Physics 142 – Homework 4 Solutions

Ch. 26, Q: 3, 9, 11, 15
Ch. 26, P: 1, 3, 6, 9, 13, 16, 17, 18, 27, 31, 33, 44, 47

Questions:

Q3: For a series combination the reciprocal of the total capacitance is equal to the sum of the reciprocal of the individual capacitances. This means that the total capacitance will be less than the individual capacitances. For a parallel combination the total capacitance is the sum of the individual capacitances, meaning the total capacitance will be larger than the individual capacitances. The larger the capacitance, the greater a charge can be built up for a given voltage (since Q = C*V), so the parallel capacitors will be more dangerous.

Q9: It can be dangerous to touch the terminals of high-voltage capacitor even after the potential has been turned off because the charge might still be on the capacitors. To ensure that the charge has dissipated one could ground the capacitors, or attach the capacitor to a resistor to dissipate the current.

Q11: We can write the energy stored in a capacitor as $U = \frac{1}{2}CV^2$, the capacitance is fixed so when the potential is doubled, the energy is quadrupled.

Q15: The electric field inside a capacitor aligns the dipoles within the dielectric, meaning the positive end of the dipole is force towards the negative plate of the capacitor, and the negative end is forces toward the positive plate. In the middle of the conductor the positive and negative ends from neighboring dipoles cancel each other out, but on the surface of the dielectric there is a positive charge near the negative plate and a negative charge near the positive plate. This effectively reduces the charge on the capacitor.

P1: We know that Q = CV, so

a) $Q = CV = (4.00\mu F)(12V) = 48\mu C$

b) $Q = CV = (4.00\mu F)(1.5V) = 6\mu C$

P3: The charge on the conductor will be evenly distributed on the sphere. First we need to know total charge on the sphere, so we can treat the sphere as a point charge and use Coulomb's Law.

$$E = k_e \frac{Q}{r^2}$$

$$4.90\times10^4 \, N/C = 8.988\times10^9 \frac{Nm^2}{C^2} \frac{Q}{(0.21m)^2}$$

$$Q = 0.24\mu C$$

So to find the charge density, we divide the total charge by the surface area.

$$\sigma = \frac{Q}{\pi r^2} = \frac{0.24\mu C}{4\pi(0.12m)^2} = 1.33\times10^{-6}\frac{C}{m^2}$$

The capacitance is given by equation 26.2

$$C = 4\pi\varepsilon_0 R = 4\pi\left(8.854\times10^{-12}\frac{C^2}{Nm^2}\right)(0.12m) = 13.4pF$$

P6: We can treat the cloud earth system as a parallel plate capacitor using equation 26.3,

$$C = \varepsilon_0\frac{A}{d} = 8.854\times10^{-12}\frac{C^2}{N\cdot m^2}\frac{(1000m)^2}{800m} = 11.1nF$$

Since we know the electric field between the earth and the ground is constant we can find the potential difference.

$$\Delta V = Ed = 3.00\times10^6\frac{N}{C}800m = 2.40\times10^9 V$$

$$Q = CV = (11.1\times10^{-9}F)(2.40\times10^9 V) = 26.6C$$

P9: We know the voltage, and the surface charge density (Q/A). We know the following things.

$$C = \varepsilon_0\frac{A}{d}$$

$$Q = CV$$

So by combing these equation

$$\frac{Q}{V} = C = \varepsilon_0\frac{A}{d}$$

$$d = \varepsilon_0\frac{V}{Q/A} = \left(8.854\times10^{-12}\frac{C^2}{Nm^2}\right)\frac{150V}{30\frac{nC}{cm^2}} = 4.42\mu m$$

P13: We can use equation 26.6 to calculate the capacitance

$$C = \frac{ab}{k_e(b-a)} = \frac{(0.07m)(0.14m)}{8.988\times10^9(0.14m-0.07m)} = 15.6pF$$

$$V = \frac{Q}{C} = \frac{4.00\mu C}{15.6pF} = 256kV$$

P16: For capacitors in parallel the total capacitance is the sum of the individual capacitances.

$$C_{tot} = C_1 + C_2 = 5.00\mu F + 12.0\mu F = 17.00\mu F$$

b) Since the capacitors are connected in parallel, each must have the same 9.00 V.

c)
$$Q_1 = C_1 V = 5.00\mu F(9.00V) = 45\mu C$$
$$Q_2 = C_2 V = 12.00\mu F(9.00V) = 108\mu C$$

P17: If the capacitors are connected in series.

$$\frac{1}{C_{tot}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{5.00\mu F} + \frac{1}{12.00\mu F} = 0.28333$$

$$C_{tot} = 3.53\mu F$$

It is easiest to do part c next, then go back and do part b.

$$Q_{tot} = C_{tot}V = 3.53\mu F \cdot 9V = 31.8\mu C$$

The charge on each capacitor must be equal to 31.8 μC, so now we can go back and calculate the voltage across each capacitor.

$$V_1 = \frac{Q}{C_1} = \frac{31.8\mu C}{5.0\mu F} = 6.36V$$

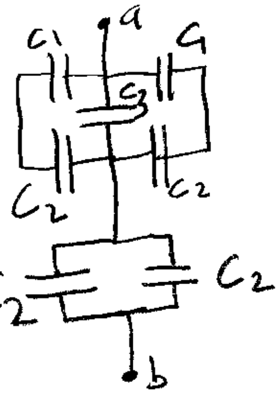
$$V_2 = \frac{Q}{C_2} = \frac{31.8\mu C}{12.0\mu F} = 2.65V$$

To check our solution we note that $V_1 + V_2 = 9.01V$, so our solution is correct to within some rounding era.

P18: In the second and third rows we can reduce the series capacitors to a single equivalent capacitor. So we have three capacitors in parallel with capacitance of C, ½ C, and 1/3 C for the first, second and third rows respectively. Since these are in parallel we can simply add the capacitance for a total capacitance of 11/6 C or 1.83 C.

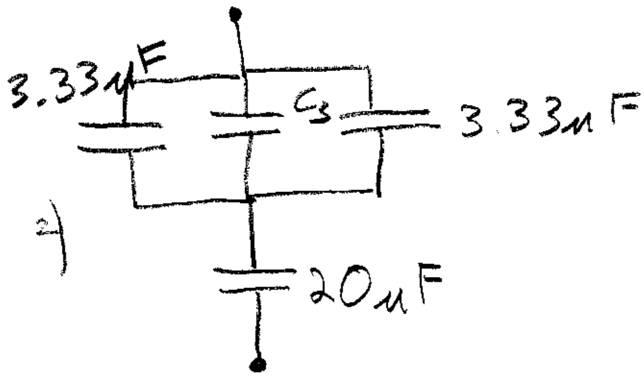
P27:

1)



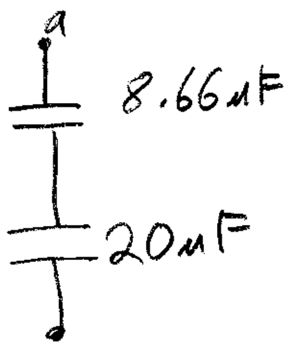
$C_1 = 5\mu F$
$C_2 = 10\mu F$
$C_3 = 2\mu F$

2)



$$\frac{1}{\frac{1}{5\mu F} + \frac{1}{10\mu F}} = 3.33\mu F$$

3)



therefore the total equivalent capacitance is:

$$\frac{1}{\frac{1}{8.66\mu F} + \frac{1}{20\mu F}} = 6.04\mu F$$

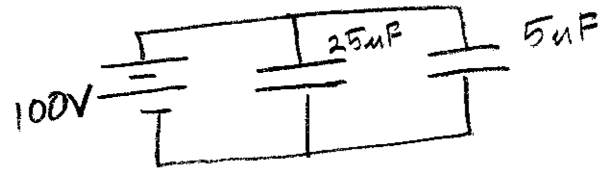
P31: We can use 26.11 to calculate the energy stored in the capacitors.

a) $U = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}3.00\mu F(12V)^2 = 216\mu J$

b) $U = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}3.00\mu F(6V)^2 = 54\mu J$

P33:

a) in parallel

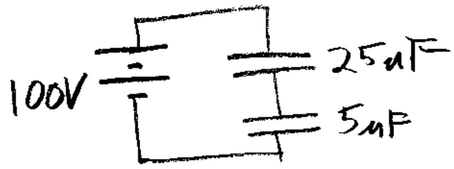


the energy in each capacitor is $\frac{1}{2}CV^2$

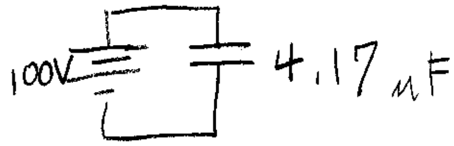
so the total energy is

$$E_p = \frac{1}{2}(25\mu F + 5\mu F)(100V)^2$$
$$= 0.15\ J$$

b) in series



is equivalent too



$$\frac{1}{\frac{1}{25\mu F} + \frac{1}{5\mu F}} = 4.17\mu F$$

so the energy is $\frac{1}{2}(4.17\mu F)(100V)^2$

$$= 268\ V$$

P44: We can combine the following equation to find the solution.

$$C = \varepsilon_0 \frac{A}{d}$$

$$Q = CV$$

$$V = E_{breakdown} d$$

We can combine these equations to solve for the charge in air κ = 1, $E_{breakdown}$ = $3\times10^6 \frac{N}{C}$

$$Q = \left(\kappa\varepsilon_0 \frac{A}{d}\right)\left(E_{breakdown} d\right) = \kappa\varepsilon_0 A E_{breakdown}$$

$$= \left(8.854\times10^{-12} \frac{C^2}{Nm^2}\right)\left(0.0005m^2\right)\left(3\times10^6 \frac{N}{C}\right) = 13.3nC$$

In polystyrene κ = 2.56, $E_{breakdown}$ = $24\times10^6 \frac{N}{C}$

$$Q = \left(\kappa\varepsilon_0 \frac{A}{d}\right)\left(E_{breakdown} d\right) = \kappa\varepsilon_0 A E_{breakdown}$$

$$= 2.56\left(8.854\times10^{-12} \frac{C^2}{Nm^2}\right)\left(0.0005m^2\right)\left(24\times10^6 \frac{N}{C}\right) = 272nC$$

P47: First we want to calculate the charge on the capacitor in air.

$$Q = CV = \left(\varepsilon_0 \frac{A}{d}\right)V = \left(8.85\times10^{-12} \frac{C^2}{Nm^2}\right)\left(\frac{0.0025m^2}{0.015m}\right)250V = 369pC$$

When the capacitor is submerged the charge don't change, because there is no plate for the charge to go, it has been disconnected from the battery and the water is insulating. The capacitance will increase by a factor of κ, therefore the voltage will decrease by a factor of κ. K is 60 for water.

$$C_{new} = 80\left(8.85\times10^{-12} \frac{C^2}{Nm^2}\right)\left(\frac{0.0025m^2}{0.015m}\right) = 118pF$$

$$V_{new} = \frac{250V}{80} = 3.13V$$

To calculate the difference in energy stored in capacitor we use 26.11

$$U = \frac{1}{2}Q(\Delta V)$$

$$\Delta U = U_f - U_i = \frac{1}{2}Q\left(V_f - V_i\right) = \frac{369pC}{2}(3.13V - 250V) = 45.5nJ$$