Hwk 4 Solutions   PHYS 142   Summer '09   Dr. Jenkins

Problem 25.55

Since work is equal to change in PE:

$$W = \int dU = \int V\,dq = \frac{k}{R}\int_0^Q q\,dq = \frac{kQ^2}{2R}$$

size is const.   only charge is changing

Question 26.3

For a simple metal sphere we are considering the potential difference between the sphere and $V=0$ at ∞. Meaning:

$$\Delta V = V_{sphere} = \frac{kq}{R}$$

$$\Rightarrow C = \frac{q}{\Delta V} = \frac{R}{k}$$

So capacitance is linearly proportional to radius

For tripling the volume:

$$V = \frac{4}{3}\pi R^3 \Rightarrow 3V = \frac{4}{3}\pi\left(3^{1/3}R\right)^3$$

Therefore increasing R by $\sqrt[3]{3}$ triples the volume;

Therefore C will also increase by $\sqrt[3]{3}$; So the answer is (d).

Question 26.4

The answer is (d). Capacitors in series add as:

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2}$$

Since $1/\text{big}$ = small, the $1/\text{small}$ term will dominate the sum; the total will be slightly less b/c adding $1/\text{big}$ will make $1/C_T$ slightly larger, which makes C_T slightly smaller.

Question 26.8

The answer is (b)

$$U = \frac{Q^2}{2C} = \frac{Q^2 d}{2\epsilon A} \Rightarrow d \to 2d \text{ means } U \to 2U$$

Question 26.9

(i) $C = \frac{\epsilon A}{d} \Rightarrow$ d increases means C decreases (b)

(ii) circuit is closed and no new charge can be added with the battery out, so the total charge won't change. (c)

(iii) $E = \frac{\sigma}{e} = \frac{Q}{\epsilon A}$; since Q stays the same, E stays the same (recall field btw capacitor plates is independent of distance) (c)

(iv) $C = \frac{Q}{\Delta V} \Rightarrow \Delta V = \frac{Q}{C} \Rightarrow$ C decreases w/ Q constant means ΔV increases

or

$\Delta V = Ed \Rightarrow$ E stays the same w/ d increases means ΔV increases. (a)

(v) $U = \frac{Q^2}{2C} \Rightarrow$ again C decreases w/ Q constant means U increases. (a)

Question 26.10

(i) C decreases by same reasoning as in 26.9 (b)

I'll come back to (ii) and (iii)...

(iv) Since the whole circuit is just the capacitor connected to the battery, ΔV must be that generated by the battery regardless of what happens with the capacitor (c)

(ii) $C = \frac{Q}{\Delta V} \Rightarrow Q = C\,\Delta V \Rightarrow$ C decreases with ΔV constant means Q decreases (b)

(iii) $E = \frac{Q}{\epsilon A} \Rightarrow$ Q decreases means E decreases (b)

(v) $U = \frac{1}{2} Q \Delta V \Rightarrow$ Q decreases w/ ΔV constant means U decreases (b)

Question 26.15

(i) $C = \kappa C_0$, $\kappa > 1 \Rightarrow$ C increases (a)

(iv) ΔV stays the same b/c it is governed again by the battery (c)

(ii) $Q = C\,\Delta V \Rightarrow$ Q increases (a)

(iii) $E = \frac{Q}{\epsilon A}$; Q and ϵ both increase by a factor of κ, so E stays the same (c)

(v) $U = \frac{1}{2} Q \Delta V \Rightarrow$ U increases (a)

Problem 26.3

(a) $E = \frac{kq}{r^2} \Rightarrow q = \frac{Er^2}{k} = \frac{(4.90\times10^4\ N/C)(0.21\ m)^2}{8.99\times10^9\ Nm^2/C^2}$

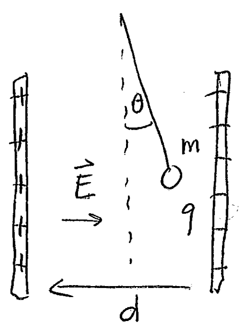
$q = 2.40\times10^{-7}\ C$

$\sigma = \frac{q}{A} = \frac{q}{4\pi R^2} = \frac{2.40\times10^{-7}\ C}{4\pi(0.12\ m)^2} = 1.33\times10^{-6}\ C/m^2$

(b) C for a conducting sphere :

$C = \frac{R}{k} = \frac{0.12\ m}{8.99\times10^9\ N\ m^2/C^2} = 1.33\times10^{-11}\ F$

Problem 26.8



$\Sigma F_y = 0$:

$T\cos\theta = mg$

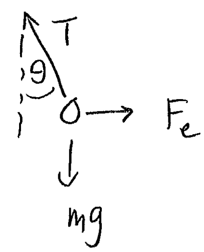
$T = \frac{mg}{\cos\theta}$

$\Sigma F_x = 0$:

$F_e = T\sin\theta = mg\tan\theta$

$\Rightarrow qE = mg\tan\theta$

$\Rightarrow E = \frac{mg}{q}\tan\theta$



$V = -\vec{E}\cdot\vec{d} = Ed = \frac{mgd}{q}\tan\theta$

Problem 26.12

(a) capacitors sum normally in parallel:

$$C_{eq} = C_1 + C_2 = 5.0\mu F + 12.0\mu F = 17.0\mu F$$

(b) the potential across each parallel branch is the same:

$$V_{battery} = V_1 = V_2 = 9.00V$$

(c) $C = \frac{Q}{V} \Rightarrow Q = CV$

$$Q_1 = C_1 V = (5.0\times10^{-6} F)(9.0V) = 4.5\times10^{-5} C$$

$$Q_2 = C_2 V = (12.0\times10^{-6} F)(9.0V) = 1.08\times10^{-4} C$$

Problem 26.13

(a) capacitors add inversely in series:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

$$C_{eq} = \frac{(5.0)(12.0)}{5.0 + 12.0}\mu F = \frac{60}{17}\mu F \approx 3.53\mu F$$

(b) find total equivalent charge first:

$$Q_{eq} = C_{eq}\Delta V = (3.53\times10^{-6} F)(9.0V) = 3.18\times10^{-5} C$$

same charge on each capacitor.

$$\Delta V_1 = \frac{Q_{eq}}{C_1} = \frac{3.18\times10^{-5} C}{5.0\times10^{-6} F} = 6.35 V$$

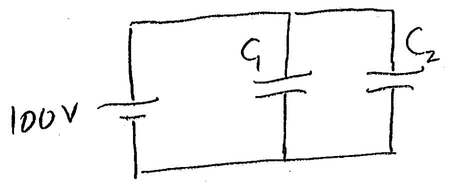
$$\Delta V_2 = \frac{Q_{eq}}{C_2} = \frac{3.18\times10^{-5} C}{12.0\times10^{-6} F} = 2.65 V$$

(c) charge is same on each,

$$Q_1 = Q_2 = Q_{eq} = 3.18\times10^{-5} C$$

Problem 26.28

(a)



$C_1 = 25.0\,\mu F$

$C_2 = 5.0\,\mu F$

$\Rightarrow C_{eq} = 30.0\,\mu F$

$U = \frac{1}{2} C V^2 = \frac{1}{2} (30.0 \times 10^{-6} F)(100\,V)^2$

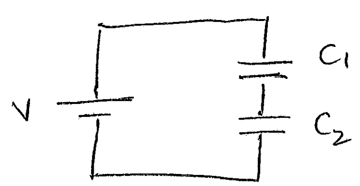
$U = 0.150\,J$

(b) need C_{eq} for series set up

$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{(25.0)(5.0)}{30.0}\,\mu F \simeq 4.17\,\mu F$

$U = \frac{1}{2} C V^2 \Rightarrow V = \sqrt{\frac{2U}{C}} = \sqrt{\frac{2(0.150 J)}{4.17 \times 10^{-6} F}}$

$V = 268\,V$



Problem 26.33

Thinking of the sphere as a capacitor (which it is, right)?

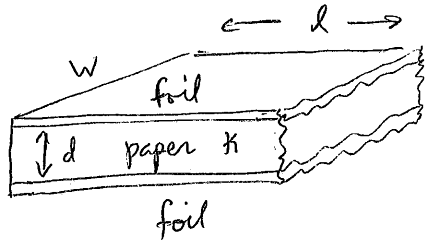
$$U = \frac{1}{2} Q V \quad ; \quad V = \frac{kQ}{R}$$

$$\Rightarrow U = \frac{kQ^2}{2R}$$

Problem 26.39

$$C = \frac{\epsilon A}{d} = \frac{K \epsilon_0 l w}{d}$$

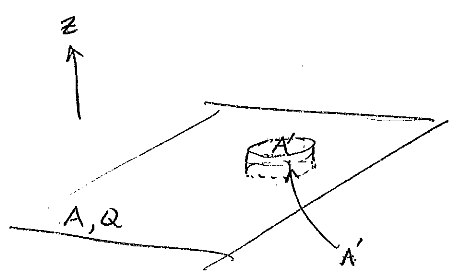
want to find l; d will be the thickness of the paper; note the thickness of the foil is not needed.



$$l = \frac{C d}{K \epsilon_0 W} = \frac{(9.50 \times 10^{-8} F)(2.50 \times 10^{-5} m)}{3.70 (8.85 \times 10^{-12})(0.070 m)}$$

$$l = 1.04 \text{ m}$$

Problem 26.44

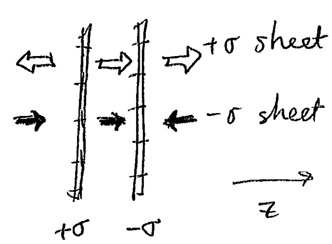


(a) $$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon}$$

$$2E\cancel{A'} = \frac{\sigma \cancel{A'}}{\epsilon} \quad ; \quad \sigma = \frac{Q}{A}$$

$$\Rightarrow \vec{E} = \frac{Q}{2\epsilon A} \hat{z}$$

(b) same capacitor argument from before:



$E_+ = E_-$

field in between is:

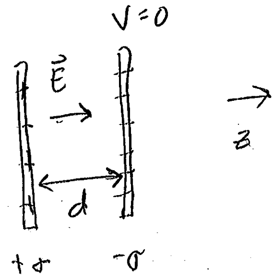
$$\vec{E} = (E_+ + E_-) \hat{z}$$

$$= \left(\frac{Q}{2\epsilon A} + \frac{Q}{2\epsilon A} \right) \hat{z}$$

$$\vec{E} = \frac{Q}{\epsilon A} \hat{z}$$

(Prob 26-44) cont.

(c) path moves from (−) plate $(V=0)$ to (+) plate



$$\Delta V = -\int_{(-)}^{(+)} \vec{E} \cdot d\vec{s} = -\vec{E} \cdot \vec{d}$$

where $\vec{d} = -d\,\hat{z}$

$$\Rightarrow \Delta V = Ed = \frac{Qd}{\epsilon A}$$

(d) $$C = \frac{Q}{\Delta V} = \cancel{Q}\left(\frac{\epsilon A}{\cancel{Q}d}\right) = \frac{\epsilon A}{d} = \frac{\kappa \epsilon_0 A}{d}$$

Question 27.3

$R = \frac{\rho l}{A}$; ρ, l the same for both wires

if $R_A = 3R_B$

(i) $$\Rightarrow \frac{\rho l}{A_A} = \frac{3\rho l}{A_B} \Rightarrow \frac{1}{A_A} = \frac{3}{A_B}$$

$$\Rightarrow A_A = \frac{A_B}{3} \Rightarrow \frac{A_A}{A_B} = \frac{1}{3}$$; answer (f)

(ii) $$\frac{A_A}{A_B} = \frac{\pi r_A^2}{\pi r_A^B} = \left(\frac{r_A}{r_B}\right)^2 = \frac{1}{3} \Rightarrow \frac{r_A}{r_B} = \frac{1}{\sqrt{3}}$$; answer (e)

Question 27.4

$l \to \frac{1}{3} l_0$; also $A \to 3A_0$

$$R_0 = \frac{\rho l_0}{A_0} \; ; \quad R = \frac{\rho l}{A} = \frac{\rho}{3A_0}\left(\frac{l_0}{3}\right) = \frac{\rho l_0}{9A_0} = \frac{1}{9} R_0$$

answer (b)

Question 27.7

(i) $I_{avg} = nqv_d A$

if I is constant, then when A decreases, v_d must increase;

answer (a)

(ii) $R = \frac{\rho l}{A}$; resistance per unit length is $\frac{R}{l} = \frac{\rho}{A}$; for const. ρ (same wire), $\frac{R}{l}$ will increase as A decreases ; answer (a)

Question 27.10

There are electrons evenly distributed throughout the wire; when the switch is turned on, the field is setup up everywhere nearly instantly, so the potential is also in place everywhere, so the electrons start moving everywhere at the same time ; $\therefore$ the electrons in the wire very near the bulb will flow through the bulb virtually immediately.

Question 27.12

$$P = \frac{\Delta V^2}{R}$$

R will be different for each :

$$R = \frac{\rho l}{A} = \frac{\rho l}{\pi r^2}$$

if $R_B = \frac{\rho l_B}{A_B}$, then $R_A = \frac{\rho l_A}{A_A} = \frac{2\rho l_B}{\pi (2r_B)^2} = \frac{\rho l_B}{2A_B} = \frac{R_B}{2}$

$$\Rightarrow P_B = \frac{\Delta V^2}{R_B} \; ; \; P_A = \frac{\Delta V^2}{R_B/2} = \frac{2\Delta V^2}{R_B}$$

$$\Rightarrow \frac{P_A}{P_B} = \frac{2}{1} = 2\times$$; answer (e)

Problem 27.4

$I_{avg} = \dfrac{\Delta q}{\Delta t}$; Δt is the period of the rotation $T = \dfrac{2\pi}{\omega}$

$\Rightarrow \quad I = \dfrac{q\omega}{2\pi}$

Problem 27.10

$I = n q v_d A$; n is # of charge carriers per unit volume ; for 1 e^- per atom, this is same as number of atoms per unit volume ; to get from density, we need the mass per atom :

molar mass of aluminum is $M = 27.0\, g/mol$

$$\frac{27.0\,g}{\cancel{mol}}\left(\frac{\cancel{mol}}{6.02\times10^{23}\text{ atoms}}\right) = 4.49\times10^{-23}\ g/\text{atom}$$

$$n = \frac{\text{density}}{\text{mass/atom}} = \frac{2.70\,g}{cm^3}\left(\frac{\text{atom}}{4.49\times10^{-23}\,g}\right) = 6.02\times10^{22}\ \text{atoms}/cm^3$$

$$n = 6.02\times10^{28}\ \text{atoms}/m^3$$

$$\Rightarrow \quad v_d = \frac{I}{nqA} = \frac{5.00\ C/s}{(6.02\times10^{28}/\cancel{m^3})(1.6\times10^{-19}\,\cancel{C})(4.0\times10^{-6}\,\cancel{m^2})}$$

$$v_d = 1.30\times10^{-4}\,m/s$$

Problem 27.13

$\rho_d = \dfrac{m}{V} = \dfrac{m}{A\ell}$ (ρ_d : mass density)

$\rho = 1.7\times10^{-8}\,\Omega\cdot m$ for Cu ; $\rho_d = 8.94\,g/cm^3 = 8940\,kg/m^3$

(a) $R = \dfrac{\rho \ell}{A}$; $A = \dfrac{m}{\rho_d \ell}$

$$\Rightarrow R = \frac{\rho \rho_d \ell^2}{m} \Rightarrow \ell = \sqrt{\frac{mR}{\rho \rho_d}} = \sqrt{\frac{(1.0\times10^{-3}\,\cancel{kg})(0.5\,\cancel{\Omega})}{(1.7\times10^{-8}\,\cancel{\Omega}\cdot \cancel{m})(8940\,\cancel{kg}/m^{\cancel{3}\,2})}}$$

$$\ell = 1.82\,m$$

(Prob 27.13) cont.

(b) from (a), plug in l to resistance eqn:

$$R = \frac{\rho l}{A} \Rightarrow A = \frac{\rho l}{R} = \frac{(1.7\times10^{-8}\ \Omega\cdot m)(1.82m)}{0.50\Omega}$$

$$A = 6.19\times10^{-8}\ m^2$$

$$\Rightarrow A = \pi r^2 \Rightarrow d = 2\sqrt{\frac{A}{\pi}} = 2\sqrt{\frac{6.19\times10^{-8}\ m^2}{\pi}}$$

$$d = 2.8\times10^{-4}\ m$$