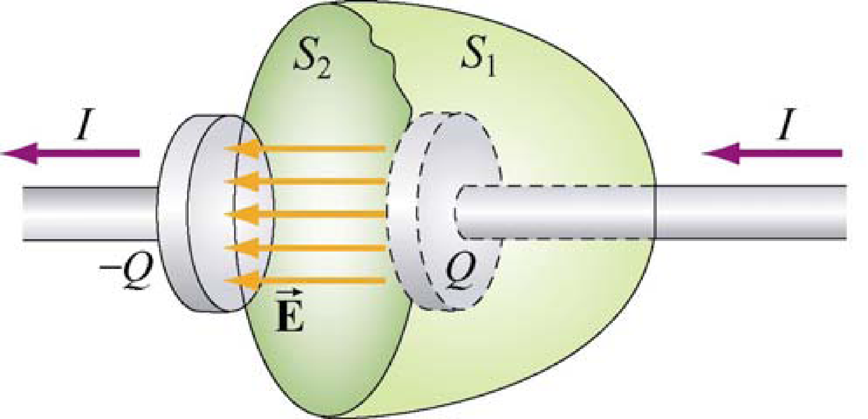
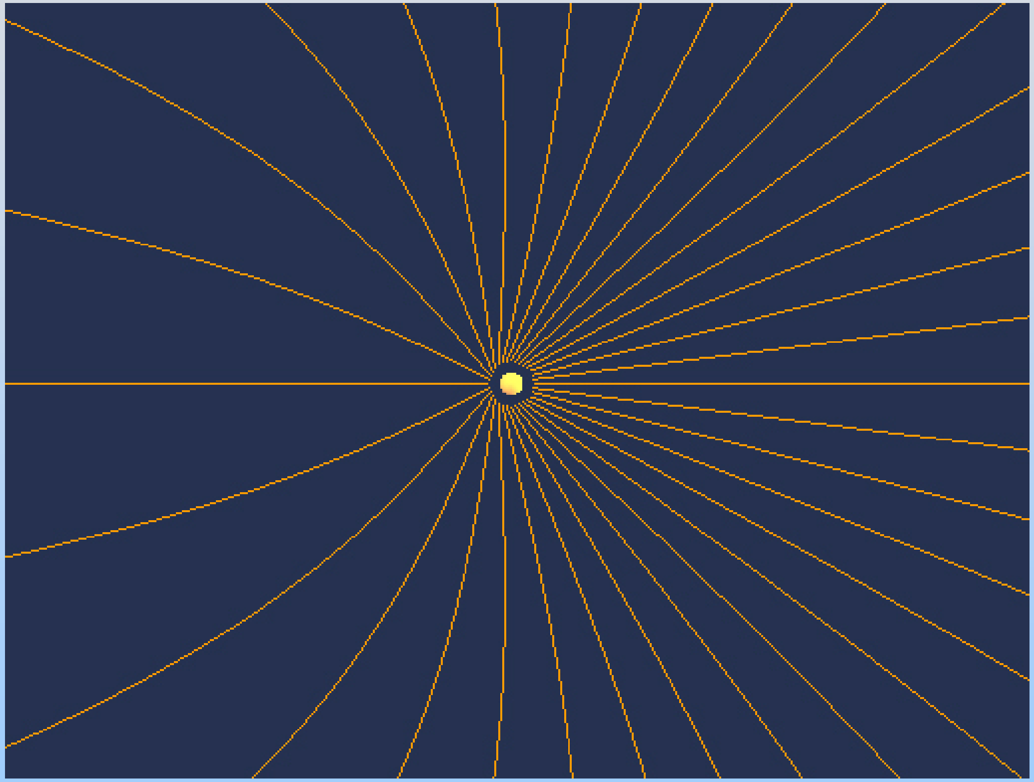
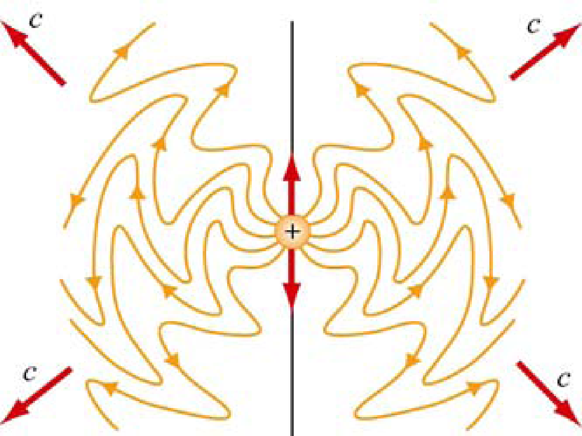


S_2
S_1
I
I
$-Q$
Q
$\vec{E}$

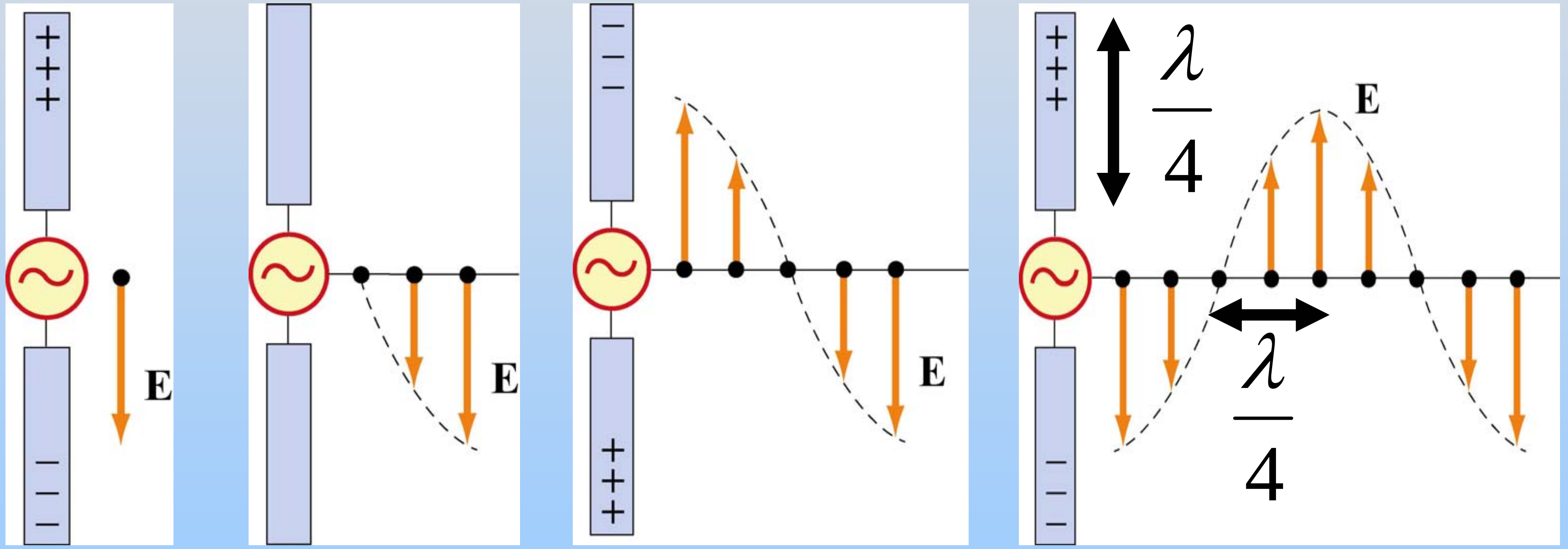
Generating Electric Dipole Radiation Applet




c
c
+
c
c

Quarter-Wavelength Antenna

Accelerated charges are the source of EM waves.
Most common example: Electric Dipole Radiation.



t = 0 t = T/4 t = T/2 t = T

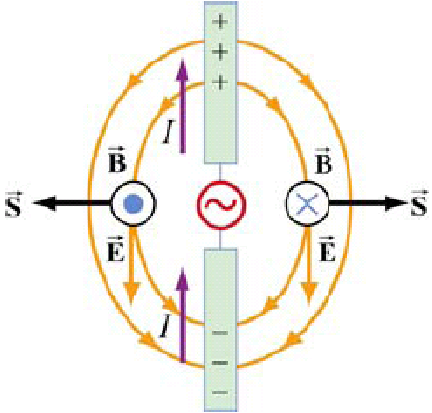
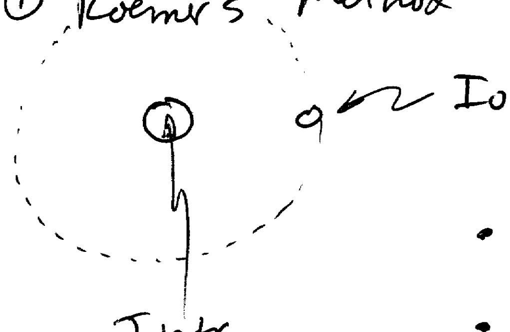


Figure 13.8.3 Electric and magnetic field lines produced by an electric-dipole antenna.

Speed of Light Measurements:

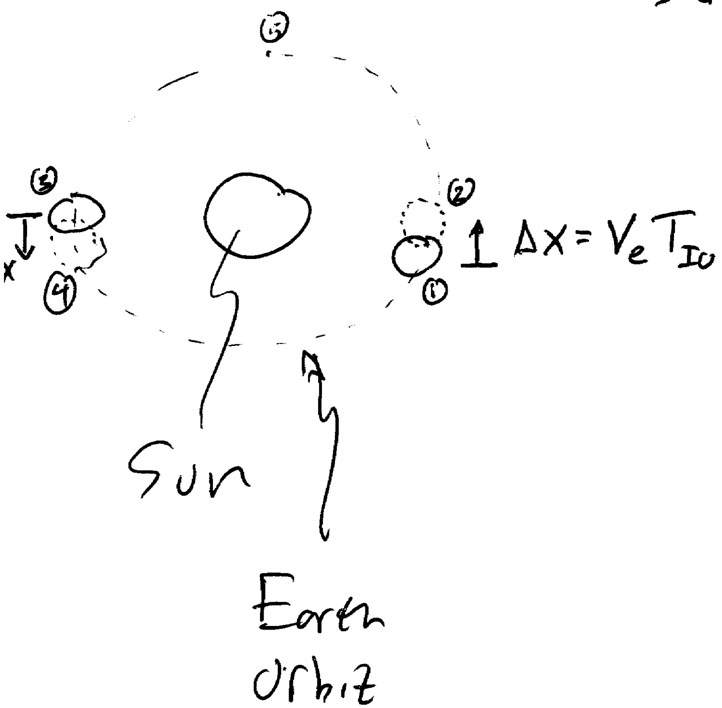
(i) Roemer's Method (1675) – analysis of apparent period of Io, Jupiter's moon:



Let speed of light $= V_L$

- When Io make one orbit, it takes a time T
- $$T'_{1-2} = T + \frac{V_e T}{V_L}, \quad T'_{3-4} = T - \frac{V_e T}{V_L} \Rightarrow V_L = \frac{2 V_e T}{\Delta T'}$$

$$\Delta T' = \left| T'_{1-2} - T'_{3-4} \right|$$



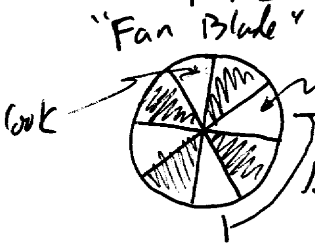
- T measured @ position (5), or

$$T = \frac{T'_{1-2} + T'_{3-4}}{2}$$

$$\Rightarrow V_L > 2.3 \cdot 10^8 \ m/s$$

Speed of Light Measurements:

② Fizeau's Method (1849)



$$\Delta\theta = \frac{2\pi}{N}$$

$$d = c\left(\frac{\Delta t}{2}\right)$$

For fast enough ω, will see candle through "same" slot as source was emitted from...

$$\Delta\theta = \frac{2\pi}{N}, \quad N = \#\text{ of slots in blade}$$

$$\omega = \frac{2\pi}{T} = \frac{\Delta\theta'}{\Delta t'} = \frac{(2\pi/N)}{\Delta t} \Rightarrow \Delta t = \frac{2\pi}{N\omega}$$

$$\text{Also, } c = \frac{2d}{\Delta t} \Rightarrow \Delta t = \frac{2d}{c}.$$

$$\Rightarrow \frac{2d}{2\pi} N\omega = c$$

$$\text{or } \boxed{c = \frac{Nd}{\pi}\omega}$$

$$\boxed{3.1 \cdot 10^{8}\ m/s}$$

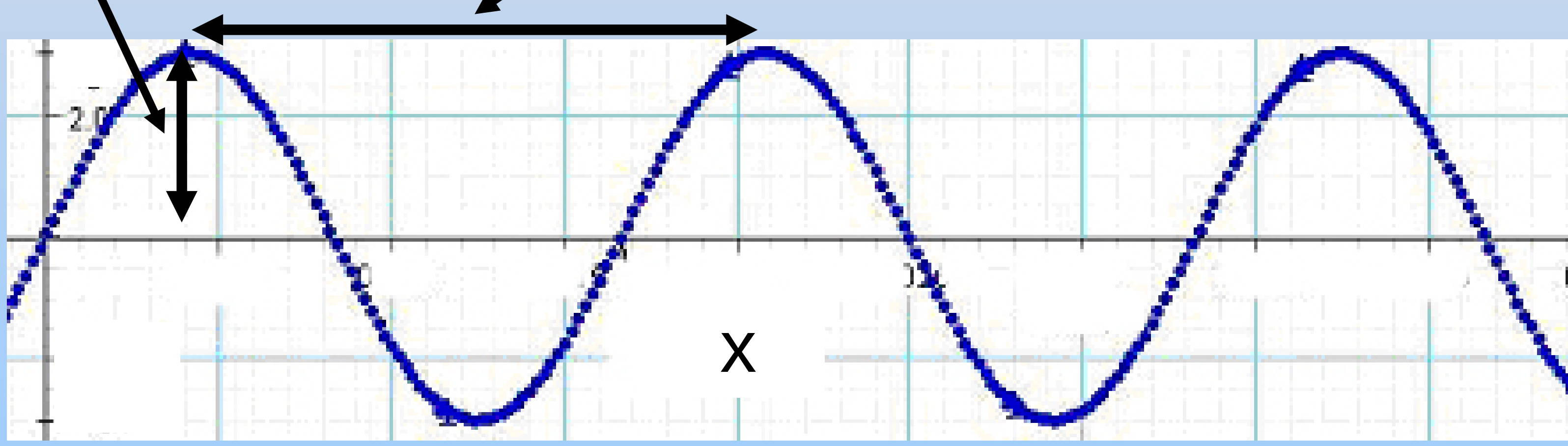
Last Time:
Traveling Waves

Traveling Sine Wave

Now consider $f(x) = y = y_0\sin(kx)$:

Amplitude (y_0)

$$\text{Wavelength } (\lambda) = \frac{2\pi}{\text{wavenumber } (k)}$$



What is $g(x,t) = f(x+vt)$? Travels to left at velocity v

$$y = y_0\sin(k(x+vt)) = y_0\sin(kx+kvt)$$

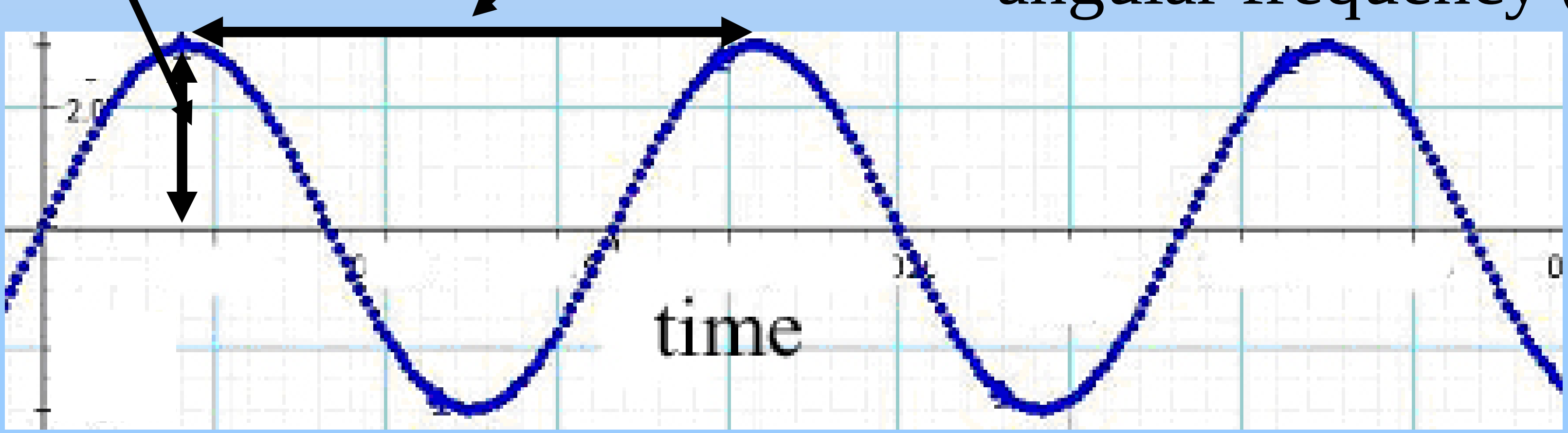
Traveling Sine Wave

$$y = y_0 \sin\left(kx + kvt\right)$$

At x=0, just a function of time: $y = y_0 \sin(kvt) \equiv y_0 \sin(\omega t)$

$$\text{Period}\,(T) = \frac{1}{\text{frequency}\,(f)} = \frac{2\pi}{\text{angular frequency}\,(\omega)}$$

Amplitude (y_0)



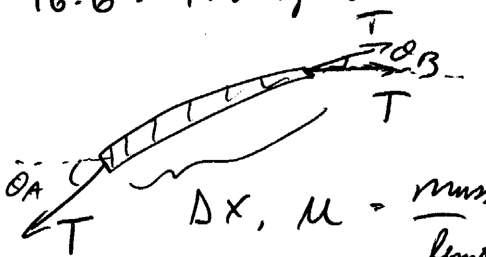
Traveling Sine Wave

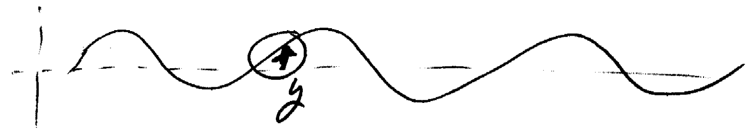
- Wavelength: λ
- Frequency : f

$$y = y_0 \sin(kx - \omega t)$$

- Wave Number: $k = \dfrac{2\pi}{\lambda}$
- Angular Frequency: $\omega = 2\pi f$
- Period: $T = \dfrac{1}{f} = \dfrac{2\pi}{\omega}$
- Speed of Propagation: $v = \dfrac{\omega}{k} = \lambda f$
- Direction of Propagation: $+x$

Section 16.6: Traveling wave on a string.





Δx, $\mu = \frac{\text{mass}}{\text{length}}$

$$\sum F_y = T\sin\theta_B - T\sin\theta_A = T(\sin\theta_B - \sin\theta_A)$$

For "small" Amplitude ⇒ small angles ⇒ $\sin\theta \sim \tan\theta$:

$$\approx T(\underbrace{\tan\theta_B}_{\text{Slope of String @ B}} - \underbrace{\tan\theta_A}_{\text{Slope of String @ A}}) = T\left(\frac{\partial y}{\partial x}\Big|_B - \frac{\partial y}{\partial x}\Big|_A\right)$$

$$m a_y = \sum F_y$$

$$\Rightarrow \mu \Delta x \frac{\partial^2 y}{\partial t^2} = T\left(\frac{\partial y}{\partial x}\Big|_B - \frac{\partial y}{\partial x}\Big|_A\right)$$

$$\rightarrow \frac{\mu}{T}\frac{\partial^2 y}{\partial t^2} = \frac{\frac{\partial y}{\partial x}\Big|_B - \frac{\partial y}{\partial x}\Big|_A}{\Delta x}$$

$$\rightarrow \frac{\partial^2 y}{\partial x^2} = \frac{\mu}{T}\frac{\partial^2 y}{\partial t^2} \quad ; \quad y = A\cos(kx - \omega t)$$

$$\Rightarrow A^2 k^2 = \frac{\mu}{T} A^2 \omega^2 \Rightarrow \frac{\omega}{k} = \sqrt{\frac{T}{\mu}}$$

But $kx - \omega t = \text{Constant} \Rightarrow \frac{dx}{dt} = \frac{\omega}{k} = \underline{\underline{V}}$

$$\therefore \frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} \quad \text{where } V = \frac{\omega}{k} = \sqrt{\frac{T}{\mu}}$$

Last Time:
Maxwell's Equations

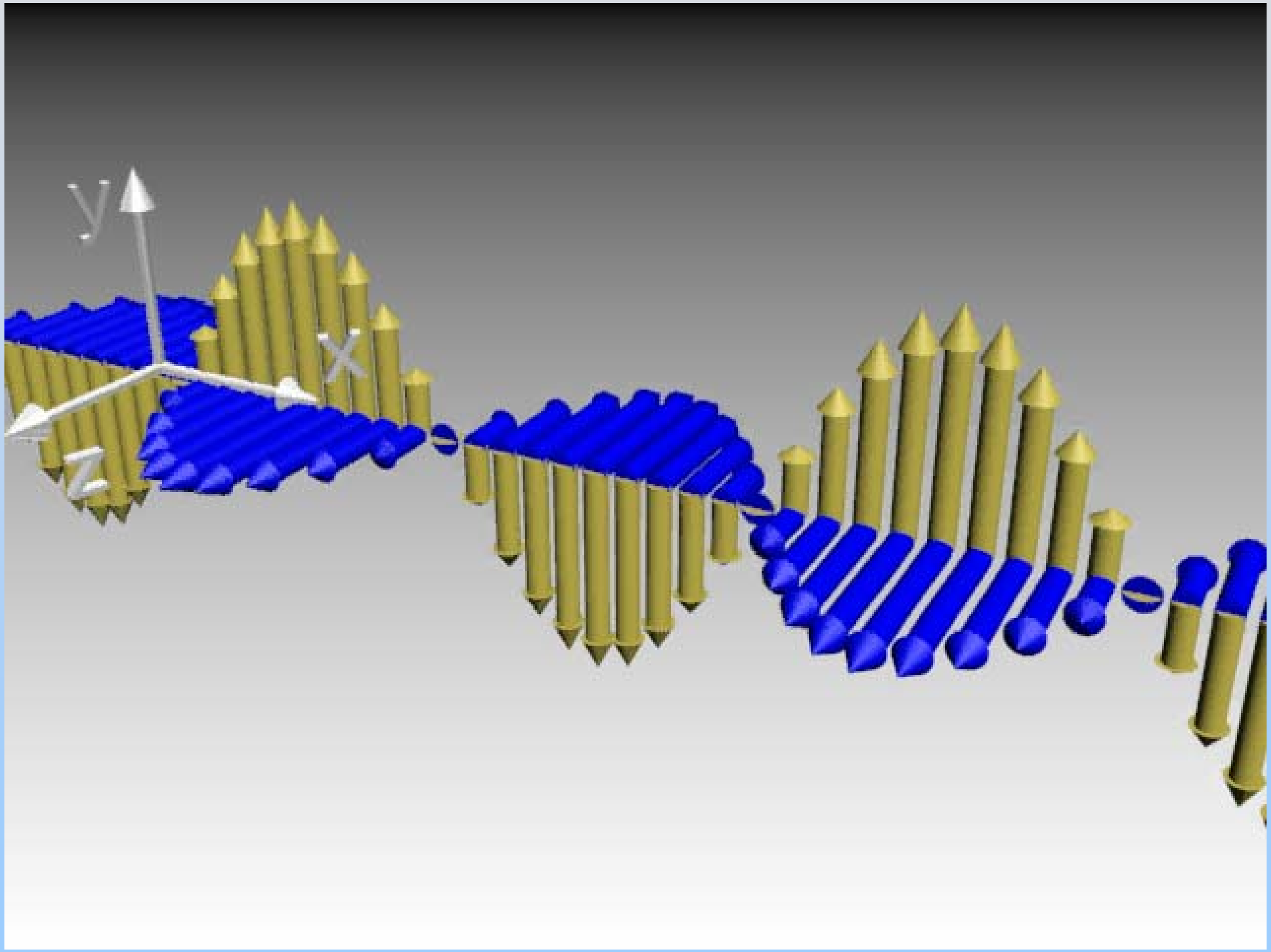
Maxwell's Equations

$\oiint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{in}}{\varepsilon_0}$	(Gauss's Law)
$\oint_C \vec{\mathbf{E}} \cdot d\,\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt}$	(Faraday's Law)
$\oiint_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$	(Magnetic Gauss's Law)
$\oint_C \vec{\mathbf{B}} \cdot d\,\vec{\mathbf{s}} = \mu_0 I_{enc} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$	(Ampere-Maxwell Law)

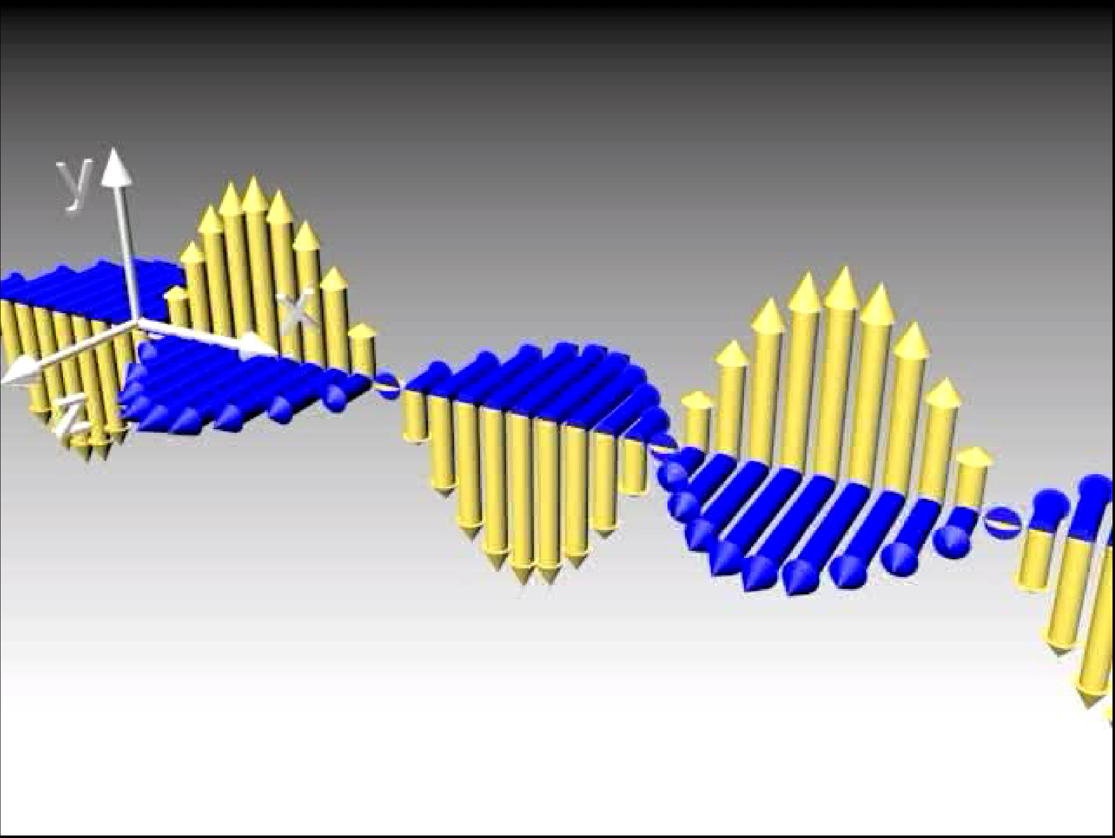
$\vec{\mathbf{F}} = q(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}})$ (Lorentz force Law)

Which Leads To…
EM Waves

Electromagnetic Radiation: Plane Waves



http://ocw.mit.edu/ans7870/8/8.02T/f04/visualizations/light/07-EBlight/07-EB_Light_320.html


y
x
z

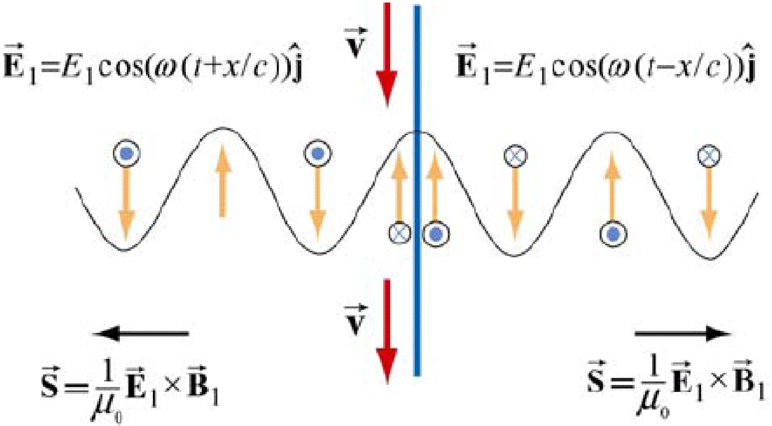


Figure 13.9.4 Electric field generated by the oscillation of a current sheet.

Traveling E & B Waves

- Wavelength: λ
- Frequency : f

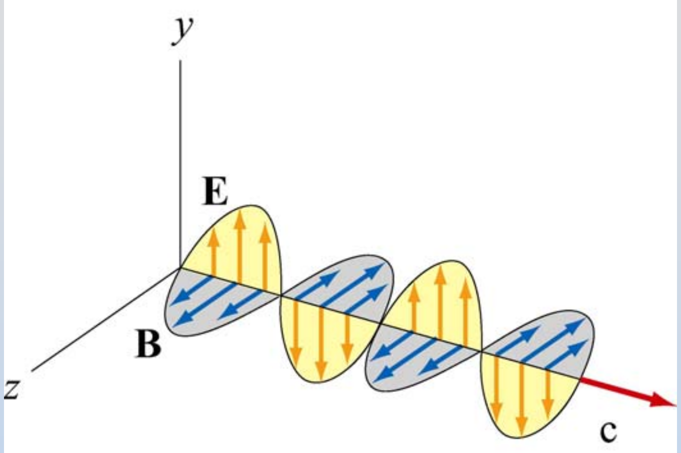
$$\vec{\mathbf{E}} = \hat{\mathbf{E}} E_0 \sin(kx - \omega t)$$

- Wave Number: $k = \dfrac{2\pi}{\lambda}$
- Angular Frequency: $\omega = 2\pi f$
- Period: $T = \dfrac{1}{f} = \dfrac{2\pi}{\omega}$
- Speed of Propagation: $v = \dfrac{\omega}{k} = \lambda f$
- Direction of Propagation: $+x$

Properties of EM Waves

Travel (through vacuum) with speed of light

$$v = c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3\times10^8 \frac{m}{s}$$



At every point in the wave and any instant of time, E and B are in phase with one another, with

$$\frac{E}{B} = \frac{E_0}{B_0} = c$$

E and B fields perpendicular to one another, and to the direction of propagation (they are **transverse**):

$$\text{Direction of propagation} = \text{Direction of } \vec{\mathbf{E}} \times \vec{\mathbf{B}}$$

PRS Questions: Direction of Propagation

How Do Maxwell’s Equations Lead to EM Waves?

Derive Wave Equation

$$\vec{\nabla} = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$$

Stoke's Thm: $\oint(\vec{\nabla}\times\vec{V})\cdot d\vec{a} = \int \vec{V}\cdot d\vec{s}$

Divergence Thm: $\int \vec{\nabla}\cdot\vec{V}\, d\mathcal{V} = \oint \vec{V}\cdot d\vec{a}$

$$\vec{\nabla}\times(\nabla\times\vec{V}) = \vec{\nabla}(\vec{\nabla}\cdot\vec{V}) - \nabla^2\vec{V}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

$(\nabla^2 = \vec{\nabla}\cdot\vec{\nabla})$

Consider Gauss's Law(s) in free space ($q=0, I=0$)

$$\oint \vec{E}\cdot da = \int(\vec{\nabla}\cdot\vec{E})\, d\mathcal{V} = \underbrace{\int \frac{\rho}{\varepsilon_0}\, d\mathcal{V}}_{Q_{inc}/\varepsilon_0}$$

$$\Rightarrow \vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$$

free space, $\Rightarrow \nabla\cdot\vec{E} = 0$

Also: $\oint \vec{B}\cdot d\vec{a} = \int \vec{\nabla}\cdot\vec{B}\, d\mathcal{V} = 0$

$$\Rightarrow \vec{\nabla}\cdot\vec{B} = 0$$

Faraday's Law:

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

Stokes Thm $\Rightarrow$ $\oint \vec{E} \cdot d\vec{s} = \oint (\vec{\nabla} \times \vec{E}) \cdot d\vec{a}$

& $-\frac{d\Phi_B}{dt} = -\frac{d}{dt} \oint B \cdot d\vec{a} = \oint -\frac{d\vec{B}}{dt} \cdot d\vec{a}$

$\therefore \quad \vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt}$

Ampere - Maxwell Law:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc} + \varepsilon_0 \mu_0 \frac{d\Phi_E}{dt}$$

Stokes Thm $\Rightarrow$ $\oint \vec{B} \cdot d\vec{s} = \oint (\vec{\nabla} \times \vec{B}) \cdot d\vec{a}$

$\mu_0 I_{enc} = \oint \mu_0 \vec{J} \cdot d\vec{a}$

$\varepsilon_0 \mu_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \mu_0 \int \frac{d\vec{E}}{d\vec{t}} \cdot d\vec{a}$

$\therefore \quad \vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \varepsilon_0 \mu_0 \frac{dE}{dt}$

	Integral Form	Differential Form
Gauss's Law	$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{inc}}{\varepsilon_0}$	$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$
Gauss's Law of Magnetism	$\oint \vec{B} \cdot d\vec{a} = 0$	$\vec{\nabla} \cdot \vec{B} = 0$
Faraday's Law	$\int \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$	$\vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt}$
Ampere-Maxwell Law	$\int \vec{B} \cdot d\vec{s} = \mu_0 I_{inc} + \varepsilon_0 \mu_0 \frac{d\Phi_E}{dt}$	$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \varepsilon_0 \mu_0 \frac{d\vec{E}}{dt}$

Free space: $\rho = 0$, $\vec{J} = 0$ $\Rightarrow$ $\vec{\nabla} \cdot \vec{E} = 0$, $\vec{\nabla} \cdot \vec{B} = 0$

$\vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt}$, $\vec{\nabla} \times \vec{B} = \varepsilon_0 \mu_0 \frac{d\vec{E}}{dt}$

General form of Wave Equation in free space:

$$\vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt} \Rightarrow \vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = -\frac{d}{dt}(\vec{\nabla} \times \vec{B})$$

$$\underbrace{\vec{\nabla}(\vec{\nabla} \cdot \vec{E})}_{0} - \nabla^2 \vec{E} = -\frac{d}{dt}\left(\mu_0 \varepsilon_0 \frac{dE}{dt}\right)$$

$$\Rightarrow \boxed{\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}}$$

$$\nabla \cdot E = 0$$
$$\nabla \cdot B = 0$$
$$\nabla \times E = -\frac{dB}{dt}$$
$$\nabla \times B = \mu_0 \varepsilon_0 \frac{dE}{dt}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \varepsilon_0 \frac{d\vec{E}}{dt} \Rightarrow \underbrace{\vec{\nabla} \times (\vec{\nabla} \times \vec{B})}_{\vec{\nabla}(\vec{\nabla} \cdot \vec{B}) - \nabla^2 \vec{B}} = \mu_0 \varepsilon_0 \frac{d}{dt} \underbrace{(\vec{\nabla} \times \vec{E})}_{-\frac{dB}{dt}}$$

$$\Rightarrow \boxed{\nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}}$$

Each component of $\vec{E}$ & $\vec{B}$ obey wave equation: Consider $\vec{E} = \hat{x} E_x(x,y,z,t) + \hat{y} E_y(x,y,z,t) + \hat{z} E_z(x,y,z,t)$

$$\nabla^2 \vec{E} = \hat{x} \frac{\partial^2}{\partial x^2} E_x + \hat{y} \frac{\partial^2}{\partial y^2} E_y + \hat{z} \frac{\partial^2}{\partial z^2} E_z$$

$$\frac{\partial^2 \vec{E}}{\partial t^2} = \hat{x} \frac{\partial^2}{\partial t^2} E_x + \hat{y} \frac{\partial^2}{\partial t^2} E_y + \hat{z} \frac{\partial^2}{\partial t^2} E_z$$

$$\therefore \frac{\partial^2 E_x}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_x}{\partial t^2} \quad \text{etc...}$$

Same for B components

Plane wave solutions:

Assume ① $\vec{E}$ only pts. in one direction everywhere ② $|\vec{E}|$ only varies with time & position x } $\vec{E} = \hat{y}\, E_y(x,t)$

$$\vec{\nabla} \times \vec{B} = \varepsilon_0 \mu_0 \frac{d\vec{E}}{dt} \quad \& \quad \vec{E} = \hat{y}\, E_y(x,t) \Rightarrow \vec{B} = \hat{z}\, B_z(x,t)$$

[Big Assumptions! However, superposition of plane wave solutions that have various amplitudes, ~~wavelengths~~ frequencies, & direction of travel will reproduce any possible solution (Exactly like any time domain waveform can be thought of as a superposition of Sin's & Cos's of various Amplitudes & frequencies; Principle of Fourier Analysis!)

$$\Rightarrow \frac{\partial^2 E_y}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2} \quad \& \quad \frac{\partial^2 B_z}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B_z}{\partial t^2}$$

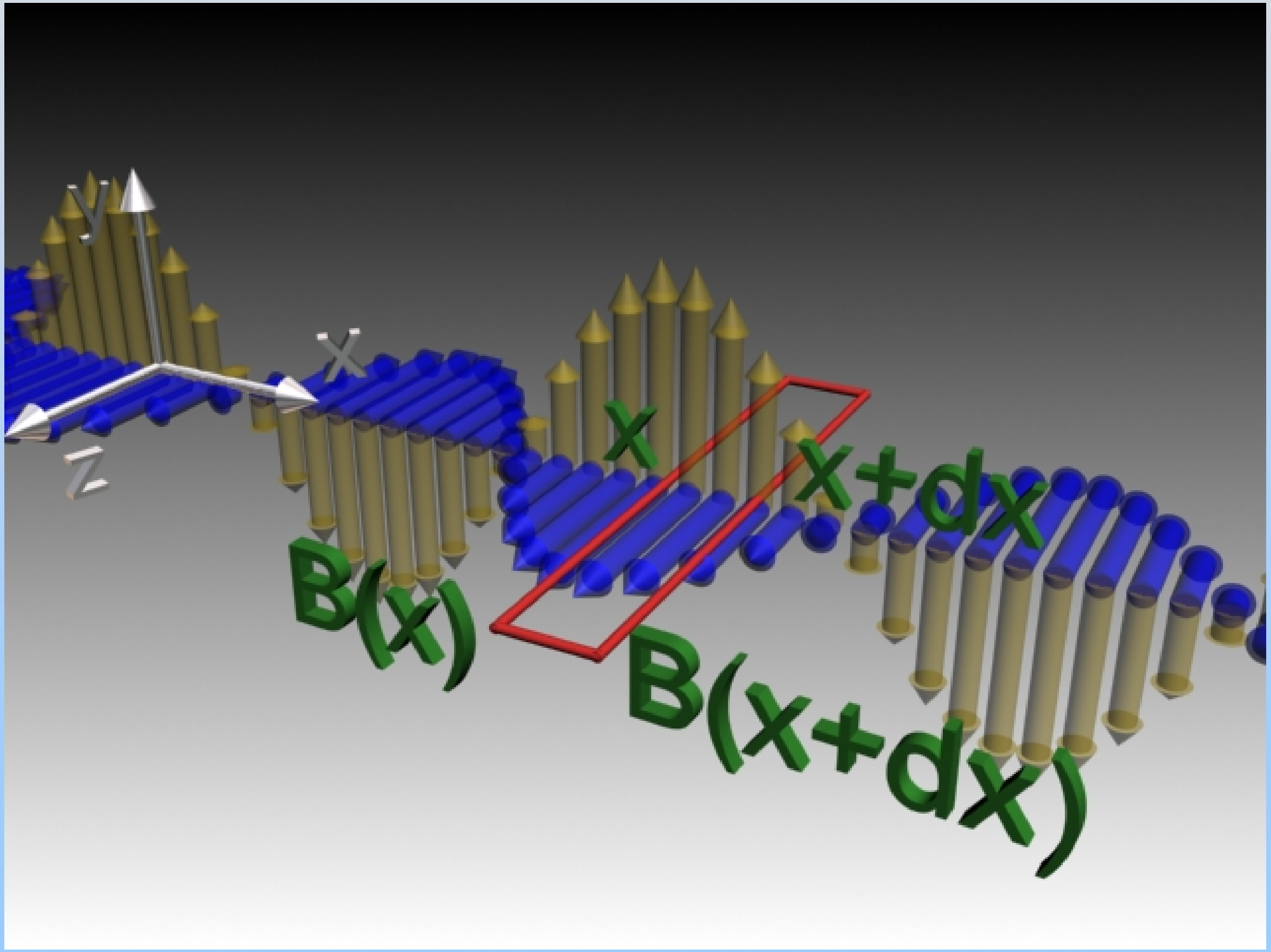
Looks like a wave equation:

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$ wave on a string] $\Rightarrow$ $v = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 3 \cdot 10^8 \frac{m}{s}$

Same as light!!!

Wave Equation

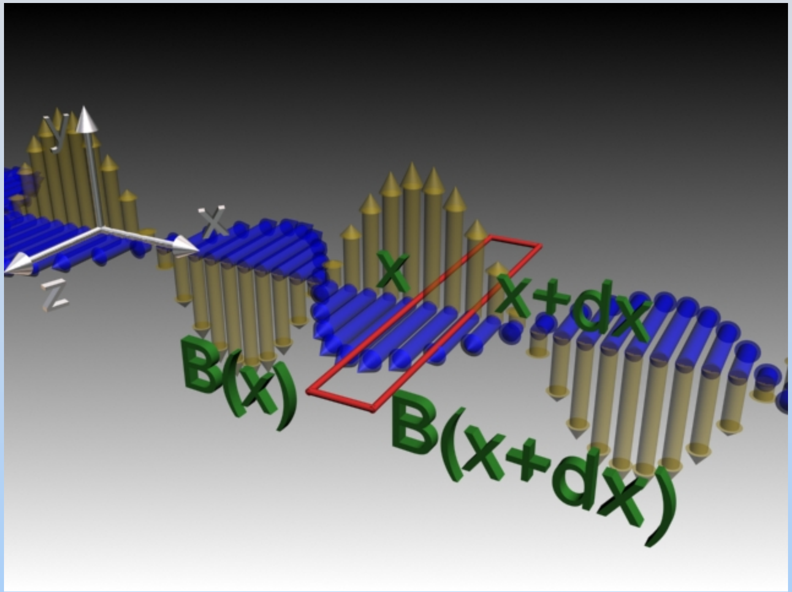
Start with Ampere-Maxwell Eq: $\oint_C \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 \varepsilon_0 \frac{d}{dt} \int \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$



Wave Equation

Start with Ampere-Maxwell Eq: $\oint_C \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 \varepsilon_0 \frac{d}{dt} \int \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$

Apply it to red rectangle:



$$\oint_C \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B_z(x,t)l - B_z(x+dx,t)l$$

$$\mu_0 \varepsilon_0 \frac{d}{dt} \int \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \mu_0 \varepsilon_0 \left(l\, dx \frac{\partial E_y}{\partial t} \right)$$

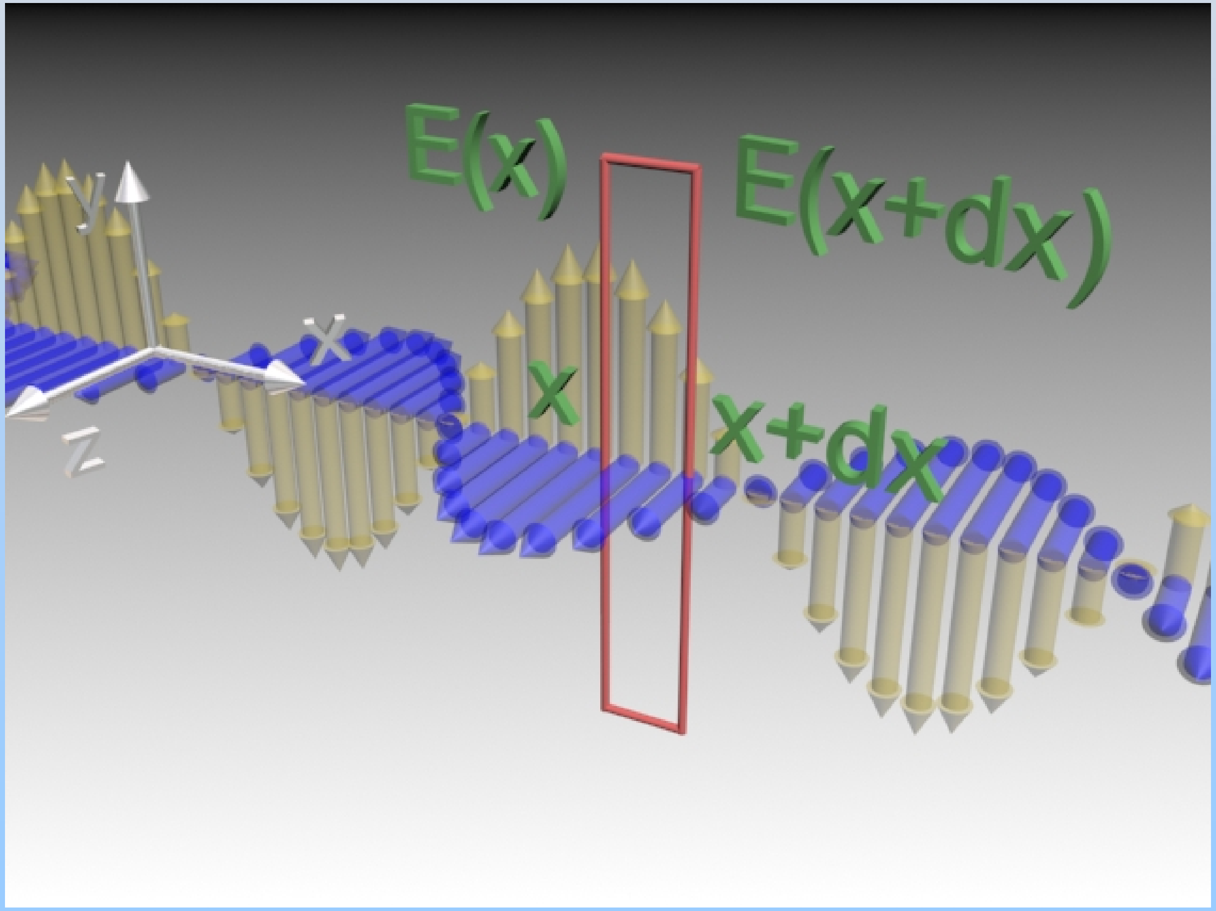
$$-\frac{B_z(x+dx,t) - B_z(x,t)}{dx} = \mu_0 \varepsilon_0 \frac{\partial E_y}{\partial t}$$

So in the limit that *dx* is very small:

$$-\frac{\partial B_z}{\partial x} = \mu_0 \varepsilon_0 \frac{\partial E_y}{\partial t}$$

Wave Equation

Now go to Faraday's Law $$\oint_C \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d}{dt}\int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$$

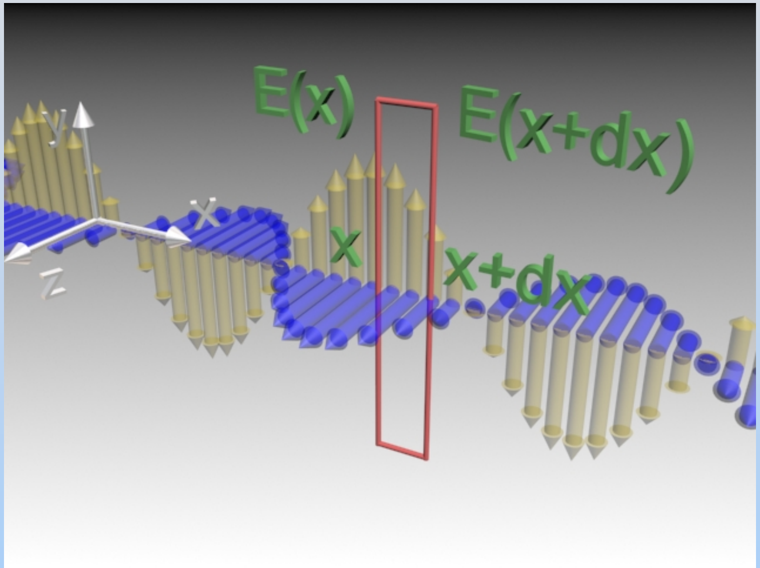


Wave Equation

Faraday's Law:

$$\oint_C \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d}{dt}\int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$$

Apply it to red rectangle:



$$\oint_C \vec{\mathbf{E}} \cdot d\,\vec{\mathbf{s}} = E_y(x+dx,t)l - E_y(x,t)l$$

$$-\frac{d}{dt}\int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = -l dx \frac{\partial B_z}{\partial t}$$

$$\frac{E_y(x+dx,t) - E_y(x,t)}{dx} = -\frac{\partial B_z}{\partial t}$$

So in the limit that *dx* is very small:

$$\boxed{\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}}$$

1D Wave Equation for E

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t} \qquad -\frac{\partial B_z}{\partial x} = \mu_0 \varepsilon_0 \frac{\partial E_y}{\partial t}$$

Take *x*-derivative of 1st and use the 2nd equation

$$\frac{\partial}{\partial x}\left(\frac{\partial E_y}{\partial x}\right) = \underline{\frac{\partial^2 E_y}{\partial x^2}} = \frac{\partial}{\partial x}\left(-\frac{\partial B_z}{\partial t}\right) = -\frac{\partial}{\partial t}\left(\frac{\partial B_z}{\partial x}\right) = \underline{\mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2}}$$

$$\boxed{\frac{\partial^2 E_y}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2}}$$

1D Wave Equation for E

$$\frac{\partial^2 E_y}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2}$$

This is an equation for a wave. Let: $E_y = f\left(x - vt\right)$

$$\left.\begin{aligned} \frac{\partial^2 E_y}{\partial x^2} &= f''\left(x - vt\right) \\ \frac{\partial^2 E_y}{\partial t^2} &= v^2 f''\left(x - vt\right) \end{aligned}\right\} \quad v^2 = \frac{1}{\mu_0 \varepsilon_0}$$

1D Wave Equation for B

$$\frac{\partial B_z}{\partial t} = -\frac{\partial E_y}{\partial x} \qquad \frac{\partial B_z}{\partial x} = -\mu_0 \varepsilon_0 \frac{\partial E_y}{\partial t}$$

Take *x*-derivative of 1st and use the 2nd equation

$$\frac{\partial}{\partial t}\left(\frac{\partial B_z}{\partial t}\right) = \underline{\frac{\partial^2 B_z}{\partial t^2}} = \frac{\partial}{\partial t}\left(-\frac{\partial E_y}{\partial x}\right) = -\frac{\partial}{\partial x}\left(\frac{\partial E_y}{\partial t}\right) = \underline{\frac{1}{\mu_0 \varepsilon_0}\frac{\partial^2 B_z}{\partial x^2}}$$

$$\boxed{\frac{\partial^2 B_z}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B_z}{\partial t^2}}$$

Electromagnetic Radiation

Both E & B travel like waves:

$$\frac{\partial^2 E_y}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2} \qquad \frac{\partial^2 B_z}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B_z}{\partial t^2}$$

But there are strict relations between them:

$$\frac{\partial B_z}{\partial t} = -\frac{\partial E_y}{\partial x} \qquad \frac{\partial B_z}{\partial x} = -\mu_0 \varepsilon_0 \frac{\partial E_y}{\partial t}$$

Here, E_y and B_z are "the same," traveling along x axis

Amplitudes of E & B

$$\text{Let } E_y = E_0 f\left(x - vt\right);\ B_z = B_0 f\left(x - vt\right)$$

$$\frac{\partial B_z}{\partial t} = -\frac{\partial E_y}{\partial x} \Rightarrow -vB_0 f'\left(x - vt\right) = -E_0 f'\left(x - vt\right)$$

$$\boxed{\Rightarrow vB_0 = E_0}$$

E_y and B_z are "the same," just different amplitudes