Hwk 7 Solutions PHYS 142 Summer '09 Dr. Jenkins

Question 31.3

(i) the vector area of the coil is in the direction normal to the plane of the coil, w/ the (+) direction determined by the right hand rule; $\therefore$ $\vec{B}\cdot\vec{A}$ is maximum when $\vec{A} = A\hat{y}$, which is the same as the coil lying in the x-z plane (c)

(ii) $\vec{B}\cdot\vec{A} = 0$ when $\vec{A} = A\hat{x}$ or $A\hat{z}$, $\therefore$ the answer is (b)

Question 31.5

$\vec{B}$ is out of the page; the flux is increasing in the same direction because the area enclosing field is increasing as the bar moves; Lenz's law says that the induced current will be such that the induced field is opposite the change in flux; $\Rightarrow$ the induced field is into the page, and $\therefore$ the induced current is clockwise by the RHR

If the bar was moving to the left, then $\frac{d\vec{A}}{dt}$ is negative, so then $\frac{d\Phi}{dt}$ is as well; $\therefore$ the opposing induced field is out of the page, so the induced current is counterclockwise.

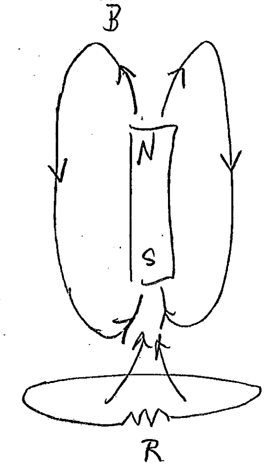
Question 31.8

the acceleration of the plate will be lessened by a inhibiting force as the plate enters the field and experiences a change in flux; the eddy current produced in the plate will experience a force by $\vec{F} = I\vec{l}\times\vec{B}$; this net force will be in the direction opposite the motion and will exist for as long as the plate is partially inside the field. Therefore, $a \neq 9.8\ m/s^2$, and $\vec{v}$ is affected.

Question 31.10

the coil will act as a solenoid when the current is switched on; when this current first comes on the field will ramp up quickly causing a sharp increase in flux through the ring, which will induce a current; this in turn will lead to a net upward force on the ring due to the coil's field, just as in problem 29.31, resulting in upward acceleration of the ring as long as $F_B > mg$

Question 31.12



(i) since field lines come into the South pole of a magnet, as the magnet falls toward the loop, Φ will increase in the upward direction, meaning the induced field is downward, which means the current is clockwise, or left through the resistor (a)

(ii) similarly, once the magnet passes through the loop, the field lines coming out of the north pole will be receding, meaning Φ will decrease in the upward direction, i.e. increase in the downward direction $\Rightarrow$ induced field is up $\Rightarrow$ current is CCW $\Rightarrow$ current is right through R (b)

(iii) the vertical components of the field lines pointing down toward the loop, increasing the flux on the north-pole side of the loop will be exactly cancelled by the upward components on the south-pole side of the loop, resulting in no net flux, and $\therefore$ no current.

Question 31.13

(i) closing the switch will cause a magnetic field in the first solenoid; as this field turns on, it will cause a change in flux in the second coil, creating an EMF. RHR gives a field to the right in the first coil, so the EMF and induced field will be to the left in the second coil, which corresponds to current to the right through R (b)

(ii) once the field through the first coil is steady, no further EMF will be generated in the second coil, so no current will flow (d)

(iii) similar to (i), except reversed: the field turning off in the first coil will cause an EMF to the right in the second coil, causing a current to the left through R (b)

Problem 31.4

(a) for the average current, we need the average EMF through the coil:

$$\mathcal{E} = \frac{-d\phi}{dt} = -N\frac{d}{dt}(\vec{B}\cdot\vec{A}) = -NA\frac{dB}{dt} = -NA\frac{\Delta B}{\Delta t}$$

$$\mathcal{E} = -12\pi(0.021m)^2\left(\frac{0-0.110T}{0.180s}\right) = 0.0102\,V$$

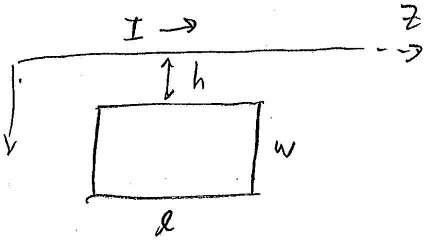
$$\Rightarrow I = \frac{\mathcal{E}}{R} = \frac{0.0102V}{2.3\Omega} = 4.42\times10^{-3}A = 4420\mu A$$

The meter is actually much more sensitive than it needs to be, perhaps even too sensitive; this current would peg the meter.

(b) The toward us, so the change in flux as the coil is pulled out will be away from us, which means the induced field is toward, corresponding to a CCW current, which will go into the positive (red) terminal of the meter, indicating (+) current.

Problem 31.8

(a) the field in the region of the loop is:



$\vec{B} = \frac{\mu_0 I}{2\pi r}$ into the page.

$\Rightarrow \Phi = \int \vec{B} \cdot d\vec{A}$; $d\vec{A} = l\,dr$ out of page

$$\Phi = -\frac{\mu_0 I l}{2\pi} \int_h^{h+w} \frac{dr}{r} = -\frac{\mu_0 I l}{2\pi} \left[\ln(h+w) - \ln(h) \right]$$

or. $\Phi = \frac{-\mu_0 I l}{2\pi} \ln\left(1 + \frac{w}{h}\right)$ (−) is into page

(b) $\mathcal{E} = -\frac{d\Phi}{dt} = \frac{\mu_0 l}{2\pi} \ln\left(1 + \frac{w}{h}\right) \frac{dI}{dt}$;

$I = a + bt \Rightarrow \frac{dI}{dt} = b$

$$\Rightarrow \mathcal{E} = \frac{(\cancel{4}^{2}\pi \times 10^{-7})(1.0\,m)}{\cancel{2\pi}} \ln\left(1 + \frac{0.1\,m}{0.01\,m}\right)(10.0\,A/s)$$

$\mathcal{E} = 4.80 \times 10^{-6}\,V$ (+) is out of page

$\Rightarrow$ induced current is CCW

Problem 31.9

(a) field inside solenoid is $B = \mu_0 n I$; at the end where the ring is, it's $B = \frac{1}{2}\mu_0 n I$ (to the right)

$\Rightarrow \mathcal{E} = -\frac{d\Phi}{dt} = -\frac{1}{2}\mu_0 n A \frac{dI}{dt}$ where A is the area of the solenoid, not the ring (field is zero outside of solenoid)

$$\mathcal{E} = -\frac{1}{2}(4\pi \times 10^{-7})(1000/m)\,\pi\,(0.03m)^2\,(270\,A/s)$$

$\mathcal{E} = -4.80 \times 10^{-4}\,V$ (−) is to the left

$$I = \frac{\mathcal{E}}{R} = \frac{4.80 \times 10^{-4}\,V}{3.0 \times 10^{-4}\,\Omega} = 1.60\,A \quad \text{CCW}$$

(Prob 31.9) cont.

(b) field at the center of a current loop is

$$B = \frac{\mu_0 I}{2r} = \frac{(4\pi \times 10^{-7})(1.60\,A)}{2(0.05\,m)} = 2.01 \times 10^{-5}\ T$$

(c) direction by RHR is to the left (same, of course, as $\mathcal{E}$)

Problem 31.11

$$\mathcal{E} = -\frac{d\Phi}{dt} = -N_{coil} A_{sol} \frac{dB_{sol}}{dt} = -N_c \pi r_s^2 \mu_0 n_s \frac{dI}{dt}$$

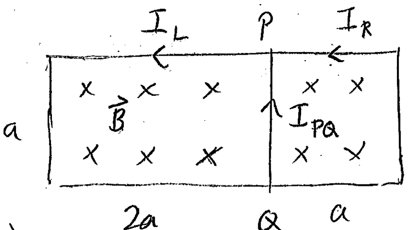
$$\frac{dI}{dt} = (5\,A)(120)\cos(120t)$$

$$\Rightarrow \quad \mathcal{E} = -15\pi(0.02m)^2(4\pi \times 10^{-7})(1000/m)(5\,A)(120/s)\cos(120t)$$

$$\mathcal{E} = -14.2\cos(120t)\ mV$$

Problem 31.13

the changing field will impart an EMF in each loop of the configuration; we can plug this EMF into a Kirchoff's loop problem and solve for I_{PQ}:



left loop:

$$\mathcal{E}_L - I_L(5ra) - I_{PQ}(ra) = 0$$

$$\mathcal{E}_L = -\frac{d\Phi_L}{dt} = -\frac{d}{dt}(\vec{B}\cdot\vec{A}) = 2a^2\frac{dB}{dt} \quad (+)$$

$r = R/l$; $\vec{A} = A$ out of page $\Rightarrow$ $\vec{B}\cdot\vec{A} = -BA$;

① $2a^2(1.0\times10^{-3}\,T/s) - 5raI_L - raI_{PQ} = 0$

right loop:

② $a^2(1.0\times10^{-3}\,T/s) - 3raI_R + raI_{PQ} = 0$

current @ P:

$I_R + I_{PQ} = I_L$; sub into ① ;

fast solution: multiply ① by 3/5 and subtract ② from it:

(Prob 31.13) cont.

$$\frac{6}{5}a^2(1.0\times10^{-3}) - 3ra(\cancel{I_R} + I_{PQ}) - \frac{3}{5}raI_{PQ} = 0$$

$$-a^2(1.0\times10^{-3}) + 3\cancel{raI_R} - raI_{PQ} = 0$$

$$\Rightarrow \frac{1}{5}a^2(1.0\times10^{-3}) - 4raI_{PQ} - \frac{3}{5}raI_{PQ} = 0 \qquad (\times 5)$$

$$\Rightarrow 23raI_{PQ} = a^2(1.0\times10^{-3})$$

$$I_{PQ} = \frac{(1.0\times10^{-3})}{23}\frac{a^{\cancel{2}}}{r\cancel{a}} = \frac{(1.0\times10^{-3})}{23}\frac{(0.65m)}{(0.1\,\Omega/m)} = 2.83\times10^{-4}\,A$$

direction is upward since sign came out (+)

Problem 31.15

$$\mathcal{E} = -\frac{d\Phi}{dt} = -N\frac{d}{dt}(\vec{B}\cdot\vec{A}) = -NA\frac{dB}{dt};$$

$$B = \mu_0 nI; \quad \frac{dB}{dt} = \mu_0 n\frac{dI}{dt};$$

$$\mathcal{E} = -250\pi(0.06m)^2(4\pi\times10^{-7})(400/m)(30.0A)(1.60/s)e^{-1.60t}$$

$$\mathcal{E} = (6.82\times10^{-2}\,V)e^{-1.60t}$$

Problem 31.21

(a) $\mathcal{E} = Blv = (1.20\times10^{-6}T)(14.0m)(70.0m/s) = 1.18\times10^{-3}\,V$

electrons in the wings would feel a force given by $\vec{F} = -e\vec{v}\times\vec{B}$, which is to the east, or right on the plane; ∴ the left wing is at a higher (+) potential.

(b) there would be no change to the results above wrt. the plane.

(c) connecting the wings with a wire would create a circuit loop oriented vertically, which would see no change in flux from the $\vec{B}$-field in question; so we can't obtain power from this potential.

Problem 31.23

(a) $|F_{app}| = |F_B| = |I\vec{l} \times \vec{B}| = IlB$

$$I = \frac{\mathcal{E}}{R} = \frac{Blv}{R}$$

$$\Rightarrow F_{app} = \frac{B^2 l^2 v}{R} = \frac{(2.50T)^2(1.2m)^2(2.0m/s)}{6.0\Omega} = 3.0N$$

direction is obviously to the right.

(b) $P = I^2 R = \frac{B^2 l^2 v^2}{R} = F_{app} v = 3.0N(2.0m/s) = 6.0W$

Problem 31.27

using the eqn for EMF in a rotating bar from ex. 31.4 :

$$\mathcal{E} = \frac{1}{2} B\omega l^2 = \frac{1}{2}(50.0\times10^{-6}T)(2.0\,rev/s)(2\pi\,rad/rev)(3.0m)^2$$

$$\mathcal{E} = 2.83\times10^{-3}\,V$$

Problem 31.28

(a) field lines come out of north pole, so moving the magnet to the left creates a change in flux to the left $\Rightarrow$ induced field to the right $\Rightarrow$ current to the right through R

(b) field to the left in coil turns on when the switch is closed $\Rightarrow$ induced field to the right in loop $\Rightarrow$ current in R is out of the page.

(c) field is into the loop before decrease; decreasing I means decreasing field $\Rightarrow$ change in flux out of the loop $\Rightarrow$ induced field is into loop $\Rightarrow$ current is to the right through R.

(d) using $\vec{F} = q\vec{v}\times\vec{B}$ wrt (+) charges, for an upward resulting force and $\vec{v}$ to the right, $\vec{B}$ must be into the page.

Problem 31.29

let x be to the right;

(a) front edge of coil is affected first:

$F = NIwB$

get I from EMF

$$\mathcal{E} = -N\frac{d\Phi}{dt} = -N\frac{d}{dt}(\vec{B}\cdot\vec{A}) = NBw\frac{dx}{dt} = NBwv \quad \text{(out of page)}$$

$$\Rightarrow I = \frac{\mathcal{E}}{R} = \frac{NBwv}{R} \quad \text{(ccw)}$$

$$\Rightarrow F = N\left(\frac{NBwv}{R}\right)wB = \frac{N^2B^2w^2v}{R}$$

direction

$\vec{F} = I\vec{w}\times\vec{B} = F(-\hat{x})$ to the left by RHR

(b) once the loop is all the way inside the field, there is no change in flux, so $\therefore$ no current and no force.

(c) flux will decrease at the same rate as it increased in part (a);

$\Rightarrow$ current will be CW, but acting on trailing edge of loop

$\Rightarrow \vec{F} = I\vec{w}\times\vec{B} = F(-\hat{x})$ still to the left

$$\Rightarrow \vec{F} = \frac{N^2B^2w^2v}{R}(-\hat{x})$$

Problem 31.30

field lines go into South pole; as magnet moves toward loop, flux increase is into the page $\Rightarrow$ induced field is out of the page $\Rightarrow$ current through loop is CCW $\Rightarrow$ current travels from b to a through R, so $V_a - V_b$ is negative.

Problem 31.33

we can get EMF from Faraday's law, then get $\vec{E}$ from

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{s}$$

$$\mathcal{E} = -\frac{d\Phi}{dt} = -\frac{d}{dt}(\vec{B} \cdot \vec{A}) = \pi r_1^2 \frac{dB}{dt} \quad \text{(out of page)}$$

even though $\vec{E}$ is time dependent, it is constant along the circumference of the loop at r_1 $\Rightarrow$ comes out of integral

$$\Rightarrow \mathcal{E} = E \oint ds = E(2\pi r_1)$$

$$\Rightarrow E(2\cancel{\pi}\cancel{r_1}) = \cancel{\pi} r_1^{\cancel{2}} \frac{dB}{dt} \Rightarrow E = \frac{r_1}{2}\frac{dB}{dt} = \frac{r_1}{2}(0.060t)$$

$$E = \frac{0.020\,m}{2}(0.0600\ T/\cancel{s})(3.00\cancel{s}) = 1.80 \times 10^{-3}\ N/C$$

direction is $\parallel d\vec{s}$ $\Rightarrow$ CCW $(\hat{\phi})$

Problem 31.34

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi}{dt} \Rightarrow E(2\cancel{\pi}\cancel{r}) = -\frac{d}{dt}(\vec{B} \cdot \vec{A}) = -\cancel{\pi} r^2 \mu_0 n \frac{dI}{dt}$$

$$\Rightarrow |E| = \frac{\mu_0 n r}{2}\frac{dI}{dt} = \frac{(4\pi \times 10^{-7})(1000/m)(0.01\,m)}{2}(100\pi)(5.0A)\cos(100\pi t)$$

$$E = (9.87 \times 10^{-3}\ V/m)\cos(100\pi t)$$

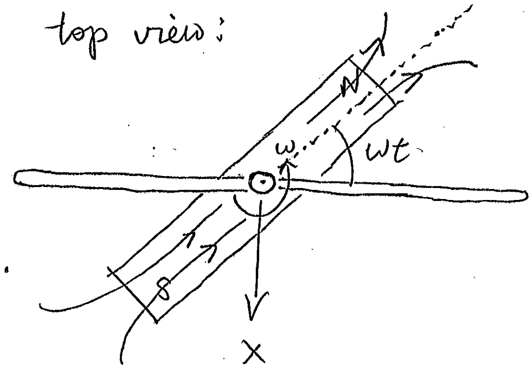
for current increasing CCW, B is increasing to the left

$\Rightarrow$ $\mathcal{E}$ is to the right $\Rightarrow$ E is <u>clockwise</u> $(-\hat{\phi})$

Problem 31.38

let Φ be positive to the right; the flux will be zero initially and will increase to the left as the magnet starts to turn (-), given the orientation of field lines;

top view:



flux goes from zero to (-) to zero to (+) in one cycle

$\Rightarrow \Phi \sim -\sin \omega t$

$$\Phi = -\Phi_0 \sin \omega t$$

$$\Rightarrow \quad I = \frac{\mathcal{E}}{R} = -\frac{1}{R}\frac{d\Phi}{dt} = \frac{\omega \Phi_0}{R} \cos \omega t$$