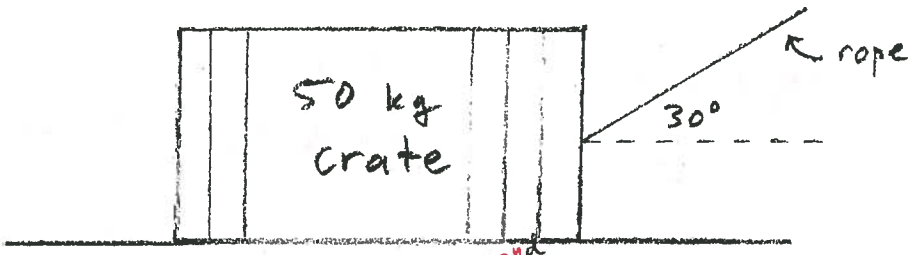
Physics 141 Summer 2010
Quiz 2 06/10/2010

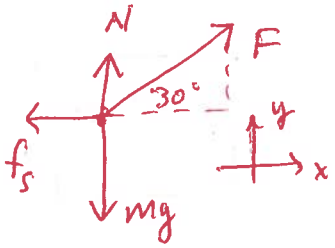
Name: Solutions

Show all work clearly and justify all answers logically/mathematically. You have 15 minutes to complete this 10 point quiz.

1. (*10 points*) A worker drags a 50 kg crate across a factory floor by pulling on a rope tied to a crate (see figure below). The worker exerts a force of 270 N on the rope, which is inclined at 30 degrees with respect to the horizontal. Assume the coefficient of kinetic friction, μ_k, between the crate and the floor is 0.5. (a) What force does the floor exert on the crate? (b) What is the acceleration of the crate? (c) What force must the worker exert to drag the box at a constant velocity?

Bonus: Deduce an expression for the most effective angle at which to drag the crate.





Apply Newton's 2nd Laws in y-direction...

a.) $\sum F_y = F\sin\theta + N - mg = 0$

$\Rightarrow N = mg - F\sin\theta$

plugging in the numbers...

$N = (9.8\ m/s^2)(50\ kg) - (270\ N)\sin 30^\circ$ ($\sin 30^\circ = 1/2$)

$= (490\ N) - 135\ N$

$$\boxed{N = 355\ \text{Newtons}}$$

b.) Apply Newton's 2nd law in x-dir.

$\sum F_x = F\cos 30^\circ - f_s = ma_x$

$F\cos 30^\circ - \mu N = ma_x$

$$\Rightarrow a_x = \frac{F\cos 30^\circ - \mu(mg - F\sin 30^\circ)}{m} = \frac{(270\ N)\sqrt{3}/2 - (0.5)(355\ N)}{50\ kg} = \boxed{1.13\ m/s^2 = a}$$

C.) Note from part b

$$a_x = \frac{1}{m}\left[F\cos\theta - \mu(mg - F\sin\theta)\right]$$

To drag the crate at constant velocity, $a = 0$.

$$\Rightarrow \quad F\cos\theta - \mu(mg - F\sin\theta) = 0.$$

$$\Rightarrow \quad F(\cos\theta + \mu\sin\theta) = \mu mg$$

$$F = \frac{\mu mg}{\cos\theta + \mu\sin\theta}$$

plugging in the numbers...

$$F = \frac{(0.5)(9.8\ m/s^2)(50\ kg)}{\sqrt{3}/2 + (0.5)(1/2)}$$

$$= \frac{245}{1.116} = 219.53\ N \simeq \boxed{220\ N}$$

Bonus: This part requires the use of calculus. Note that for acceleration to be maximized, the quantity $\cos\theta + \mu\sin\theta$ should be maximized.

(~~$F\cos\theta$~~ $F(\cos\theta + \mu\sin\theta) - \mu mg = Ma$)

$\Rightarrow$ ~~the~~ Think of $\cos\theta + \mu\sin\theta$ as a function of θ. This function has an extremum when $\frac{df}{d\theta} = 0$

$$\frac{d}{d\theta}(\cos\theta + \mu\sin\theta) = 0 \Rightarrow -\sin\theta + \mu\cos\theta = 0$$

$$\Rightarrow \boxed{\mu = \tan\theta \text{ (gives best angle at which to } \ldots \text{)}}$$