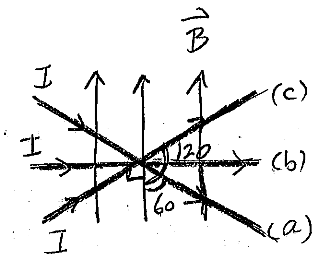
Hwk 6 Solutions PHYS 142 Summer '09 Dr. Jenkins

Question 29.12

A current loop in a magnetic field will feel some torque, given the proper orientation. There will be some equilibrium position in which the torque will go to zero, but any amount of rotation out of the eq position would reveal the torque.

Problem 29.27



force on a wire in a mag-field

$\vec{F} = I\vec{l} \times \vec{B}$

$\Rightarrow \quad F = IlB\sin\theta$

(a) $F_{60} = (5.0A)(2.8m)(.390T)\underbrace{\sin 60}_{\frac{\sqrt{3}}{2}}$

$= 4.73N$

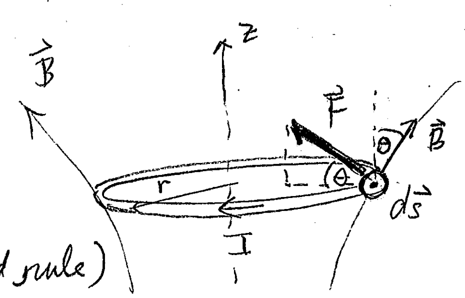
(b) $F_{90} = (5.0A)(2.8m)(.390T)\underbrace{\sin 90}_{1}$

$= 5.46\ N$

(c) $\sin 120 = \sin 60$

$\Rightarrow \quad F_{120} = F_{60} = 4.73\ N$

Problem 29.31



force on each bit of ring is:

$d\vec{F} = I\,d\vec{s} \times \vec{B}$

magnitude is $\quad F = I\,ds\,B\sin\theta$

direction is as shown at right (rt-hand rule)

$\vec{F}$ has radially inward component and upward z-component.

radial components will all cancel; for z-comp:

$F_z = IB\sin\theta \int_{ring} ds \qquad$ (trivial integral)

$\Rightarrow \quad \vec{F} = 2\pi r I B \sin\theta\ \hat{z}$

Problem 29.32

(a) $\vec{F}_{ab} = I\,\vec{ab} \times \vec{B} = 0 \quad (\sin 180 = 0)$

$\vec{F}_{bc} = I\,\vec{bc} \times \vec{B} = (5.0\,A)(.40\,m)(0.020\,T) \sin 90 \;(-\hat{x})$

(direction from RHR)

$\vec{F}_{bc} = -0.0400\,N\,\hat{x}$

for $\vec{cd}$, the length is 40 cm times $\sqrt{2}$ (45-45-90):

$\vec{F}_{cd} = (5.0A)(.40\,m)\cancel{\sqrt{2}}(0.020T)\,\overset{1/\sqrt{2}}{\cancel{\sin 45}}\,(-\hat{z})$

$\vec{F}_{cd} = -0.0400\,N\,\hat{z}$

$\vec{F}_{da} = (5.0\,A)(0.40m)\sqrt{2}\,(0.020\,T)\,\overset{1}{\cancel{\sin 90}}\,(\overset{1/\sqrt{2}}{\cancel{\cos 45}}\,\hat{x} + \overset{1/\sqrt{2}}{\cancel{\sin 45}}\,\hat{z})$

(check direction from RHR)

$\vec{F}_{da} = 0.0400\,N\,(\hat{x} + \hat{z})$

(b) since the net force on a closed loop in a uniform mag-field is zero, you could find three of the forces and get the 4th by taking $-\vec{F}_{net}$ from the other three.

Problem 29.35

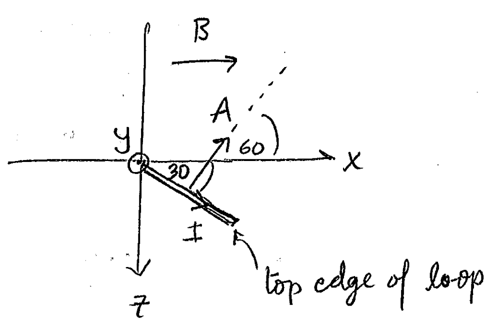
torque on current loop:

$\vec{\tau} = \vec{\mu} \times \vec{B} \;;\; \vec{\mu} = NI\vec{A}$

get direction of $\vec{A}$ from RHR (w/ fingers in current direction)

mag: $\tau = NIAB \sin 60$

direction is into page (RHR)



$\tau = 100(1.20A)(0.40m)(0.30m)(0.80\,T)\,\frac{\sqrt{3}}{2}$

$\Rightarrow \tau = 9.98\,N\cdot m$ into page $\Rightarrow$ clockwise rotation

Problem 29.38

(a) max torque magnitude:

$\vec{\tau} = \vec{\mu} \times \vec{B} \Rightarrow \tau_{max} = \mu B \sin 90 = IAB$

$\tau_{max} = (5.0A)(3.0\times10^{-3}T)\left[\pi\left(\frac{.10m}{2}\right)^2\right] = 1.18\times10^{-4}\ N\cdot m$

(b) $U = -\vec{\mu}\cdot\vec{B} = -I\vec{A}\cdot\vec{B}$

range from $\vec{A}\parallel\vec{B}$ to $-\vec{A}\parallel\vec{B}$ (antiparallel: $\cos 180 = -1$)

$U_{min} = -IAB$ (parallel)

$= -1.18\times10^{-4} J$

$U_{max} = IAB$ (anti parallel)

$= 1.18\times10^{-4} J$

notice the two vectors want to be aligned such that $\vec{\tau} = 0$, so being completely out of alignment (opposite directions) has the most potential energy

$\Rightarrow -1.18\times10^{-4} J \le U \le 1.18\times10^{-4} J$

Problem 29.42

(a) Hall voltage formula:

$\Delta V_H = \frac{IB}{nqt}$; given a B-field and the corresponding ΔV_H, we can find the value for $\frac{I}{nqt} \rightarrow \frac{nqt}{I}$; this is enough info to get a field strength for a known ΔV_H:

$\frac{B}{\Delta V_H} = \frac{nqt}{I} = \frac{0.080T}{0.70\times10^{-6}V} = 1.14\times10^5\ T/V$

$\Rightarrow B_{unknown} = 1.14\times10^5\ T/V\ (0.330\times10^{-6}\ V) = 0.0377\ T$

(b) $t = 2.00mm$, and we know I, q ; So solve for n:

$\frac{nqt}{I} = 1.14\times10^5 T/V \Rightarrow n = \frac{(0.12A)(1.14\times10^5 T/V)}{(1.6\times10^{-19}C)(0.0020m)}$

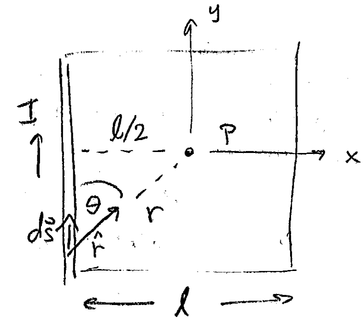
$n = 4.29\times10^{25}\ /m^3$

Problem 30.3

(a) I'll rederive this formula b/c the one in the text is quirky and difficult to generalize:

for the field around a finite straight wire, starting w/ Biot-Savart law:

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{s} \times \hat{r}}{r^2}$$



looking at one of the four sides of the loop independently:

both the cross product and field-around a wire versions of the RHR tell us the field inside the loop is into the page (for all four of the sides);

for the magnitude:

$$d\vec{s} \times \hat{r} = ds\,(1)\sin\theta$$

$$r = \frac{l/2}{\sin\theta} \quad ; \quad s = \frac{-l/2}{\tan\theta} = -\frac{l}{2}\cot\theta$$ (look at position wrt axes for sign)

$$\Rightarrow ds = \frac{l}{2}\csc^2\theta\, d\theta = \frac{l\, d\theta}{2\sin^2\theta}$$

$$\Rightarrow \frac{d\vec{s} \times \hat{r}}{r^2} = \frac{l\, d\theta}{2\sin^2\theta}\left(\frac{\sin^2\theta}{(l/2)^2}\right)\sin\theta = \frac{1}{(l/2)}\sin\theta\, d\theta$$

$$\Rightarrow B = \frac{2\mu_0 I}{4\pi l}\int_{\theta_1}^{\theta_2}\sin\theta\, d\theta = \frac{2\mu_0 I}{4\pi l}(\cos\theta_1 - \cos\theta_2)$$

notice unlike the text, this is in terms of $\cos\theta$, and in terms of the actual angle between $d\vec{s}$ and $\hat{r}$ relevant in the cross product; note θ_1 and θ_2 are numbered according to which one the current reaches first.

we need four of these fields added together for this loop:

$$\vec{B} = \frac{2\mu_0 I}{\pi l}(\cos 45 - \cos 135) = \frac{2(4\pi \times 10^{-7})(10.0A)}{\pi(0.40m)}\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right)$$

$$\vec{B} = 2.83 \times 10^{-5}\,T \text{ into page}$$

(Prob 30.3) cont.

(b) $4l$ is the circumference now:

the field at the center of a single loop is:

$$B = \frac{\mu_0 I}{2R}$$

$$4l = 2\pi R \quad \Rightarrow \quad R = \frac{2l}{\pi}$$

$$\Rightarrow \vec{B} = \frac{\mu_0 \pi I}{4l} = \frac{(4\pi \times 10^{-7})\pi(10.0\,A)}{4(0.40\,m)} = 2.47 \times 10^{-5}\,T \text{ into page}$$

Problem 30.4

think of this as a long straight wire and a loop added together.

the field from the straight part then is $B_w = \frac{\mu_0 I}{2\pi R}$

and for the loop, it's $B_l = \frac{\mu_0 I}{2R}$

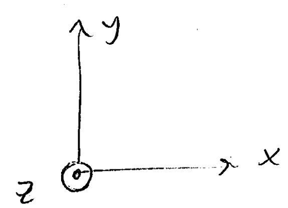
both are into the page by RHR:

$$\Rightarrow \vec{B} = \frac{\mu_0 I}{2R}\left(1 + \frac{1}{\pi}\right) \text{ into page}$$

Problem 30.7

field from each wire will be given by $B = \frac{\mu_0 I}{2\pi r}$, the net field will be:

$$\vec{B}_{net} = \frac{\mu_0 I_2}{4\pi a}\hat{y} + \frac{\mu_0 I_1}{2\pi a}\hat{y}$$



and they said:

$$\vec{B}_{net} = \frac{2\mu_0 I_1}{2\pi a}(\pm\hat{y})$$

the two options will come from the two possible resultant directions:

(a) (+) sign

$$\frac{2\mu_0 I_1}{\cancel{2\pi a}} = \frac{\mu_0 I_2}{2\,\cancel{4\pi a}} + \frac{\mu_0 I_1}{\cancel{2\pi a}} \quad \Rightarrow \quad 2I_1 = \frac{I_2}{2} + I_1$$

(Prob 30.7) cont.

(a)
$$I_1 = \frac{I_2}{2} \Rightarrow I_2 = 2I_1$$

for $\vec{B}_2$ in $+\hat{y}$ direction as I arbitrarily decided, and no (−) sign in the answer, I_2 must be out of the page ($\hat{z}$)

(b) (−) sign

$$-2I_1 = \frac{I_2}{2} + I_1 \Rightarrow -3I_1 = \frac{I_2}{2} \Rightarrow I_2 = -6I_1$$

(−) sign here means I_2 is in opposite direction from what was chosen, i.e. into the page ($-\hat{z}$)

Problem 30.10

the straight lengths of wire contribute 0 field because $\theta = \{0°, 180°\}$

$\Rightarrow d\vec{s} \times \hat{r} = 0$

for the curved piece, the distance to each $d\vec{s}$ (r) is const and the angle is always 90° because $\hat{r}$ is radially inward and $d\vec{s}$ is tangent to the circle always.

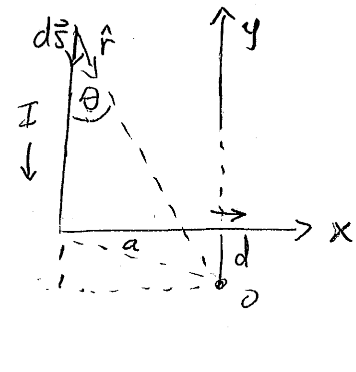
$$\Rightarrow \quad B = \frac{\mu_0 I}{4\pi r^2} \int ds \quad ; \quad ds = r\,d\theta$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{4\pi r} \int_0^{30°} d\theta = \frac{\mu_0 I}{4\pi r}\left(\frac{\pi}{6}\right) = \frac{(4\pi \times 10^{-7})(3.0\,A)}{24(0.60m)}$$

$$\vec{B} = 2.62 \times 10^{-7}\,T \text{ into page}$$

Problem 30.14

the going and coming segments will produce the same field, using eqn from problem 3:



$$B = \frac{\mu_0 I}{2\pi r_\perp}(\cos\theta_1 - \cos\theta_2)$$

left side:

$$\vec{B}_1 = \frac{\mu_0 I}{2\pi a}\left(\cos 0 - \frac{d}{\sqrt{a^2+d^2}}\right)\hat{z}$$

(Prob 30.14) cont.

bottom:

$$\vec{B}_2 = \frac{\mu_0 I}{2\pi d}\left(\frac{a}{\sqrt{a^2+d^2}} + \frac{+a}{\sqrt{a^2+d^2}}\right) = \frac{\mu_0 I}{\pi d}\left(\frac{a}{\sqrt{a^2+d^2}}\right)(-\hat{z})$$

$$\vec{B}_{net} = 2\vec{B}_1 + \vec{B}_2$$

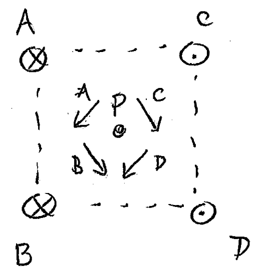
this is sufficient b/c the algebra left is weird and not enlightening.

Problem 30.23

since all currents are the same, and the distance is the same from each wire to the center, the magnitude of the field from wire is the same as the others:

$$B = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi \times 10^{-7})(5.0\,A)}{2\pi(0.20\,m)\sqrt{2}/2} = 7.07\times10^{-6}\,T$$

now look at directions:



x-components cancel

all 4 y-comps add:

$$\vec{B}_{net} = \cancel{4}^{\,2}(7.07\times10^{-6}\,T)\,\cancel{\sin 45}^{\,\sqrt{2}/2}\,(-\hat{y})$$

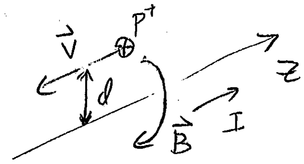
$$\vec{B}_{net} = -2.0\times10^{-5}\,T\;\hat{y}$$

Problem 30.24

the proton's velocity generates a force on it from the magnetic field of the wire given by $\vec{F} = q\vec{v}\times\vec{B}$, where $\vec{B}$ is the usual field around a wire: $\vec{B} = \frac{\mu_0 I}{2\pi d}\,\hat{\phi}$ (cyl coords)

since $\vec{v} \perp \vec{B} \Rightarrow \vec{v}\times\vec{B} = vB(-\hat{z}\times\hat{\phi}) = vB\,\hat{r}$

so $\vec{F} = qvB\,\hat{r}$ balances with $m\vec{g}$ of the proton:



$$mg = qvB = \frac{qv\mu_0 I}{2\pi d} \Rightarrow d = \frac{qv\mu_0 I}{2\pi mg}$$

$$d = \frac{(1.6\times10^{-19}C)(2.3\times10^{4}m/s)(4\pi\times10^{-7})(1.20\times10^{-6}A)}{2\pi(1.67\times10^{-27}kg)(9.8\,m/s^2)} = 0.054\,m$$

Problem 30.25

using Ampere's law, and loops at the radii in question:

(a) $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$

$B_a (2\pi r_a) = \mu_0 I_{in}$

$\vec{B}_a = \dfrac{(4\pi \times 10^{-7})(1.0A)}{2\pi (1.0 \times 10^{-3} m)} = 2.0 \times 10^{-4} T$ upward

(b) $B_b (2\pi r_b) = \mu_0 (I_{in} - I_{out})$

$\vec{B}_b = \dfrac{(4\pi \times 10^{-7})(2.0A)}{2\pi (3.0 \times 10^{-3} m)} = 1.33 \times 10^{-4} T$ downward

Problem 30.27

(a) treat like a single wire w/ 100·2A current; use formula from Ampere's law for field inside a wire:

$B = \dfrac{\mu_0 I r}{2\pi R^2}$

force on one wire at $r = 0.2cm$ from field of other 99:

$B = \dfrac{(4\pi \times 10^{-7}) 99 (2.0A)(0.002m)}{2\pi (0.005m)} = 3.17 \times 10^{-3} T \hat{\phi}$

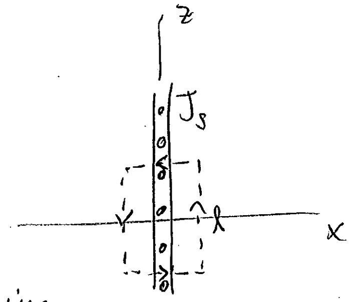
$\vec{F} = I\vec{l} \times \vec{B} = I l B \sin 90 (-\hat{r})$ $\quad (\vec{l} = l\hat{z};\ \hat{z} \times \hat{\phi} = -\hat{r})$

force per unit length: $\dfrac{F}{l} = IB = (2.0A)(3.17 \times 10^{-3} T)$

$\vec{F} = 6.34 \times 10^{-3} N/m$ inward

(b) $B \sim r$, so B gets bigger at larger r, meaning its max at $r = R$; so B would be strongest at $r = R$, so for the same current I, the force would be largest at $r = R$ also.

Problem 30.33



using Ampere's law:

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$$

B still must be parallel to current producing surface, as usual, so $\vec{B}\cdot d\vec{l} = 0$ for the two legs of the loop $\perp$ to the surface and $\vec{B}\cdot d\vec{l} = B\,dl$ for the two legs $\parallel$ to the surface

$$\Rightarrow \oint_{loop} \vec{B}\cdot d\vec{l} = 2Bl = \mu_0 I_{enc}$$

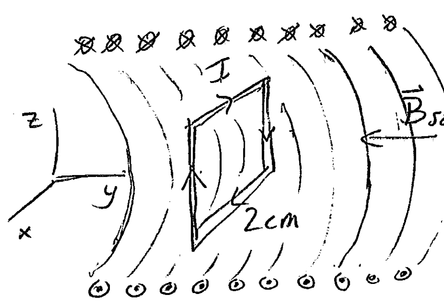
if current sheet is being thought of as one-dimensional, then total current enclosed is:

$$I_{enc} = J_s l$$

$$\Rightarrow \quad 2B\cancel{l} = \mu_0 J_s \cancel{l} \quad \Rightarrow \quad B = \frac{\mu_0 J_s}{2}$$

for the direction, consider the current coming out of the page for one small length $d\vec{l}$, as if it were just a single wire; the RHR gives then that the field would be in the $+\hat{z}$ direction to the right of sheet and in the $-\hat{z}$ direction to the left. the same method applied at <u>each</u> $d\vec{l}$ shows that the field must run parallel to the sheet and perpendicular to the wire everywhere, going upward to the right and downward to the left.

Problem 30.37



$$\vec{B}_{sol} = \mu_0 n I\,(-\hat{y})$$

$$\vec{B}_{sol} = (4\pi\times10^{-7})(3000/m)(15.0A)(-\hat{y})$$

$$\vec{B}_{sol} = -5.65\times10^{-2}\,T\ \hat{y}$$

(Prob 30.37) cont.

the force on each side of the loop is given by

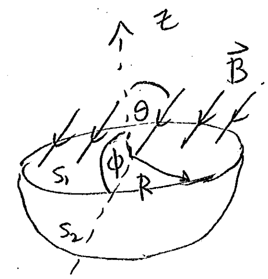
$\vec{F} = I\vec{\ell} \times \vec{B}$; magnitude is (same for all 4 sides)

$F = I\ell B = (0.20A)(0.02m)(5.65 \times 10^{-2} T) = 2.26 \times 10^{-4} N$

using RHR for the cross-product, we see then, that the force is radially outward on all four sides.

this should mean that the torque is zero, which can be reemphasized by $\vec{\tau} = \vec{\mu} \times \vec{B}$, since $\vec{\mu} = I\vec{A} = I\ell^2(-\hat{y}) \Rightarrow \vec{\mu} \parallel \vec{B} \Rightarrow \vec{\tau} = 0$

Problem 30.40



$\Phi_M = \vec{B} \cdot \vec{A}$

(a) $\vec{A}_1 = \pi R^2 \hat{z}$

$\Phi_1 = BA\cos\phi = -\pi BR^2 \cos\theta$

(b) Since no current is enclosed, $\Phi_{TOT} = 0 \Rightarrow \Phi_1 + \Phi_2 = 0$

$\Rightarrow \Phi_2 = -\Phi_1 = \pi BR^2 \cos\theta$

(which is much easier than the $\int \vec{B} \cdot d\vec{A}$ integral you would have to do the other way)

Problem 30.41

(a) $\vec{B}$ outside of the solenoid is zero, so the flux is just that through the part of the disk inside the solenoid:

$\Phi = \vec{B} \cdot \vec{A}_{sol} = \mu_0 \left(\frac{N}{\ell}\right) I (\pi r_{sol}^2) = (4\pi \times 10^{-7})\left(\frac{300}{0.3m}\right)(12.0A)\,\pi\left(\frac{0.025m}{2}\right)^2$

$\Phi = 7.40 \times 10^{-6}$ Wb

(b) $\Phi = \mu_0 \frac{N}{\ell} I [\pi(r_{out}^2 - r_{in}^2)] = (4\pi \times 10^{-7})\left(\frac{300}{0.3m}\right)(12.0A)\,\pi\left[(0.008m)^2 - (0.004m)^2\right]$

$\Phi = 2.27 \times 10^{-6}$ Wb