Question 23.12
An electric field is the force field per unit charge emitted by a source; any charged object emersed in an E-field will experience a net force equal to the sum of the forces caused by all sources present. This net force is, as always, a single-valued vector quantity (one magnitude, one direction); therefore, the E-field will similarly be a single-valued vector quantity. Since field lines are essentially a "contour plot" of these E-field vectors in a region of space, crossing lines would indicate the nonsensical presence of two separate net forces on any charged object placed at the point of intersection, which is physically and mathematically impossible.

Question 23.14
The answer is *(a) zero*. Each infinitesimal segment of the ring will generate an identical electric field, and, given the circular distribution of these segments, each will have a counterpart on the opposite side of the ring such that the field vectors produced by the two will be exactly equal in magnitude and opposite in direction at the center of the ring, thereby canceling with each other, resulting in a zero net field vector.

Problem 23.7

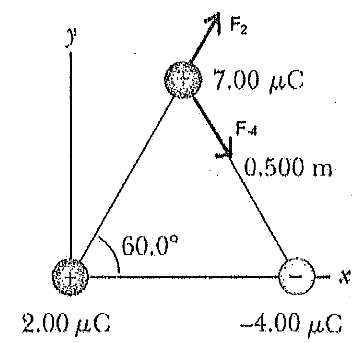


Figure P23.7 Problems 7 and 14.

First calculate the magnitude of the individual forces from each of the other two particles:

$$F_2 = \frac{kqq_0}{r^2} = \frac{(8.99\times10^9\,\mathrm{N\cdot m^2/C^2})\,(7.00\times10^{-6}\,\mathrm{C})\,(2.00\times10^{-6}\,\mathrm{C})}{(0.500\,\mathrm{m})^2} = 0.503\,\mathrm{N}$$

$$F_{-4} = \frac{(8.99\times10^9\,\mathrm{N\cdot m^2/C^2})\,(7.00\times10^{-6}\,\mathrm{C})\,(4.00\times10^{-6}\,\mathrm{C})}{(0.500\,\mathrm{m})^2} = 1.01\,\mathrm{N}$$

Now calculate x and y components; note the x components are both positive, but the y-component of the -4μC force is negative:

$$F_x = 0.503\cdot\cos 60^\circ + 1.01\cdot\cos 60^\circ = 0.755\,\mathrm{N}$$
$$F_x = 0.503\cdot\sin 60^\circ - 1.01\cdot\sin 60^\circ = -0.436\,\mathrm{N}$$

$$\Rightarrow \qquad |F| = \sqrt{0.755^2 + 0.436^2} = 0.872\,\mathrm{N}$$
$$\text{and} \qquad \theta = \tan^{-1}\left(\frac{-0.436}{0.755}\right) = -30.0^\circ$$

So the resultant net force is 0.872 N @ 330°.

Problem 23.8

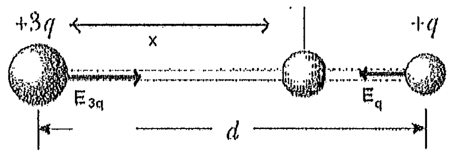


Figure P23.8

We want to find the point between the two outer beads where the E-field goes to zero. Working in one dimension, we can disregard all but the signs of the vector nature of the problem. If we let the distance from the $3q$ bead to the third bead be x, then the distance from the q bead to the charge will be $d - x$:

$$E_{3q} = \frac{3kq}{x^2}; \qquad E_q = -\frac{kq}{(d-x)^2}$$

The net field at the position of the third bead is:

$$E_{net} = \frac{3kq}{x^2} - \frac{kq}{(d-x)^2},$$

so the net field (i.e. the net *force*) will be zero if:

$$\frac{3kq}{x^2} = \frac{kq}{(d-x)^2} \quad \text{or} \quad \frac{3}{x^2} = \frac{1}{(d-x)^2}$$

$$\Rightarrow \quad \sqrt{3}\,(d-x) = \pm x \quad \Rightarrow \quad x\left(1 \pm \frac{1}{\sqrt{3}}\right) = d$$

The (+) solution gives a value for $x < d$:

$$x = \frac{d}{1 + \frac{1}{\sqrt{3}}} \approx 0.634d$$

This equilibrium is a stable one if the third bead is also positively charged, resulting in only repulsive forces.

Problem 23.10

Since the forces here are equal in magnitude and both attractive, the vertical components of the two forces cancel out, leaving the net force in the horizontal $-x$ direction. This force will go to zero as $x \to 0$, and will point in the $+x$ direction as the particle moves along the $-x$ axis. Therefore, when $x \ll d$, the particle will oscillate about the origin, resulting in simple harmonic motion along the x-axis.

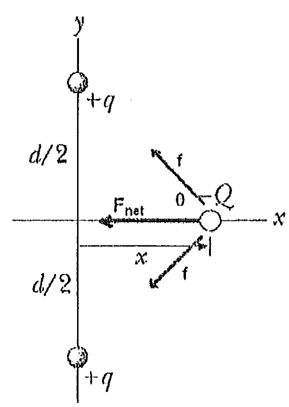


Figure P23.10

(a) To show this mathematically, we will start with the electric force equation:

$$f = \frac{kqQ}{r^2} = \frac{kqQ}{x^2 + \left(\frac{d}{2}\right)^2}$$

Since $x \ll d$, we can approximate the denominator, resulting in a constant value for f:

$$f \approx \frac{4kqQ}{d^2}$$

The net force is, then, two times the x-component of f in the $-\hat{\imath}$ direction:

$$F_{net} = -\frac{8kqQ \cdot \cos\theta}{d^2}$$

We can write:

$$\cos\theta = \frac{adj}{hyp} = \frac{x}{\sqrt{x^2 + \left(\frac{d}{2}\right)^2}} \approx \frac{2x}{d}$$

$$\Rightarrow \qquad F_{net} = -\frac{16kqQ \cdot x}{d^3}$$

Now we rewrite the left-hand side:

$$F_{net} = ma \qquad \Rightarrow \qquad \frac{d^2x}{dt^2} = -\frac{16kqQ}{md^3} \cdot x$$

Which we recognize as the differential equation for a harmonic oscillator with:

$$\omega^2 = \frac{16kqQ}{md^3} \qquad \Rightarrow \qquad T = \frac{2\pi}{\omega} = \frac{\pi}{2}\sqrt{\frac{md^3}{kqQ}}$$

(b) Velocity is maximum at the origin, while acceleration is maximum at $x = a$;

$$v_{max} = \omega a_{max} = \sqrt{\frac{16kqQ}{md^3}} \cdot \frac{16kqQa}{md^3} = 64a\left(\frac{kqQ}{md^3}\right)^{3/2}$$

Problem 23.12

This problem is entirely similar to Problem 23.8 with different charges and $d \to 1.00$m. For $q_1 = -2.50\mu$C and $q_1 = 6.00\mu$C:

$$E_1 = -\frac{kq_1}{x^2}; \qquad E_2 = -\frac{kq_2}{(1.00 - x)^2}$$

The net field will be zero where:

$$E_{net} = E_1 + E_2 = 0$$

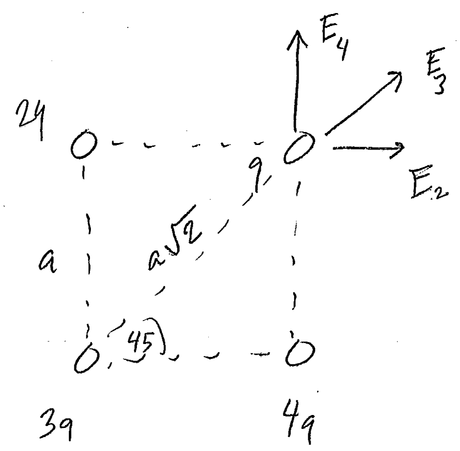
$$\Rightarrow \qquad -\frac{kq_1}{x^2} = \frac{kq_2}{(1.00 - x)^2} \qquad \Rightarrow \qquad \frac{2.50}{x^2} = \frac{6.00}{(1.00\text{m} - x)^2}$$

$$\Rightarrow \qquad (1.00\text{m} - x)^2 = 2.40x^2 \qquad \Rightarrow \qquad 1.00\text{m} - x = \pm 1.55x$$

$$\Rightarrow \qquad x = 0.392\text{m}; \; x = -1.82\text{m}$$

Here we have two solutions that both seem reasonable, but examining the original distribution of charges, we see that the fields will *add* together in between the two charges and only be capable of canceling to the *left* of both charges. Therefore we require the second answer as the correct solution, 1.82 m to the left of the -2.50μC charge. Note that our (incorrect) assumption that x would be positive (i.e. between the charges) in our original setup corrects itself in the math.

Problem 23.17



(a) Total E-field is the vector sum of the fields generated by the other 3 charges:

$$\vec{E}_2 = \frac{2kq}{a^2}\,\hat{i}$$

$$\vec{E}_4 = \frac{4kq}{a^2}\,\hat{j} \qquad |E_3| = \frac{3kq}{2a^2}$$

$$\Rightarrow E_{3x} = \frac{3kq}{2a^2}\cos 45 = \frac{3\sqrt{2}\,kq}{4a^2}$$

$$E_{3y} = \frac{3kq}{2a^2}\sin 45 = \frac{3\sqrt{2}\,kq}{4a^2}$$

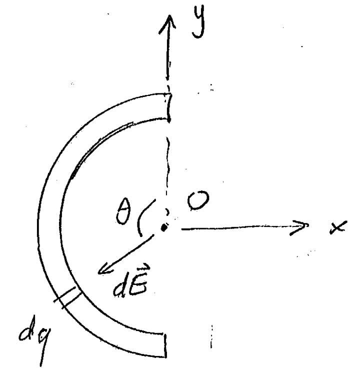
$$\Rightarrow E_{net\,x} = E_2 + E_{3x} = \left(2 + \frac{3\sqrt{2}}{4}\right)\frac{kq}{a^2}$$

$$E_{net\,y} = E_4 + E_{3y} = \left(4 + \frac{3\sqrt{2}}{4}\right)\frac{kq}{a^2}$$

$$\Rightarrow (3.06\,\hat{i} + 5.06\,\hat{j})\,\frac{kq}{a^2} \quad \Rightarrow \quad \vec{E}_{net} = 5.91\,\frac{kq}{a^2} \text{ @ } 58.8^\circ$$

(b) $$\vec{F}_{net} = q\,\vec{E}_{net} = 5.91\,\frac{kq^2}{a^2} \text{ @ } 58.8^\circ$$

Problem 23.27



$$\vec{E} = k\int \frac{dq}{r^2}\,\hat{r}$$

due to symmetry, E_y will be zero.

for E_x, note definition of θ:

$$E_x = \int dE \sin\theta = k\int \frac{dq}{r^2}\sin\theta$$

$$dq = \lambda r\, d\theta$$

$$\Rightarrow E_x = \frac{k\lambda}{r}\int_0^{\pi} \sin\theta\, d\theta = \frac{k\lambda}{r}\left[\cos 0 - \cos\pi\right]$$

$$E_x = \frac{2k\lambda}{r}$$

to get the proper numbers in here, we need to redefine λ and r in terms of L and q:

$$\lambda = \frac{q}{L} \;;\quad L = \pi r \Rightarrow r = \frac{L}{\pi}$$

$$\Rightarrow E_x = \frac{2\pi k q}{L^2} = \frac{2\pi\,(8.99\times10^9\ \mathrm{N\,m^2/C^2})(-7.5\times10^{-6}\ \mathrm{C})}{(0.14\ \mathrm{m})^2}$$

$$\Rightarrow \vec{E} = -2.16\times10^7\ \mathrm{N/C}\ \hat{i}$$

Problem 23.30

(a) entire surface is $A = \underset{\text{(caps)}}{2\pi r^2} + \underset{\text{(side)}}{2\pi r l} = 2\pi r(r+l)$

$\sigma = 15.0 \times 10^{-9}\ C/m^2$

$Q = \sigma A = (15.0 \times 10^{-9}\ C/m^2)\, 2\pi\, (0.025 m)(0.025 m + 0.060 m)$

$Q = 2.00 \times 10^{-10}\ C$

(b) lateral surface only $A = 2\pi r l$

$Q = (15.0 \times 10^{-9}\ C/m^2)\, 2\pi (0.025 m)(0.060 m)$

$Q = 1.41 \times 10^{-10}\ C$

(c) $\rho = 500 \times 10^{-9}\ C/m^3$; volume is $V = \pi r^2 l$

$Q = \rho V = (500 \times 10^{-9}\ C/m^3)\, \pi\, (0.025 m)^2 (0.06 m)$

$Q = 5.89 \times 10^{-11}\ C$

Problem 23.34

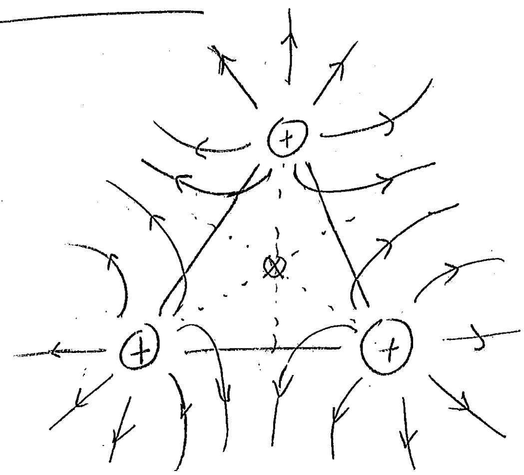
(a) simply take the ratio of field lines

$\frac{q_1}{q_2} = \frac{-3}{9} = -\frac{1}{3}$

(b) q_1 is negative b/c the lines are entering the charge ; q_2 is positive b/c the lines are exiting.

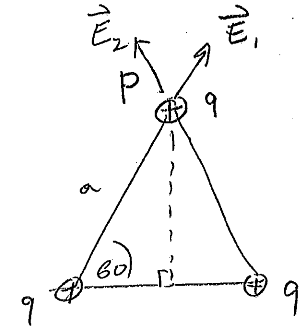
Problem 23.35

(a)



the field goes to zero at the geometric center of the triangle.

(b) $|E_1| = |E_2| = \frac{kq}{a^2}$



x-components cancel by symmetry.

$$\vec{E} = 2E_y\,\hat{j} = \frac{2kq}{a^2}\sin 60^\circ\,\hat{j} = \frac{\sqrt{3}kq}{a^2}\hat{j} \simeq 1.732\frac{kq}{a^2}\hat{j}$$

Problem 23.36

(a) use $\vec{F} = q\vec{E} = m\vec{a} \Rightarrow \vec{a} = \frac{q\vec{E}}{m}$

$q = +1.609\times10^{-19}\,C$; $m = 1.67\times10^{-27}\,kg$ for proton

$$\vec{a} = \frac{(1.609\times10^{-19}C)}{1.67\times10^{-27}kg}(-6.00\times10^{5}\,N/C)\,\hat{\imath} = -5.78\times10^{13}\,\hat{\imath}\ m/s^2$$

(b) use kinematics for (b) and (c):

$$\cancelto{0}{v_f^2} - v_0^2 = 2a\Delta x \Rightarrow v_0 = \sqrt{-2(-5.78\times10^{13}\,m/s^2)(0.07\,m)}$$

$$\Rightarrow \vec{v}_0 = 2.84\times10^{6}\,\hat{\imath}\ m/s$$

(Problem 23.36) cont.

(c) $\cancel{v_f}^{\,0} = v_0 + at \Rightarrow t = \frac{-v_0}{a} = \frac{-2.84\times10^{6}\ m/s}{-5.76\times10^{13}\ m/s^2}$

$t = 4.92\times10^{-8}\ s$

Problem 23.37

(a) similar to prob. 36:

$a = \frac{qE}{m} = \frac{(1.609\times10^{-19}\,C)(640\ N/C)}{1.67\times10^{-27}\ kg} = 6.17\times10^{10}\ m/s^2$

(b) $v_f = \cancel{v_0}^{\,0} + at \Rightarrow t = \frac{v_f}{a} = \frac{1.20\times10^{6}\ m/s}{6.17\times10^{10}\ m/s^2}$

$t = 1.95\times10^{-5}\ s$

(c) $\Delta x = \cancel{v_0}^{\,0}t + \frac{1}{2}at^2 = \frac{1}{2}(6.17\times10^{10}\ m/s^2)(1.95\times10^{-5}\,s)^2$

$\Delta x = 11.7\ m$

(d) $K = \frac{1}{2}mv_f^2 = \frac{1}{2}(1.67\times10^{-27}\,kg)(1.20\times10^{6}\ m/s)^2$

$K = 1.20\times10^{-15}\ J$

Problem 23.39

use work energy thm: $W = -\Delta K$

$W = Fd$; $F = qE = +eE$

$eEd = K$ (change is $-K$ b/c particle is coming to rest)

$E = \frac{K}{ed}$; direction is in direction of motion for e^-.

Problem 23.49

each block has charge $\frac{Q}{2}$

electric force in equilibrium w/ spring force:

$$F_E = F_s \Rightarrow \frac{k q_1 q_2}{r^2} = k x$$

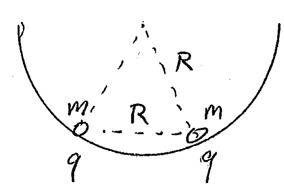
$$\Rightarrow \frac{k\left(\frac{Q}{2}\right)^2}{L^2} = k(L - L_0)$$

solve for Q: $\left(\frac{Q}{2}\right)^2 = \frac{kL^2}{k}(L - L_0)$

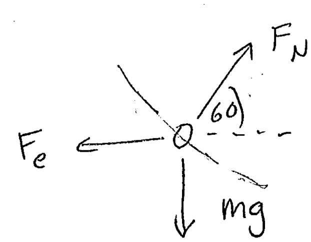
$$\Rightarrow Q = 2L\sqrt{\frac{k}{k}(L - L_0)}$$

Problem 23.56

from PHYS 142, using:



$\vec{F}_{net} = 0$



$\Sigma F_y = 0$:

$F_N \sin 60 - mg = 0$

$$\Rightarrow F_N = \frac{mg}{\sin 60}$$

$\Sigma F_x = 0$:

$$F_N \cos 60 - F_e = 0 \Rightarrow mg \cot 60 = \frac{k q^2}{R^2}$$

$$\cot 60 = \frac{1}{\sqrt{3}} \; ; \Rightarrow q = R\sqrt{\frac{mg}{\sqrt{3}\, k}}$$