Hwk 11 Solutions PHYS 142 Summer '09 Dr. Jenkins

Question 37.1

(a) for constructive interference, the path length difference must be a multiple of whole wavelengths (or zero), i.e. waves are in phase w/ each other; $\delta = m\lambda$ where $m = 0, 1, 2, \ldots$

(b) for destructive, the waves must be out of phase 180° to cancel, so the path difference is a half a wavelength plus $2\pi n$, i.e.

$$\delta = (n + \tfrac{1}{2})\lambda \quad \text{where } n = 0, 1, 2, \ldots$$

Question 37.5

The answer is (c) the bright fringes get closer together. Since wavelength decreases by a factor of $1/n_{water} = 3/4$, and the positions of the fringes are proportional to λ, the fringes will be closer together with all other parameters constant.

Question 37.8

Since light reflecting off the outer surface is phase shifted 180°, and light off the inner surface isn't shifted, the two will interfere destructively when the thickness is a multiple of $\lambda/2$, but the same result arises when the thickness approaches zero, as is the case just before the bubble pops. At this point the front and back surfaces are essentially the same, so the two sets of reflected light recombine destructively almost immediately with no addition path difference.

Question 37.11

Assuming R is adequately large, the back (bottom) surface of the lens and the top of the flat glass will be extremely close together near the center of the lens in the neighborhood of the very small area of contact. Since the light reflecting up from the back surface of the lens has no phase shift, and that from the top of the glass will be shifted 180°, the two will interfere destructively when the distance between the two is ~0, just as with the soap film about to break in the previous question. Hence, there is a dark spot in the middle. The surrounding rings being noncircular is simply revealing very subtle imperfections in the surfaces of the two objects that are otherwise unnoticeable.

Problem 37.7

(a) for the first non-central max, $m=1$:

$$x_m = \frac{m\lambda L}{d} \Rightarrow x_1 = \frac{(546.1\times10^{-9}\,m)(1.20\,m)}{0.25\times10^{-3}\,m} = 2.62\,mm$$

(b) difference between x_n for $n=1,2$:

$$x_n = \left(n+\tfrac{1}{2}\right)\frac{\lambda L}{d}$$

$$\Rightarrow x_2 - x_1 = \left(1+\cancel{\tfrac{1}{2}} - 0 - \cancel{\tfrac{1}{2}}\right)\frac{(546.1\times10^{-9}\,m)(1.20\,m)}{0.25\times10^{-3}\,m} = 2.62\,mm$$

Problem 37.13

for a phase difference ϕ, geometry tells us that

$$\frac{\delta}{\lambda} = \frac{\phi}{2\pi}\ ; \quad \text{since } \delta = d\sin\theta \text{ also, then}$$

$$\phi = \frac{2\pi}{\lambda} d\sin\theta$$

(a) $$\phi = \frac{2\pi}{500\times10^{-9}\,m}(0.12\times10^{-3}\,m)\sin(0.50°) = 13.2\,rad$$

$\Rightarrow$ equivalent to a 34° phase shift

(b) use small angle approx: $\sin\theta \approx \frac{x}{L}$

$$\phi = \frac{2\pi}{500\times10^{-9}\,m}(0.12\times10^{-3}\,m)\left(\frac{0.005\,m}{1.20\,m}\right) = 2\pi$$

i.e. constructive interference (bright fringe here)

(c) $$\sin\theta = \frac{\phi\lambda}{2\pi d} = \left(\frac{1}{3}\right)\frac{500\times10^{-9}\,m}{2\pi(0.12\times10^{-3}\,m)}$$

$\Rightarrow \theta = 0.0127°$

(d) path difference this time, not phase:

$$\delta = \frac{\lambda}{4} \Rightarrow \frac{\lambda}{4} = d\sin\theta \Rightarrow \sin\theta = \frac{500\times10^{-9}m}{4(0.12\times10^{-3}m)}$$

$\Rightarrow \theta = 0.0597°$

Problem 37.18

$$I = I_0\cos^2\left(\frac{\pi d\sin\theta}{\lambda}\right) \simeq I_0\cos^2\left(\frac{\pi d x}{\lambda L}\right)$$

for fraction of max intensity:

$$\frac{I}{I_0} = \cos^2\left[\frac{\pi(0.180\times10^{-3}\,m)(0.0060\,m)}{(656.3\times10^{-9}\,m)(0.80\,m)}\right] = 0.968$$

Problem 37.23

(a) the most strongly reflected light is that which undergoes constructive interference due to the reflection from the near and far surfaces of the film; formula for constructive interference in a thin film:

$$2nt = (m+\tfrac{1}{2})\lambda \Rightarrow \lambda = \frac{2nt}{m+\frac{1}{2}} = \frac{2(1.45)(280\times10^{-9}m)}{m+\frac{1}{2}}$$

$m=0$ gives $\lambda_0 = 1620\,nm$ (IR ✗)
$m=1$ gives $\lambda_1 = 541\,nm$ (green ✓) ← dominant color in reflection is green
$m=2$ gives $\lambda_2 = 325\,nm$ (UV ✗)

(Prob 37.23) cont.

(b) most strongly transmitted light is that which undergoes destructive interference ; formula is :

$$2nt = m\lambda \Rightarrow \lambda = \frac{2nt}{m} = \frac{2(1.45)(280\times10^{-9}m)}{m}$$

$m=1$ gives $\lambda_1 = 812$ nm (IR ✗)
$m=2$ gives $\lambda_2 = 406$ nm (violet ✓) ← dominant color transmitted is violet
$m=3$ gives $\lambda_3 = 271$ nm (UV ✗)

Problem 37.27

since n is increasing at each interface ($1 < 1.30 < 1.50$), the 180° phase shift happens to light reflected from both surfaces ; this means the conditions on the path differences flip ; ∴ we need what is typically the constructive eqn to find max destructive interference (minimum reflection) :

$$2nt = (m+\tfrac{1}{2})\lambda \Rightarrow t = (m+\tfrac{1}{2})\frac{\lambda}{2n}$$

for min thickness, let $m=0$:

$$\Rightarrow t = \frac{1}{2}\cdot\frac{500\text{ nm}}{2(1.30)} = 96.2\text{ nm}$$

☆ Problem 37.10 (RECOMMENDED)

looking at $m\lambda = d\sin\theta$, we can solve for m and round down to find the number of maxima on each side :

$$m = \frac{(0.320\text{ mm})}{500\times10^{-6}\text{mm}}\cancel{\sin 30}^{\frac{1}{2}} = 320.0$$

So the $m=320$ maximum is at 30° (no rounding necessary) ; including the other side (−30°) and the one central max, there are 641 maxima in $-30° < \theta < 30°$ range.

☆ ## Problem 37.25 (REC)

we have reversal of the cons. and dest. eqns again b/c $1 < 1.25 < 1.33$ (phase flip at both interfaces)

So then, for the red condition (640 nm):

$$2nt = m\lambda_{cons} \Rightarrow t = \frac{m(640\text{ nm})}{2(1.25)}$$

and the green (512 nm):

$$2nt = (m+\tfrac{1}{2})\lambda_{des} \Rightarrow t = \frac{(m+\frac{1}{2})(512\text{ nm})}{2(1.25)}$$

need m value for these λ's ; set eqns =, "2n"s cancel:

$$\Rightarrow 640\,m = 512\,(m+\tfrac{1}{2}) \Rightarrow 128\,m = 256 \Rightarrow m = 2$$

$$\Rightarrow t = \frac{\cancel{2}(640\text{ nm})}{\cancel{2}(1.25)} = 512\text{ nm}$$

Question 38.3

Since $\sin\theta \sim \frac{1}{a}$, cutting a in half will make the pattern, and ∴ the central max, 2x as wide, answer (d)

Question 38.9

The answer is (a) polarized such that horizontally-oriented electric fields are absorbed. Brewter's law tells us that reflected light is polarized (partially) ⊥ the plane of incidence, meaning in the plane of the reflecting surface; since most reflection is off horizontal surfaces (water, car hoods, etc.), sunglasses should be polarized vertically so that these reflected rays are maximally absorbed.

Question 38.11

The answer is (a) no polarization occurs. diffraction is a spatial phenomenon with no electromagnetic implications.

Problem 38.6

(a) do the usual math in two dimensions; given L, λ, and dimensions which are actually $2X_1$ and $2Y_1$ from the $X_n = \frac{n\lambda L}{a}$ eqn (remember lab, measuring from 1st dark fringe on one side to first on the other essentially gives width of central max, here we have width and height):

$$a_x = \frac{(1)\lambda L}{X_1} = \frac{(632.8\times10^{-9}\,m)(4.50\,m)}{(0.110\,m/2)} = 5.18\times10^{-5}\,m$$

$$a_y = \frac{(1)\lambda L}{y_1} = \frac{(632.8\times10^{-9}\,m)(4.5\,m)}{(0.0060\,m/2)} = 9.49\times10^{-4}\,m$$

(b) aperture is taller than it is wide, maximum is wider than it is tall, b/c $\{x_n, y_n\} \sim \frac{1}{a}$

Problem 38.42

Since central max is wider than others, it must be a single slit pattern (this widening is not present in multi-slit patterns)

$$a = \frac{n\lambda L}{x_n}$$

from center of x_1 to center of x_{-1} is $10.3\text{ cm} - 7.5\text{ cm} = 2.8\text{ cm}$

$$\Rightarrow x_1 = \frac{2.8\text{ cm}}{2} = 1.4\text{ cm}$$

$$a = \frac{(1)(632.8\times10^{-9}\,m)(2.60\,m)}{1.4\times10^{-2}\,m} = 1.18\times10^{-4}\,m$$

Problem 38.22

$$m\lambda = d\sin\theta \Rightarrow d = \frac{m\lambda}{\sin\theta}$$

$$d = \frac{(632.8\text{ nm})}{\sin 20.5^\circ} = 1.81\times10^{3}\text{ nm} = 1.81\,\mu m$$

Problem 38.35

Intensity after passing through polarizer at angle θ is $I = I_0 \cos^2\theta$:

(a) "reduced by a factor of x" means $\frac{I}{I_0} = \frac{1}{x}$.

$\Rightarrow \frac{1}{3.0} = \cos^2\theta \Rightarrow \theta = 54.7°$

(b) $\frac{1}{5.0} = \cos^2\theta \Rightarrow \theta = 63.4°$

(c) $\frac{1}{10.0} = \cos^2\theta \Rightarrow \theta = 71.6°$

Problem 38.36

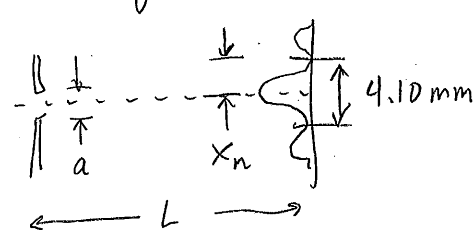
Use Brewster's angle formula: $n = \tan\theta_p$

angle given is Brewster's angle by definition,

$\Rightarrow n = \tan 48.0° = 1.11$

☆ Problem 38.2 (REC)

just like lab we did; the distance given is from x_1 to x_{-1};



$\Rightarrow x_1 = \frac{4.10 \text{ mm}}{2} = 2.05 \text{ mm}$

$x_n = \frac{n\lambda L}{a} \Rightarrow \lambda = \frac{x_n a}{nL}$

$\lambda = \frac{(2.05 \times 10^{-3} \text{ m})(0.550 \times 10^{-3} \text{ m})}{(1)(2.06\text{m})} = 547 \text{ nm}$

* **Problem 38.7** (REC)

$$I = I_0 \left(\frac{\sin \alpha}{\alpha} \right)^2 \quad \text{where} \quad \alpha = \frac{\pi a}{\lambda} \sin\theta \simeq \frac{\pi a x}{\lambda L}$$

$$\alpha = \frac{\pi (0.40 \text{ mm})(4.10 \text{ mm})}{(546.1 \times 10^{-6} \text{ mm})(1200 \text{ mm})} = 7.86 \text{ rad}$$

$$\frac{I}{I_0} = \left(\frac{\sin(7.86)}{7.86} \right)^2 = 0.0162$$

* **Problem 38.11** (REC)

use Rayleigh's criterion for angular resolution:

$$\sin\theta = \frac{\lambda}{a} = \frac{500 \times 10^{-6} \text{ mm}}{0.50 \text{ mm}} \Rightarrow \theta = 1.00 \times 10^{-3} \text{ rad}$$

* **Problem 38.19** (REC)

$$m\lambda = d \sin\theta \Rightarrow \sin\theta = \frac{m\lambda}{d}$$

$$2000 \text{ lines/cm} = \frac{1}{d} \Rightarrow d = \left(\frac{1}{2000} \right) \text{ cm}$$

$$\sin\theta_{red,1} = \frac{(1)(640 \times 10^{-7} \text{ cm})}{\left(\frac{1 \text{ cm}}{2000} \right)} = 640 \times 10^{-7} (2000)$$

$$\Rightarrow \theta_{red,1} = 0.128 \text{ rad} = 7.35°$$

* **Problem 38.34** (REC)

½ original intensity passes through 1st filter, and light is now vertically polarized:

$$I_f = I' \cos^2 30° = \left(\tfrac{1}{2} I_0 \right) \left(\tfrac{\sqrt{3}}{2} \right)^2 = \tfrac{1}{2} \left(\tfrac{3}{4} \right) I_0 = \underline{\tfrac{3}{8} I_0}$$