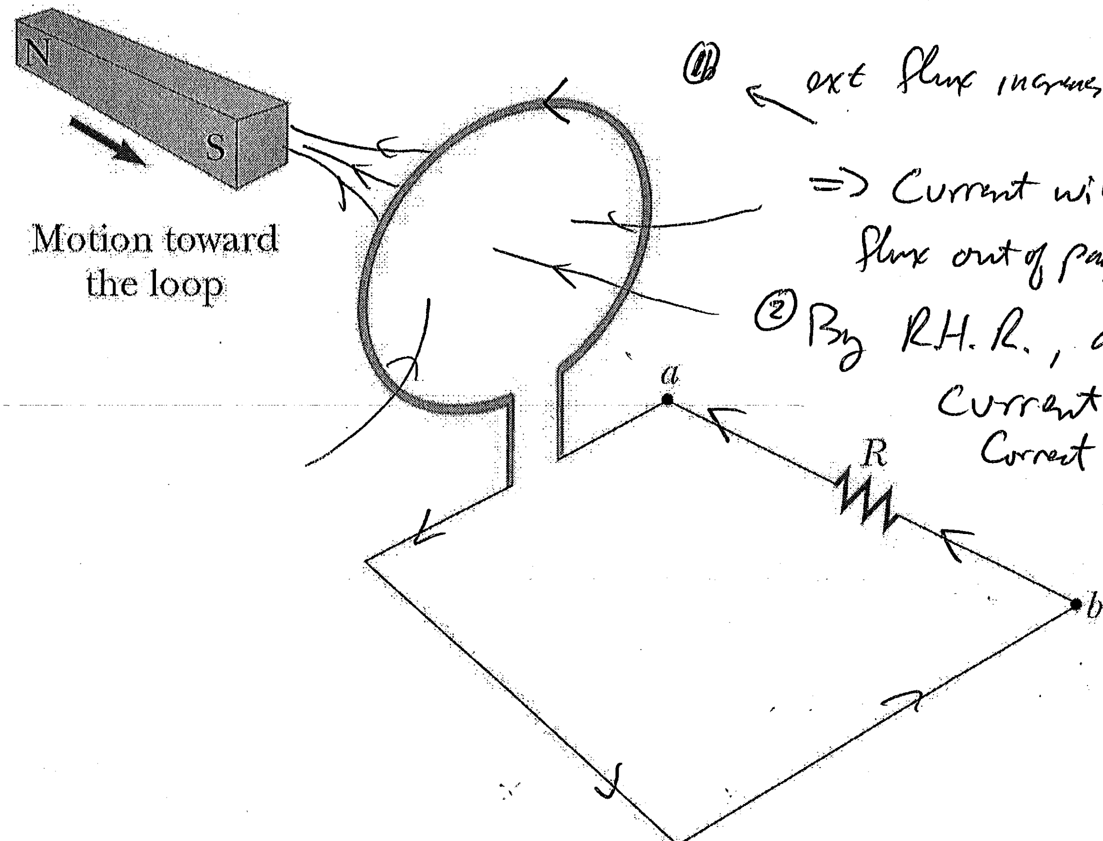
1. [4 pts] In the figure below, the bar magnet is moved toward the loop. Is the potential difference "Va –Vb" positive, negative, or zero? Explain in detail.

(1a) Loop will act like opposing magnet



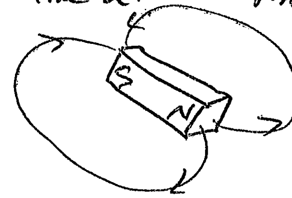
(1b) ← ext flux increasing into page

⇒ Current will create B-field flux out of page

(2) By R.H.R., a CCW current produces the Correct B-field flux

or

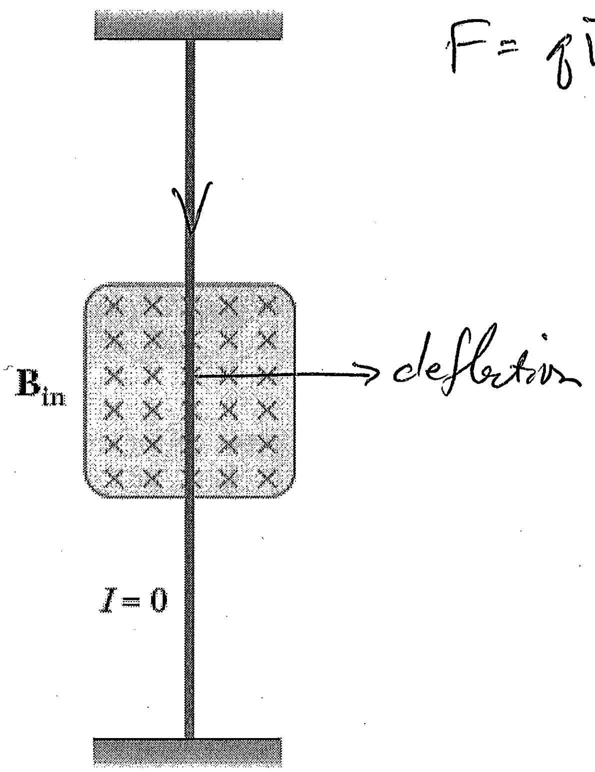
makes loop look like an opposing magnet



(3) Voltage drop across a Resistor

⇒ $V_b > V_a$ ∴ $V_a - V_b < 0$

2. [3 pts] Which way will the wire deflect if a current is applied downward through the orange wire? Explain.



$F = q\vec{v} \times \vec{B}$ or $\vec{F} = I\vec{l} \times \vec{B}$

$I\vec{l}$ downward
$\vec{B}$ inward
$\Rightarrow I\vec{l} \times \vec{B}$ RHR gives to the right.

3. [4 pts] Which of the following causes the fringes in a two-slit interference pattern to be farther apart? $\theta_{min} \sim \frac{\lambda}{d}$

(a) [1 pts] Decreasing or increasing the wavelength of the light ?

(b)[1 pts] Decreasing or increasing the distance between the slit and the screen?

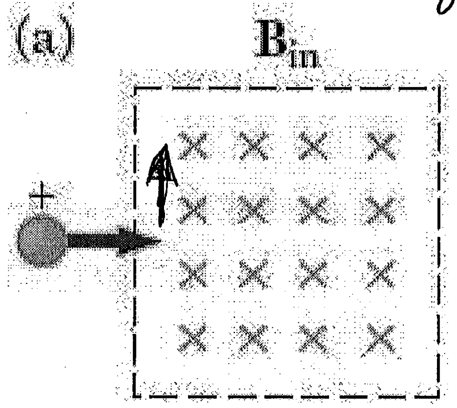
(c)[1 pts] Decreasing or increasing the slit spacing?

(d) [1 pts] Immersing the entire apparatus in a dielectric oil or leaving the apparatus in air?

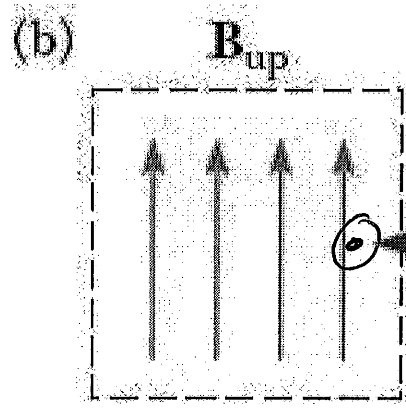
$\theta_{min} \sim \frac{\lambda/n}{d}$; increasing n decreases θ_{min}

4. [4 pts] Determine the *initial* direction of the acceleration of the charged particles as they enter a region with a uniform magnetic field. The initial velocity of the charged particles are depicted as red vectors. Draw your answer on the diagram as a vector. $\vec{F} = q\,\vec{v} \times \vec{B}$

+/or state your answer clearly



UP



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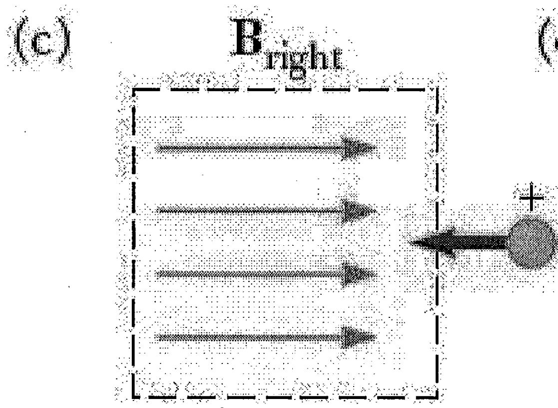
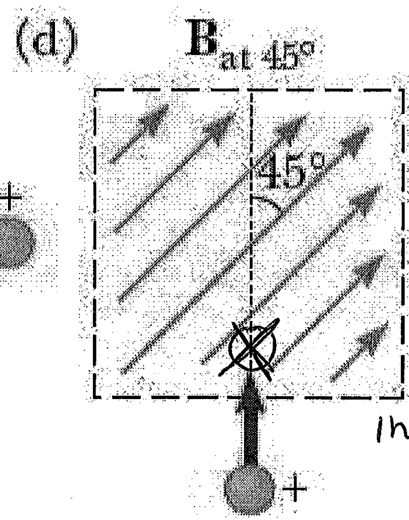


$\vec{V} \parallel \vec{B} \Rightarrow \vec{F} = 0$

<u>no</u> <u>acceleration</u>



into page

5. [6 pts] A possible means of space flight is to place a perfectly reflecting aluminized sheet into orbit around the Earth and then use the light from the Sun to push this "solar sail." Suppose a sail of area 1.00 x 10^5 m^2 and mass 1000 kg is placed in orbit facing the Sun. Assume a solar intensity of 1340 W/m^2 and ignore the forces of gravity.
(a) [3 pts] What force is exerted on the sail?
(b) [3 pts] What is the sail's acceleration?

(a) pressure, $P = \frac{2S}{c}$, $S = 1340\ W/m^2$, $c = 3 \cdot 10^8\ m/s$

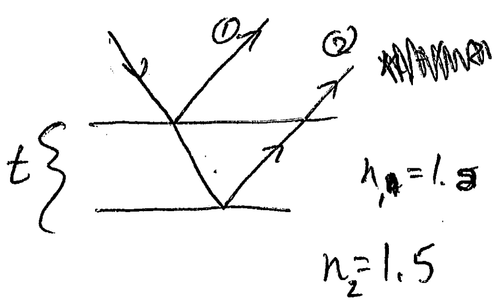
$\Rightarrow F = 2\left(\frac{S}{c}\right)(\text{Area}) = 2\ \frac{1340}{3 \cdot 10^8}\ 10^5 = \frac{2 \cdot 1.3}{3}\ N$

$\Rightarrow F = 0.87\ N$

(b) $a = \frac{F}{m} = \frac{.87\ N}{1000\ kg} = 8.7 \cdot 10^{-4}\ m/sec^2$

6. [8 pts] A dielectric coating (index of 1.2) is to be deposited on a pair of glasses (index of 1.5) so that light of wavelength ~~580~~ 600nm is minimally reflected at normal incidence. What thickness should be deposited?

State "air"



$\Delta\phi = \underbrace{(\pi + k\delta)}_{②} - \underbrace{\pi}_{①} = (m+\tfrac{1}{2})2\pi$

$\delta = 2t,\ k = \frac{2\pi}{\lambda/n_1}$

$\Rightarrow k\delta = (m+\tfrac{1}{2})2\pi$

$\Rightarrow \frac{2\pi}{(\lambda/n_1)} \cdot 2t = (m+\tfrac{1}{2})2\pi \Rightarrow 2n_1 t = \lambda(m+\tfrac{1}{2})$

$\Rightarrow t = \frac{1}{2}\left(\frac{\lambda}{n_1}\right)(m+\tfrac{1}{2})$; $\frac{1}{2}\frac{\lambda}{n_1} = \frac{1}{2}\ \frac{600\ nm}{1.2} = 250\ nm$

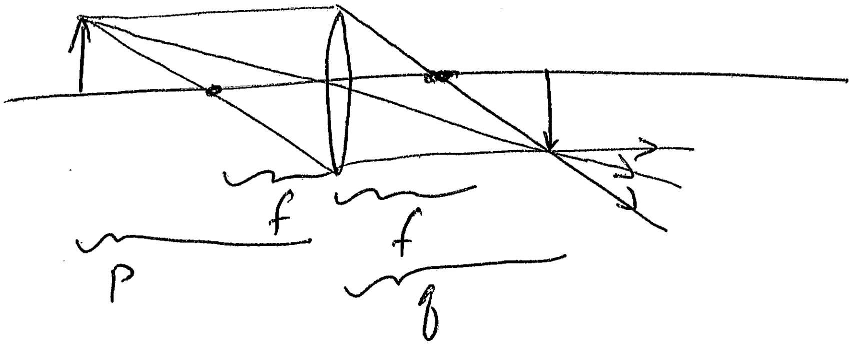
$\therefore$ $t = (m+\tfrac{1}{2})\ 250\ nm$, $m = 0, 1, 2, 3, 4 \ldots$

$t = 125, 375, \ldots\ nm$

7. [10 pts] A converging lens of focal length 10.0 cm forms images of
   (a) [5 pts] an object placed 20 cm from the lens.
   (b) [5 pts] an object placed 5 cm form the lens.

For each case (a) and (b), construct a ray diagram and calculate the image distance as well as the magnification. Describe the images (real, virtual, upright, upside down).

a)



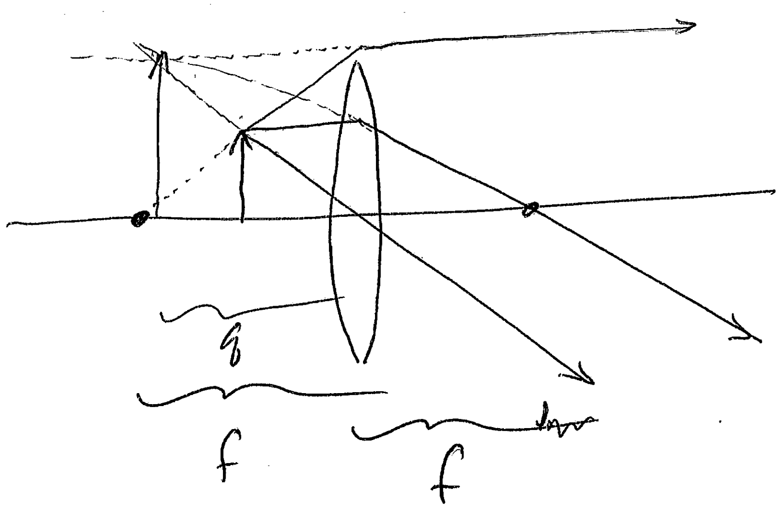
$p > 0$
$f > 0$

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \Rightarrow \frac{1}{q} = \frac{1}{10} - \frac{1}{20} = \frac{1}{20} \Rightarrow \boxed{q = 20\,\text{cm} > 0}$$

$$M = -\frac{q}{p} = \frac{-20}{20} = -1$$

Image is real, upside down, & same size as object.

b)



$p > 0$
$q > 0$

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \Rightarrow \frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{10} - \frac{1}{5} = -\frac{1}{10}$$

$$\therefore \boxed{q = -10\,\text{cm} < 0}$$

$$M = \frac{-q}{p} = \frac{10}{5} = +2$$

Image is virtual, right side up, & twice as big as object

8. [5 pts] The lunar rover and part of the lunar lander were left behind on the moon. Assume the two large objects are located 100 meters apart. A conspiracy buff does not believe that the moon landing was real and wants to prove that these two large objects are not on the moon. What minimum diameter lens is required to optically resolve the two objects if the distance to the moon is 3.8×10^8 m? Ignore the optical distortions that may occur due to the Earth's atmosphere, and assume a wavelength of 600nm.

$$\sin \Delta\theta = 1.22 \frac{\lambda}{D} \approx \tan \Delta\theta = \frac{d}{r}$$

$d = 100m$

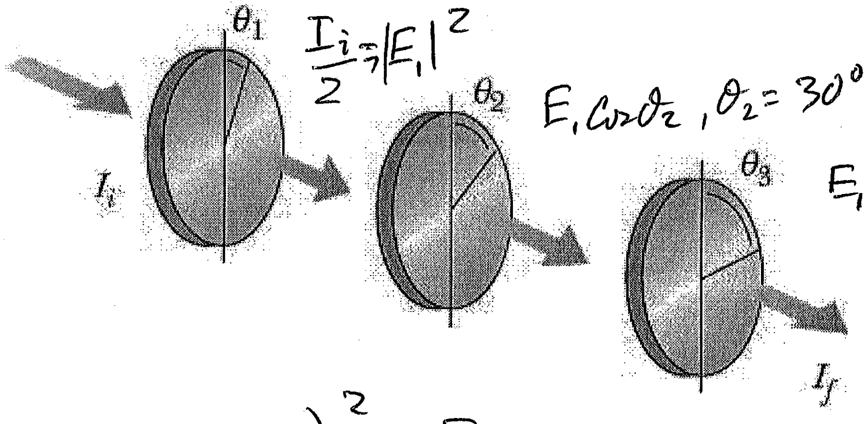
$r = 3.8 \cdot 10^8 m$

$\lambda = 600nm$

$$\Rightarrow D = \frac{1.22\,\lambda \cdot r}{d} = \frac{1.22 (6 \cdot 10^{-7})\, 3.8 \cdot 10^8}{10^2}\ m$$

$$= \frac{1.22 \cdot 6 \cdot 3.8}{10} = \boxed{2.8\,m}$$

9. [5 pts] Unpolarized light of intensity I_i is incident onto 3 polarizers. The first polarizer is aligned to θ_1=0 degrees such that the light is vertically polarized upon transmission. The second polarizer is aligned to θ_2=30 degrees. The final polarizer is aligned to θ_3=90 degrees so that the final polarization is aligned horizontally upon transmission. What is the final intensity compared to the initial intensity, I_f / I_i?



$\frac{I_i}{2} = |E_1|^2$

$E_1 \cos\theta_2, \ \theta_2 = 30°$

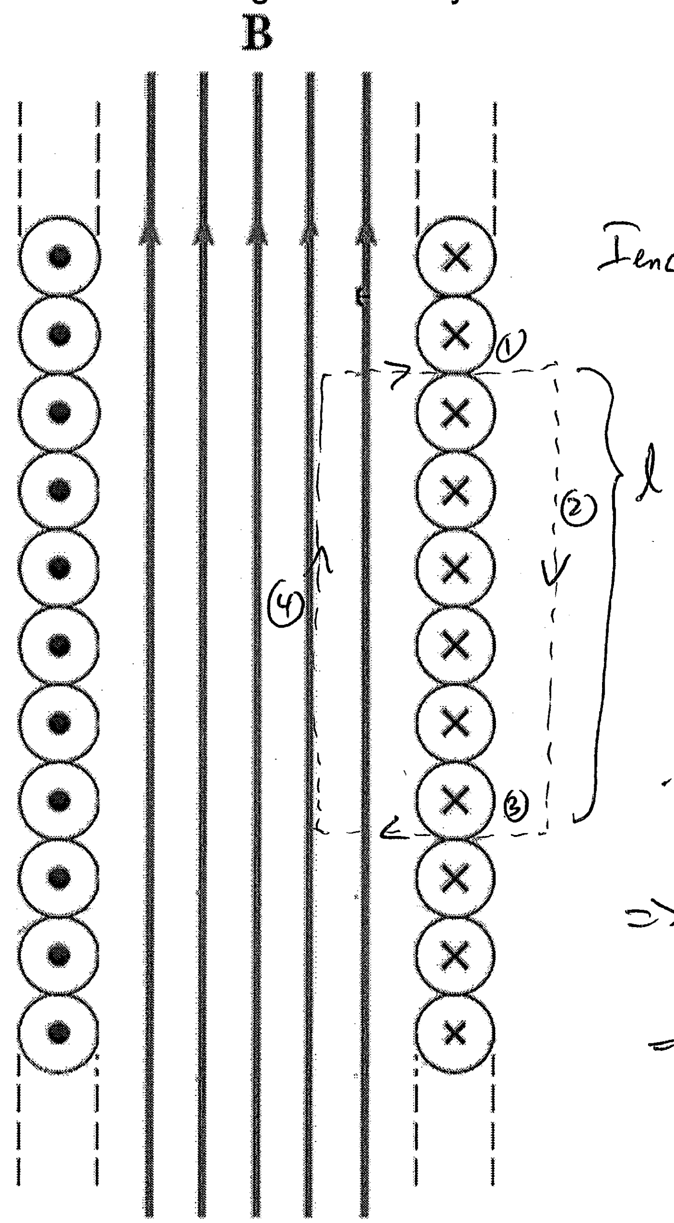
$E_1 \cos\theta_2 \cos\theta_3$

$\Delta\theta_3 = 60°$
$(90° - 30°)$

$$I_f = \left(|E_1| \cos\theta_2 \cos\theta_3\right)^2 = \frac{I_i}{2} \underbrace{\cos^2 30°}_{\left(\frac{\sqrt{3}}{2}\right)^2} \underbrace{\cos^2 60°}_{\left(\frac{1}{2}\right)^2}$$

$$\Rightarrow \frac{I_f}{I_i} = \frac{1}{2} \frac{3}{4} \frac{1}{4} = \frac{3}{32} \Rightarrow .094$$

10. [15 pts] Use Ampere's law to derive the magnetic field inside an infinitely long solenoid with n turns per length, current I in the wire, and radius r of the solenoid. On the diagram below depicting a cross sectional view of the solenoid, draw your Amperian loop. Explain in detail how you perform the integral around your chosen Amperian loop.



$$\int \vec{B}\cdot d\vec{s} = \mu_0 I_{enc}$$

$$I_{enc} = n \cdot l I$$

$$\int \vec{B}\cdot d\vec{s} = \overbrace{\int_{1} \vec{B}\cdot d\vec{s} + \int_{3} \vec{B}\cdot d\vec{s}}^{=0 \text{ since } B \perp ds} + \underbrace{\int_{2} \vec{B}\cdot d\vec{s}}_{=0 \text{ since } B=0} + \underbrace{\int_{4} \vec{B}\cdot d\vec{s}}_{B \parallel ds \text{ \& constant} \Rightarrow B\cdot l}$$

$$\therefore \int \vec{B}\cdot d\vec{s} = \mu_0 I_{enc}$$

$$\Rightarrow B\cdot l = \mu_0 n l I$$

$$\Rightarrow \underline{\underline{B = \mu_0 n I}}$$

11.[15 pts] EM Traveling waves:

(a) [2 pts] Write down all four Maxwell's equations and the Lorentz force law.

$$\oint \vec{E} \cdot d\vec{a} = \frac{q_{inc}}{\varepsilon_0} \qquad \oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{B} \cdot d\vec{a} = 0 \qquad \oint \vec{B} \cdot d\vec{l} = \mu_0 I + \varepsilon_0 \mu_0 \frac{d\Phi_E}{dt}$$

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

(b) [2 pts] Write down the wave equations for both electric and magnetic fields propagating in 'free space' (vacuum) as plane waves traveling in the **+z-direction** which can be derived directly from Maxwell's equations.

$$\frac{\partial^2 E}{\partial z^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$

$$\frac{\partial^2 B}{\partial z^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B}{\partial t^2}$$

(c) [2 pts] Write down a solution to the above wave equation for the electric field given a specific wave number "k", frequency "ω", and amplitude "E_0" for the plane wave.

$$\vec{E}(z,t) = E_0 (\hat{y}) \cos(kz - \omega t)$$

(d) [4 pts] Substitute your solution from part (c) into the wave equation of part (b) and derive the speed of the traveling wave in terms of the electric and magnetic permiabilities of free space (ε_0 and μ_0).

$$\frac{\partial^2 E}{\partial z^2} = -E_0 k^2 \cos(kz - \omega t), \qquad \frac{\partial^2 E}{\partial t^2} = -E_0 \omega^2 \cos(kz - \omega t)$$

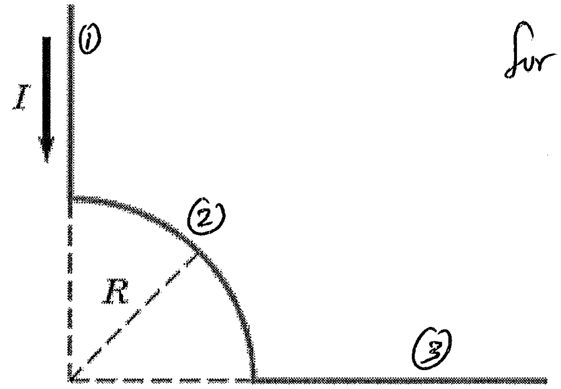
$$\therefore \frac{\partial^2 E}{\partial z^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} \Rightarrow k^2 = \mu_0 \varepsilon_0 \omega^2 \Rightarrow V_\phi = \frac{\omega}{k} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

(e) [5 pts] Given your plane wave solution from part (c), what is the electric field at time t=0 at point z=0, x=π/2, and y= π/4?

$$\vec{E} = E_0 \hat{y}$$

12. [15 pts] The segment of wire shown below carries a current I in the direction given by the arrow, where the radius of the circular arc is R. Determine the magnitude and direction of the magnetic field at the origin. Derive your results from the Biot-Savart law.

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{s} \times \hat{r}}{r^2}$$



for ① + ③ :

$$d\vec{s} \times \hat{r} = 0$$

for ② :

$$d\vec{s} \times \hat{r} = ds \text{ into page}$$

$$= R\, d\theta$$

$\therefore$ for ② : $B = \frac{\mu_0}{4\pi} I \int_0^{\pi/2} \frac{R\, d\theta}{R^2} = \frac{\mu_0}{4\pi} I \frac{\pi}{2R}$

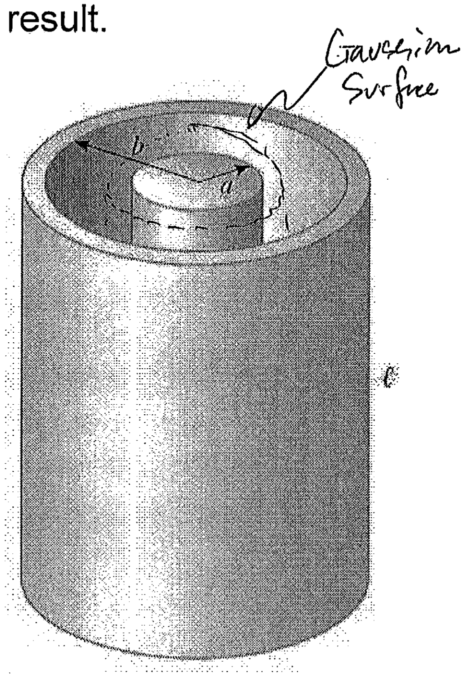
$$\Rightarrow \vec{B} = \frac{\mu_0}{8} \frac{I}{R} \text{ into page}$$

13. [15 pts] A solid cylindrical conductor of radius "a" and positive charge "Q" is coaxial with a cylindrical shell of negligible thickness, radius b>a, and negative charge "-Q". Its length is "ℓ".

(a) [5 pts] Find the electric field between the cylinders for a<r<b. Derive your result.

(b) [5 pts] Find the potential difference between the two cylinders. Derive your result.

(c) [5 pts] Find the capacitance of this cylindrical capacitor. Derive your result.



a) $\oint \vec{E} \cdot d\vec{a} = \frac{q_{inc}}{\varepsilon_0}$

$\Rightarrow E \cdot 2\pi r \ell = \frac{Q}{\varepsilon_0}$

$\Rightarrow E = \frac{Q}{2\pi\varepsilon_0} \frac{1}{r\ell}$ radially outward

b) $|\Delta V| = \int_a^b E \cdot dr = \frac{Q}{2\pi\varepsilon_0 \ell} \int_a^b \frac{dr}{r}$

$= \frac{Q}{2\pi\varepsilon_0 \ell} \ln b/a$

c) $C = \left| \frac{Q}{\Delta V} \right| = \frac{2\pi\varepsilon_0 \ell}{\ln(b/a)}$