Hwk 8 Solutions PHYS 142 Summer '09 Dr. Jenkins

Problem 32.5

want to find $\frac{dI}{dt}$; use $\mathcal{E}_L = -L\frac{dI}{dt}$ $\Rightarrow$ $\frac{dI}{dt} = -\frac{\mathcal{E}}{L}$

$$L = \mu_0 \frac{N^2}{\ell} A = (4\pi\times10^{-7}) \frac{(450)^2}{0.16\,m}(3.00\times10^{-4}\,m^2) = 4.16\times10^{-4}\,H$$

$$\Rightarrow \frac{dI}{dt} = \frac{-1.75\times10^{-4}\,V}{4.16\times10^{-4}\,H} = -0.421\,A/s$$

Problem 32.6

$\mathcal{E}_L = -L\frac{dI}{dt}$; $\frac{dI}{dt} = 2.0t - 6.0$

(a) $\mathcal{E}_1 = -90.0\times10^{-3}\,H\,(2.0(1.0s) - 6.0) = 0.360\,V$

(b) $\mathcal{E}_4 = -0.0900\,H\,(2(4.0s) - 6.0) = -0.180\,V$

(c) $0 = -0.090\,[2t_0 - 6]$

$\Rightarrow 0.540 = 0.180 t_0$

$\Rightarrow t_0 = 3.0\,s$

Problem 32.7

(a) $B = \mu_0 n I = (4\pi\times10^{-7})\left(\frac{450}{0.12m}\right)(0.040A) = 1.88\times10^{-4}\,T$

(b) $\Phi = \vec{B}\cdot\vec{A} = BA = (1.88\times10^{-4}T)\,\pi\left(\frac{0.015m}{2}\right)^2 = 3.33\times10^{-8}\,Wb$

(c) $L = \frac{N\Phi}{I} = \frac{450(3.33\times10^{-8}\,Wb)}{0.040\,A} = 0.375\,mH$

(d) the field and flux would change if the current changed, but the current actually divides out of the expression for L such that L is independent of I ; this can be seen by plugging for Φ.

Problem 32.12

if the inductor were not present, the current would be (constant):

$$I = \frac{\mathcal{E}}{R} = \frac{12\,V}{24\,\Omega} = 0.50\,A$$

w/ the inductor, the current will approach this value asymptotically; looking at

$$I = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right) = \frac{\mathcal{E}}{R}\left(1 - e^{-Rt/L}\right)$$

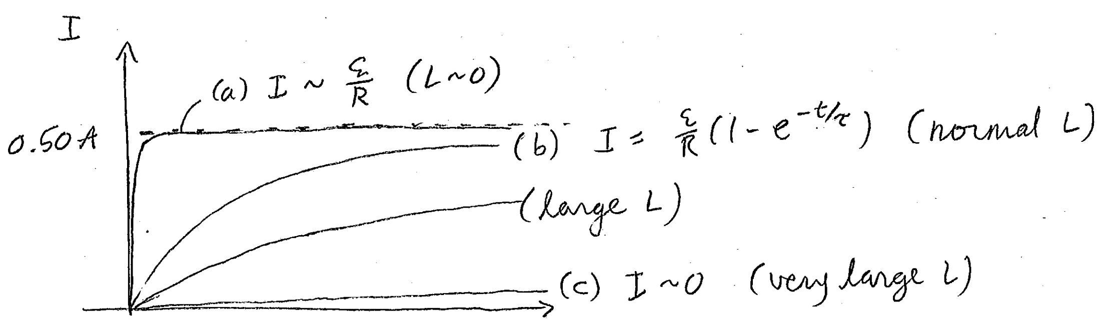
(a) $L \sim 0 \Rightarrow Rt/L \to \infty \Rightarrow e^{-Rt/L} \to \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0$

so the turn on will be almost instantaneous, with an essentially constant current of $\sim 500\,mA$;

(b) normal case;

(c) $L \to \infty \Rightarrow Rt/L \sim 0 \Rightarrow e^{-Rt/L} \sim \frac{1}{e^{0}} = 1$

so the turn on will be extremely slow, i.e. the current will take "forever" to reach $500\,mA$;



Problem 32.13

(a) $\tau = \frac{L}{R} = \frac{8.0 \times 10^{-3}\,H}{4.0\,\Omega} = 2.0 \times 10^{-3}\,s = 2.0\,ms$

(b) $I(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right)$

$\Rightarrow I(250\,\mu s) = \frac{6.0\,V}{4.0\,\Omega}\left[1 - \exp\left(-\frac{2.50 \times 10^{-4}\,s}{2.0 \times 10^{-3}\,s}\right)\right]$

$I(250\,\mu s) = 0.176\,A$

(Prob 32.13) cont.

(c) value if inductor was not present :

$$I_0 = \frac{\mathcal{E}}{R} = \frac{6.0V}{4.0\Omega} = 1.50\,A$$

(d) $0.8 I_0 = I_0 (1 - e^{-t/\tau})$

$$\Rightarrow 0.8 = 1 - e^{-t/2.0ms}$$

$$e^{-t/2.0ms} = 0.2$$

$$\frac{-t}{2.0ms} = \ln 0.2 = -1.609$$

$$\Rightarrow t = 3.22\ ms$$

Problem 32.14

$$\mathcal{E}_L = -L\frac{dI}{dt}$$

$$I(t) = \frac{\mathcal{E}}{R}(1 - e^{-t/\tau}) \Rightarrow \frac{dI}{dt} = \frac{\mathcal{E}}{\cancel{R}}\left(\frac{\cancel{R}}{L}\right)e^{-t/\tau} = \frac{\mathcal{E}}{L}e^{-t/\tau}$$

$$\Rightarrow \mathcal{E}_L = -L\left.\frac{dI}{dt}\right|_{0.200s} = 120V\, e^{-(9.0\Omega)(0.20s)/7.0H} = 92.8\,V$$

Problem 32.15

(a) could get $\mathcal{E}_L$ by finding t for $I(t) = 2.0A$, then find $\left.\frac{dI}{dt}\right|_t$, but it's much easier to just use Kirchoff loop rule :

$$V_R = IR = 2.0A(8.0\Omega) = 16.0V$$

$$\mathcal{E} - V_R - V_L = 0 \Rightarrow V_L = \mathcal{E} - V_R = 36.0V - 16.0V = 20.0V$$

$$\Rightarrow \frac{V_R}{V_L} = \frac{16.0V}{20.0V} = 0.8$$

(b) $V_R = (4.50A)(8.0\Omega) = 36.0\,V$

$$V_L = \mathcal{E} - V_R = 0\,V$$

Problem 32.28

(a) after a long time, current goes to ohm's law value :

$$I = \frac{\mathcal{E}}{R} = \frac{10.0V}{5.0\Omega} = 2.0A$$

total power from battery :

$$P = I\mathcal{E} = (2.0A)(10.0V) = 20.0W$$

(b) to the resistor :

$$P_R = I^2 R = (2.0A)^2(5.0\Omega) = 20.0W$$

(c) resistor is using all power

$$\Rightarrow P_L = 0$$

(d) $$U = \frac{1}{2}LI^2 = \frac{1}{2}(10.0H)(2.0A)^2 = 20.0J$$

Problem 32.33

(a) $$M = \frac{N_B \Phi_{AB}}{I_A} = \frac{700(9.0\times10^{-5}Wb)}{3.50A} = 18.0mH$$

(b) $$L_A = \frac{N_A \Phi_A}{I_A} = \frac{400(3.00\times10^{-4}Wb)}{3.50A} = 34.3mH$$

(c) $$\mathcal{E}_B = -M\frac{dI_A}{dt} = -(0.0180H)(0.50A/s) = -9.00mV$$

Problem 32.34

(a) $$M = \frac{N_2 \Phi_{12}}{I_1} = \frac{N_2 B_1 A_2}{I_1} = \frac{\mu_0 N_2 N_1 \cancel{I}(\pi R_2^2)}{l\cancel{I}} = \frac{\mu_0 \pi N_1 N_2 R_2^2}{l}$$

(b) $$M = \frac{N_1 \Phi_{21}}{I_2} = \frac{N_1 B_2 A_2}{I_2} = \frac{\mu_0 N_1 N_2 \cancel{I}(\pi R_2^2)}{l\cancel{I}} = \frac{\mu_0 \pi N_1 N_2 R_2^2}{l}$$

(c) Since the field outside the smaller coil 2 is zero, the flux through coil 1 is also dependent on A_2 ; hence the two values are the same.

Problem 34.1

(a) looking at Gauss's law in its simplest form:

$$\Phi_E = \frac{q_{enc}}{\epsilon_0}$$

$$\Rightarrow \frac{d\Phi_E}{dt} = \frac{1}{\epsilon_0}\frac{dq}{dt} = \frac{I}{\epsilon_0} = \frac{0.10\,A}{8.85\times10^{-12}} = 11.3\times10^{9}\ N/C\cdot s$$

(b) displacement current is

$$I_d = \epsilon_0 \frac{d\Phi_E}{dt} = \cancel{\epsilon_0}\left(\frac{I}{\cancel{\epsilon_0}}\right) = I = 0.10\,A$$

Problem 34.2

(a)

$$\frac{d\Phi_E}{dt} = \frac{d}{dt}(\vec{E}\cdot\vec{A}) = \frac{I}{\epsilon_0}$$

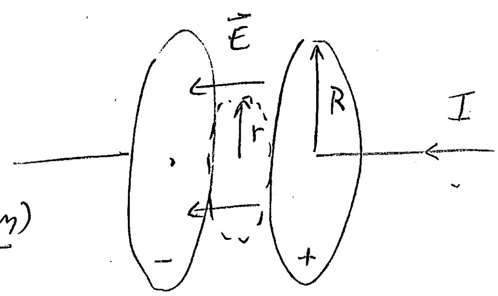
$$\Rightarrow A\frac{dE}{dt} = \frac{I}{\epsilon_0} \Rightarrow \frac{dE}{dt} = \frac{I}{\pi R^2\epsilon_0} = \frac{0.20\,A}{\pi(0.10\,m)^2(8.85\times10^{-12})}$$

$$\frac{dE}{dt} = 7.19\times10^{11}\ V.m/s$$

(b) ampere's law, displacement current version:

$$\oint \vec{B}\cdot d\vec{s} = \mu_0\epsilon_0\frac{d\Phi_E}{dt}$$

$$B(2\pi r) = \mu_0\epsilon_0 A_r\frac{dE}{dt} = \mu_0\pi r^2\left(\frac{I}{\pi R^2}\right)$$



$$\Rightarrow B = \frac{\mu_0 I r}{2\pi R^2} = \frac{(4\cancel{\pi}^{2}\times10^{-7})(0.20\,A)(0.05\,m)}{\cancel{2\pi}(0.10\,m)^2}$$

$$\Rightarrow B = 2.00\times10^{-7}\ T$$

note this expression for the field is the same as for the field inside a current-carrying wire.