdot\cos\theta = \frac{\mu_0}{4\pi} I \frac{ds\cos\theta}{R^2+x^2}$$

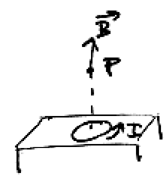
$$\cos\theta = \frac{R}{r} = \frac{R}{(R^2+x^2)^{1/2}}$$

$$\Rightarrow \int dB_x = \frac{\mu_0}{4\pi} I \frac{R}{(R^2+x^2)^{3/2}} \underbrace{\int ds}_{2\pi R}$$

$$\Rightarrow B_x = \frac{\mu_0}{4\pi} I \frac{R^2\cdot 2\pi}{(R^2+x^2)^{3/2}}$$

$$\Rightarrow \boxed{B = \frac{\mu_0 I R^2}{2(R^2+x^2)^{3/2}}}$$

b)



(2) out of the table

extra):

$$x \gg R \Rightarrow x^2 \gg R^2$$

$$\Rightarrow \boxed{B = \frac{\mu_0 I R^2}{2x^3}}$$ (1)

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(2) setting up; symmetry, x-dir...

(4) calculation...

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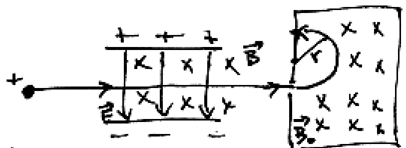
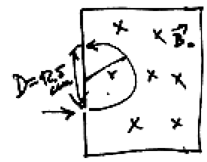
4. (10 pts) A velocity selector has an electric potential difference of 100V between its top and its bottom, which are separated by a vertical distance of 5cm. It also has a uniform magnetic field of 0.01T applied in the horizontal direction. An ion with charge +1e travels horizontally through the velocity selector, but its trajectory is perpendicular to the magnetic field. Despite the electric potential difference and the magnetic field, the ion travels in a straight line through the velocity selector, comes out of it, and enters a mass spectrometer, which has its own magnetic field of 0.2T, perpendicular to the ion's trajectory. As soon as the ion enters the mass spectrometer, its path becomes circular (due to the 0.2T magnetic field), it soon turns around, and hits the same wall it came in through, but 12.5cm away from the entry point.
What ion is this? :-)
(You may simply state your answer in terms of the ion's mass, but 2 pts of extra credit will be given to the correct identification of the ion.)
(For circular motion: $a = v^2/r$)





(2) picture

– since the ion is going straight:

$F_e = F_b \Rightarrow qE = qvB \Rightarrow v = \frac{E}{B}\left(= \frac{E}{0.01T}\right)$

$E = \frac{\Delta V}{d}\left(= \frac{100V}{5cm}\right)$

(3) forces cancel out (balance)

– in the mass spectrometer:

(2) relating circular motion to velocity from (a)

$a = \frac{v^2}{r}$ – circular motion

(2) mass equation

$a = \frac{F_{bo}}{m} = \frac{qvB_0}{m}$

$\Rightarrow \frac{v^2}{r} = \frac{qvB_0}{m}$

$\Rightarrow m = \frac{rqB_0}{v}$

$r = \frac{D}{2}\left(= \frac{12.5\,cm}{2}\right)$

$v = \frac{E}{B} = \frac{\Delta V}{d \cdot B}$

$\Rightarrow m = \frac{D}{2}\frac{qB_0}{\frac{\Delta V}{d \cdot B}}$

$\Rightarrow \boxed{m = \frac{DdqBB_0}{2\Delta V}}$

$m = \frac{(0.125)(0.05)(1.6\times10^{-19})(0.01)(0.2)}{(2)(100)}$

(1)

$m = 10^{-26} = \boxed{10\times10^{-27}\,kg.}$ ✓

$\boxed{\frac{m}{m_p} = \frac{10}{1.67} \approx 6}$

$\varepsilon_0 = 8.85\times10^{-12}\,C^2/(Nm^2)$ $e = 1.6\times10^{-19}\,C$ $m_e = 9.1\times10^{-31}\,kg$ $m_p = 1.67\times10^{-27}\,kg$ $\mu_0 = 4\pi\times10^{-7}\,Tm/A$

So our ion has atomic mass of 6 and a charge of +1 $\Rightarrow$ Li^+ ← (2)