Hwk 2 Solutions PHYS 142 Summer '09 Dr. Jenkins

Question 24.2

If there is more outgoing flux than incoming, there must be (better be) a net positive charge enclosed.

Question 24.5

(a) There will of course be field lines through the four surfaces enclosing the wire, but there will also be lines along the two cap ends, through which the wire travels; therefore the answer is (a) zero.

(b) The flux through the end caps is zero b/c no lines of field go through them; therefore the answer is (b) two.

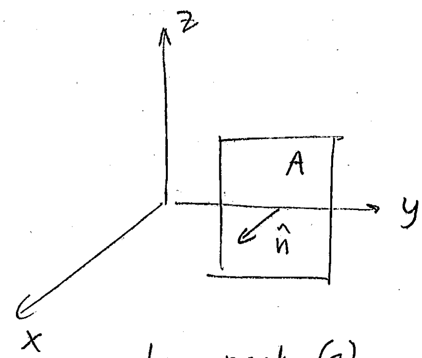
Question 24.8

Gauss's law cannot uniquely determine the field without knowing the charge distribution. A gaussian surface needs to be properly oriented with respect to a charge distribution such that the field is constant over the surface; so without a specific charge distribution, we can't choose a surface, and therefore cannot describe its surface area quantitatively.

Problem 24.3

Knowing that the direction of an area $\vec{A}$ is given by the normal vector oriented in a "sensible" positive direction (think right hand rule here):

(a) $\vec{A} = A\hat{i}$



for part (a)

we can use the basic definition of the dot product:

$\Phi_e = \vec{E} \cdot \vec{A} = (a\hat{i} + b\hat{j}) \cdot A\hat{i}$

$\Phi_e = Aa$

(b) $\vec{A} = A\hat{j}$

$\Phi_e = (a\hat{i} + b\hat{j}) \cdot A\hat{j} = bA$

(c) $\vec{A} = A\hat{k}$

$\Phi_e = (a\hat{i} + b\hat{j}) \cdot A\hat{k} = 0$

Problem 24.4

(a) $A = (0.1\,m)(0.3\,m) = 0.030\,m^2$

$\Phi_e = \vec{E} \cdot \vec{A} = EA\cos 180° = 7.8 \times 10^4\,N/C\,(0.03\,m^2)(-1)$

$\Phi_e = -2.34 \times 10^3\ N \cdot m^2/C$

(b) for the width of the surface:

$10\,cm = w\cos 60° \Rightarrow w = \dfrac{10.0\,cm}{\cos 60°} = 20.0\,cm$

$\Rightarrow A = (0.2\,m)(0.3\,m) = 0.060\,m^2$

$\Phi_e = EA\cos 60° = 7.8 \times 10^4\,N/C\,(0.06\,m^2)(\tfrac{1}{2}) = 2.34 \times 10^3\,Nm^2/C$

(c) no flux through other surfaces; $\therefore$ total flux through the box is zero.

Problem 24.5

Since the field is vertical, there will be the same amount of flux through the base as through the four faces, so we can just find the flux through the base and change the sign:

$$\Phi_b = EA\cos 180 = 52.0\,N/C\,(6.0\,m)^2\,(-1) = -1.87\times10^3\ N\cdot m^2/C$$

$$\Rightarrow \Phi_{faces} = 1.87\times10^3\ N\cdot m^2/C$$

Problem 24.6

(a) $$E = \frac{kQ}{r^2} \Rightarrow Q = \frac{Er^2}{k} = \frac{(890\,N/C)(0.75\,m)^2}{8.99\times10^9\ N\,m^2/C^2}$$

$$\Rightarrow Q = -5.57\times10^{-8}\,C$$

negative b/c field points inward.

(b) if field is constant on a spherical Gaussian surface, then the charge distribution inside must be spherically symmetric.

Problem 24.9

from Gauss's law:

$$\Phi_e = \frac{Q_{enc}}{\epsilon_0}$$

(a) $$\Phi = \frac{-2Q+Q}{\epsilon_0} = \frac{-Q}{\epsilon_0}$$

(b) $$\Phi = \frac{-Q+Q}{\epsilon_0} = 0$$

(c) $$\Phi = \frac{-2Q-Q+Q}{\epsilon_0} = \frac{-2Q}{\epsilon_0}$$

(d) $$\Phi = 0$$

Problem 24.10

(a) $\Phi = \frac{Q_{enc}}{\epsilon_0} = \frac{12.6 \times 10^{-6} C}{8.85 \times 10^{-12} C^2/N \cdot m^2} = 1.36 \times 10^6 N \cdot m^2/C$

(b) just half the total flux:

$\Phi_{hem} = \frac{1}{2}(1.36 \times 10^6 N \cdot m^2/C) = 6.78 \times 10^5 N \cdot m^2/C$

(c) independent of radius b/c $E \sim \frac{1}{r^2}$, $A \sim r^2$; ∴ radial dependence cancels out in flux; any number of field lines will pass through a spherical shell, regardless of its size

Problem 24.13

total charge enclosed is $5.00 \mu C - 6(1.00 \mu C) = -1.00 \times 10^{-6} C$

total flux through cube:

$\Phi = \frac{Q_{enc}}{\epsilon_0}$

1/6 goes through each face:

$\Phi_{face} = \frac{Q_{enc}}{6\epsilon_0} = \frac{-1.00 \times 10^{-6} C}{6(8.85 \times 10^{-12} C^2/N \cdot m^2)} = -1.88 \times 10^4 N \cdot m^2/C$

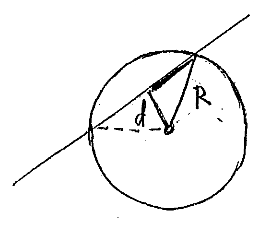
Problem 24.15

for $R < d$:

$\Phi = 0$ b/c no charge is enclosed

for $R > d$:

length of line inside sphere is (by Pyth. thm):



$L = 2\sqrt{R^2 - d^2}$

$\Rightarrow Q_{enc} = \lambda L = 2\lambda\sqrt{R^2 - d^2}$

$\Rightarrow \Phi_e = \frac{Q_{enc}}{\epsilon_0} = \frac{2\lambda\sqrt{R^2 - d^2}}{\epsilon_0}$

Problem 24.17

(b) total flux is :

$$\Phi = \frac{Q_{enc}}{\epsilon_0} = \frac{170\times10^{-6}\,C}{8.85\times10^{-12}\,C^2/Nm^2} = 1.92\times10^{7}\,N\cdot m^2/C$$

(a) One face :

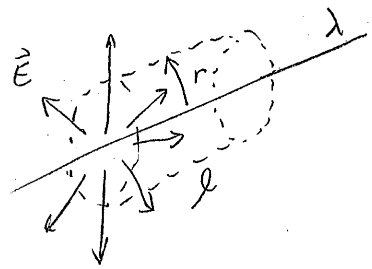
$$\Phi_{face} = \frac{\Phi}{6} = 3.20\times10^{6}\,N\cdot m^2/C$$

(c) total flux wouldn't change (b), but flux through each face would be inversely proportional to distance from charge (think of field lines through each)

Problem 24.20

from Gauss's law, using a cylindrical surface :

$$\int E\cdot dA = \frac{Q_{enc}}{\epsilon_0}$$



no flux through ends ; $Q_{enc} = \lambda\ell$;

E const over curved surface :

$$\Rightarrow EA = \frac{\lambda\ell}{\epsilon_0}$$

$$A = 2\pi r\ell \Rightarrow E = \frac{\lambda\ell}{2\pi r\ell\epsilon_0} \Rightarrow \vec{E} = \frac{\lambda}{2\pi\epsilon_0 r}\,\hat{r}$$

$\lambda = -90.0\,\mu C/m$

(a) $r = 10.0\,cm$

$$\Rightarrow \vec{E}_{10} = \frac{-90.0\times10^{-6}\,c/m}{2\pi(8.85\times10^{-12})(0.010\,m)} = -1.62\times10^{7}\,N/C\,\hat{r}$$

(b)

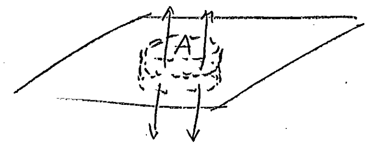
$$\vec{E}_{20} = \frac{-90.0\times10^{-6}\,C/m}{2\pi(8.85\times10^{-12})(0.020\,m)} = -8.09\times10^{6}\,N/C\,\hat{r}$$

(c)

$$\vec{E}_{100} = -1.62\times10^{5}\,N/C\,\hat{r}$$

Problem 24.21

from gauss's law, using pillbox surface



$$\int \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

$$2E\cancel{A} = \frac{\sigma \cancel{A}}{\epsilon_0}$$

$$E = \frac{\sigma}{2\epsilon_0} = \frac{9.00\times10^{-6}\,C/m^2}{2(8.85\times10^{-12})} = 5.08\times10^5\,N/C \quad (\text{upward})$$

Problem 24.23

field from plate is $E = \frac{\sigma}{2\epsilon_0}$

force of gravity and electrostatic forces are equal and opposite:

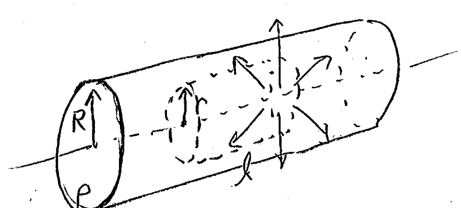
$$F_g = F_e \Rightarrow mg = qE = \frac{q\sigma}{2\epsilon_0}$$

$$\Rightarrow \sigma = \frac{2\epsilon_0 mg}{q} = \frac{2(8.85\times10^{-12})(0.010g)(9.8\,m/s^2)}{-0.70\times10^{-6}C}$$

$$\sigma = -2.48\times10^{6}\,C/m^2$$

Problem 24.27

using Gauss law and surface shown:



$$\int \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

$$E(2\pi r l) = \frac{Q_{enc}}{\epsilon_0} \quad (\text{no flux through ends})$$

$$Q_{enc} = \rho V \;;\; V = \pi r^2 l \Rightarrow Q_{enc} = \rho \pi r^2 l$$

$$\Rightarrow E(2\cancel{\pi}\cancel{r}\cancel{l}) = \frac{\rho \cancel{\pi} r^{\cancel{2}} \cancel{l}}{\epsilon_0}$$

$$\vec{E} = \frac{\rho r}{2\epsilon_0}\hat{r}$$

Problem 24.31

(a) wire is virtually "infinite" in length when compared to cylinder; can ∴ use field for very long straight wire:

$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0 r}\hat{r} \quad ; \quad \lambda = \frac{Q_{TOT}}{L_{TOT}} = \frac{2.0\times10^{-6}C}{7.0m}$$

$$\vec{E} = \frac{2.0\times10^{-6}C \quad \hat{r}}{2\pi(8.85\times10^{-12})(7.0m)(0.10m)} = 5.14\times10^{4} N/C\ \hat{r} \text{ (radially outward)}$$

(b)

$$\Phi = EA\cos 0° = (5.14\times10^{4} N/C)(2\pi)(0.10m)(0.02m)(1)$$

$$\Phi = 646\ N\cdot m^2/C$$

Problem 24.32

for the Al, field for a conductor is $E = \frac{\sigma_{Al}}{\epsilon_0}$; and $\sigma_{Al} = \frac{Q}{2A}$ (2 because charge will spread out over both sides of the plate); for the glass; field for an insulator is the usual $E = \frac{\sigma_g}{2\epsilon_0}$, but $\sigma_g = \frac{Q}{A}$ (b/c the the charge is on one side only)

$$\Rightarrow E_{Al} = \frac{Q}{2A\epsilon_0} \quad \text{and} \quad E_g = \frac{Q}{2A\epsilon_0}$$

So the fields are the same.

Problem 24.34

(a) field at 12 cm is the field <u>inside</u> the sphere which is always zero for a conductor.

(b) by gauss law, field is the same as for a point charge at the center:

$$\vec{E} = \frac{kq}{r^2}\hat{r} = \hat{r}\frac{(8.99\times10^{9})(40\times10^{-9}C)}{(0.17m)^2} = 1.24\times10^{4} N/C\ \hat{r}$$

(c) $$\vec{E} = \hat{r}\frac{(8.99\times10^{9})(40\times10^{-9}C)}{(0.75m)^2} = 639\ N/C\ \hat{r}$$

(d) charge on conductor is <u>always</u> <u>only</u> on the surface, so it would not change answers in (a) – (c).

Problem 24.43

(a) Field is the same point charge field as usual, so we basically just need to find an expression for this area (some proportion of the whole surface area of a sphere, $4\pi R^2$)

use spherical coords to set up an integral for the area:

$$A = R^2 \int_0^{2\pi} d\phi \int_0^{\theta} \sin\theta' \, d\theta'$$

$$= 2\pi R^2 \left(-\cos\theta' \Big|_0^{\theta} \right)$$

$$= 2\pi R^2 (1 - \cos\theta)$$

$$\Rightarrow \Phi = EA = \left(\frac{kQ}{R^2}\right) 2\pi R^2 (1 - \cos\theta)$$

$$= \frac{1}{\cancel{2}\, \cancel{4}\pi\epsilon_0} \frac{Q}{\cancel{R^2}} (\cancel{2\pi R^2})(1 - \cos\theta)$$

$$\Phi = \frac{Q}{2\epsilon_0} (1 - \cos\theta)$$

(b) hemisphere

$\cos 90 = 0$

$$\Rightarrow \Phi = \frac{Q}{2\epsilon_0}$$

(c) whole sphere

$\cos 180 = -1$

$$\Rightarrow \Phi = \frac{Q}{2\epsilon_0}(1+1) = \frac{Q}{\epsilon_0}$$

Problem 24.47

(a) use gauss law for each

for $r < a$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} \qquad Q_{enc} = \rho V$$

$$E(4\pi r^2) = \frac{Q_{enc}}{\epsilon_0} \qquad V = \frac{4}{3}\pi r^3$$

(Prob 24.47) cont.

(a) cont.

$\Rightarrow \quad Q_{enc} = \frac{4}{3}\pi\rho r^3$

$\Rightarrow \quad E(4\pi r^2) = \frac{4}{3}\pi r^3 \frac{\rho}{\epsilon_0}$

$\Rightarrow \quad E = \frac{\rho r}{3\epsilon_0}$

$a<r<b$

$Q_{enc} = Q$

$\Rightarrow \quad E = \frac{Q}{4\pi\epsilon_0 r^2} = \frac{kQ}{r^2}$

$b<r<c$

$E = 0$ (inside conductor)

$r>c$

$Q_{enc} = Q$ again (conductor isn't carrying any charge)

$\Rightarrow \quad E = \frac{kQ}{r^2}$

(b)

since $E=0$ for $b<r<c$, a gaussian surface of this radius must enclose zero total charge; that means charge induced on inner surface, q_b, must be equal and opposite to Q $\Rightarrow$ $q_b = -Q$

$\sigma_b = \frac{q_b}{A_b} = \frac{-Q}{4\pi b^2}$

if the total charge on outer sphere is zero, then q_b and q_c must cancel w/ each other $\Rightarrow$ $q_c = -q_b = Q$

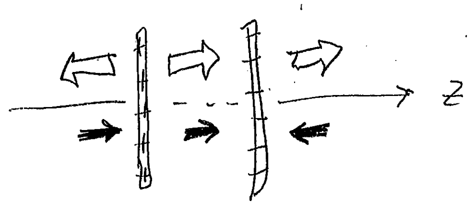
$\Rightarrow \quad \sigma_c = \frac{Q}{4\pi c^2}$

Problem 24.50

the field due to each sheet separately will be the usual for an "infinite" sheet

$E = \frac{\sigma}{2\epsilon_0}$ w/ field away from $+\sigma$ sheet and toward $-\sigma$ sheet.

looking at a handy diagram w/ white arrows for $+\sigma$ sheet and black arrows for $-\sigma$ sheet



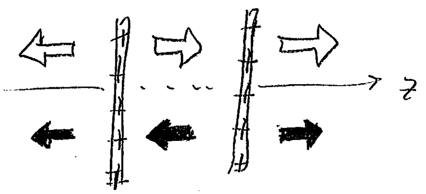
we see that outside of both plates, the fields cancel, and in between we have double the field to the right:

$E_{left} = E_{right} = 0$; $\vec{E}_{between} = \frac{\sigma}{\epsilon_0}\hat{z}$

(a) (c) (b)

Problem 24.51

using our same diagram again we see that the field cancels in the middle and doubles on the outside:



$\vec{E}_{left} = \frac{-\sigma}{\epsilon_0}\hat{z}$; $\vec{E}_{right} = \frac{\sigma}{\epsilon_0}\hat{z}$; $\vec{E}_{between} = 0$