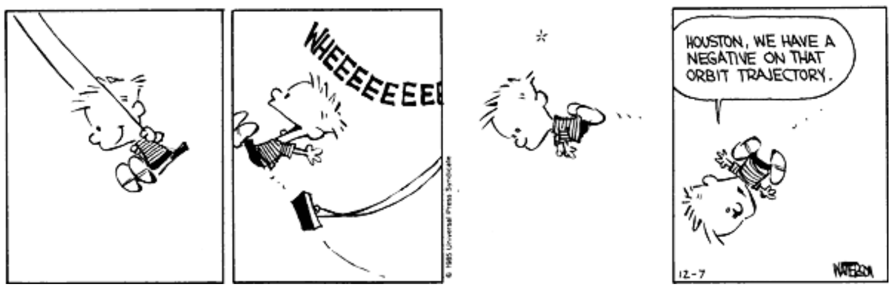
March 31, 2017 Physics 132 Prof. E. F. Redish

- **Theme Music: Benny Goodman**
  Swing, Swing, Swing
- **Cartoon: Bill Watterson**
  Calvin & Hobbes



3/31/17 Physics 132 **1**

Outline

- The makeup exam
- Recap: the math of the harmonic oscillator
- The long pendulum
  – The small angle approximation
  – The physics
- Damped and driven oscillators

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Makeup exam 1

Makeup exam 1 change in grade

#1	#2	#3	#4	#5
55%	47%	63%	87%	65%

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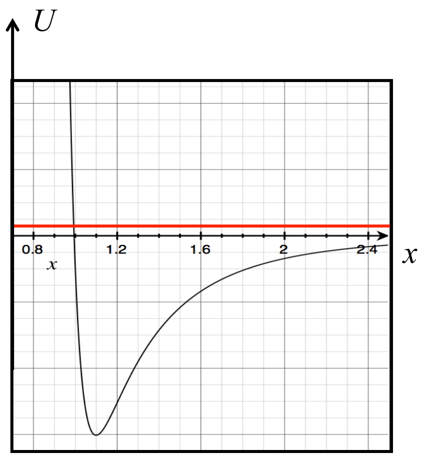
Example from the makeup exam (that a lot of folks had trouble with)

- Clicker question from last term

Two atoms have the interaction PE shown in the graph at the right. They start approaching each other with a very small KE (shown by the red line). How close will they get to each other before bouncing back?

A. 1.0 nm
B. 1.1 nm
C. Something else



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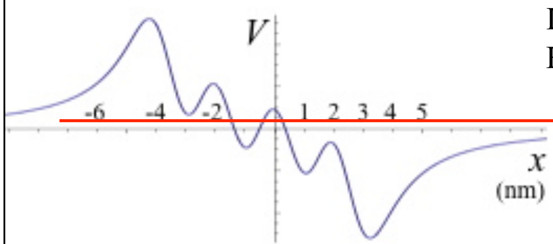
Example from the makeup exam (that a lot of folks had trouble with)

- Clicker question from Exam1 (MU)



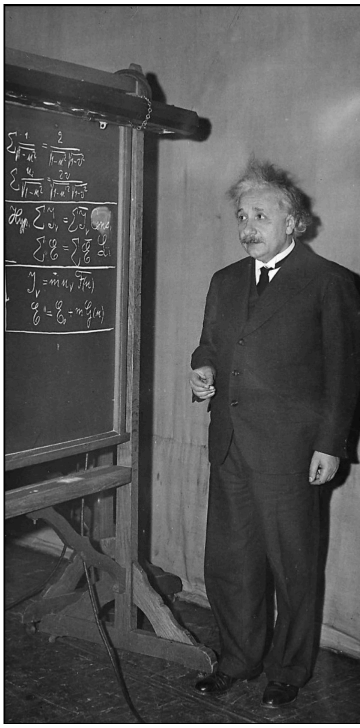
A positive ion approaches an array of charges having the electric potential shown in the graph at the left It approaches with a small KE. How close will it get?

A. 0.5 nm
B. 2.0 nm
C. 3.5 nm
D. Something else

5

A lot of folks are still having trouble with the basic ideas of estimations

1. Give enough words so we can follow your reasoning.
2. Decide what it is you need to calculate (formulas). Cross-check formulas to be sure they're correct.
3. Do you estimations from things you know (Know your basics!) and tell how you got them.
4. Keep units throughout and USE THEM to make sure you've done the correct calculation.
5. Don't keep more than 2-3 sig figs anywhere in the calculation.
6. NEVER write an untrue equation. ("= as →")
7. Be sure you answer all the questions asked!
8. Throughout, treat your numbers as MEASUREMENTS and check them for reasonableness!



The Equation of the Day

The Damped Oscillator

$$\frac{d^2x}{dt^2}+\gamma\frac{dx}{dt}+\omega_0^2 x=0$$

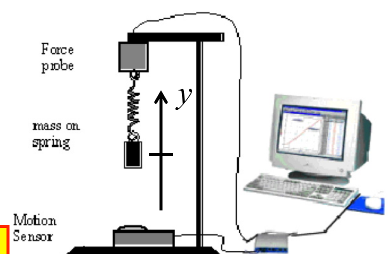
$$x(t)=Ae^{-\gamma t/2}\cos(\omega_1 t+\phi)$$

$$\omega_1=\sqrt{\omega_0^2-\frac{\gamma^2}{4}}$$

7 **Physics 132**

Analyzing the forces: hanging mass on a spring

- FBD:
  What are the forces acting on the mass?

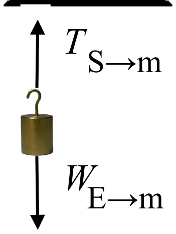


$$F_m^{net}=T_{s\to m}+W_{e\to m}$$
$$=k(L_0-l_0)-mg$$
$$=0 \qquad \Rightarrow \qquad k(L_0-l_0)=mg$$

At equilibrium

$$F_m^{net}=T_{s\to m}+W_{e\to m}$$
$$=k(-y+L_0-l_0)-mg$$
$$=-ky$$

At any stretch



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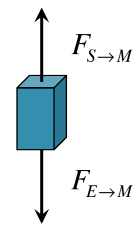
Summary with Equations: Mass on a spring



Measured from where?

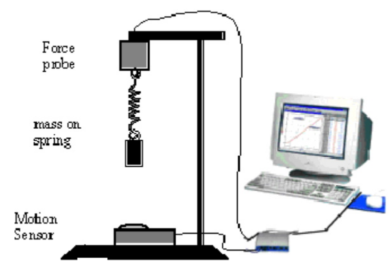
$$a = \frac{1}{m}F^{net} \qquad F^{net} = -ky$$

$$a = \frac{d^2y}{dt^2} = -\omega_0^2 y \qquad \omega_0^2 = \frac{k}{m}$$

$$y(t) = A\cos(\omega_0 t + \phi)$$

$$\omega_0 = \frac{2\pi}{T}$$

Interpret!



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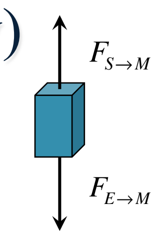
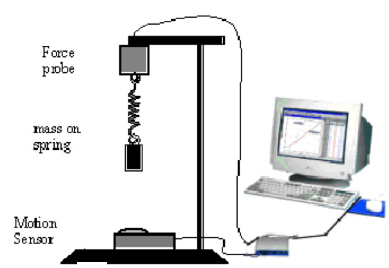
Summary with Equations: Mass on a spring (Energy)



Measured from where?

$$E = \tfrac{1}{2}mv^2 + mgh + \tfrac{1}{2}k(\Delta L)^2$$

$$E_i = E_f$$



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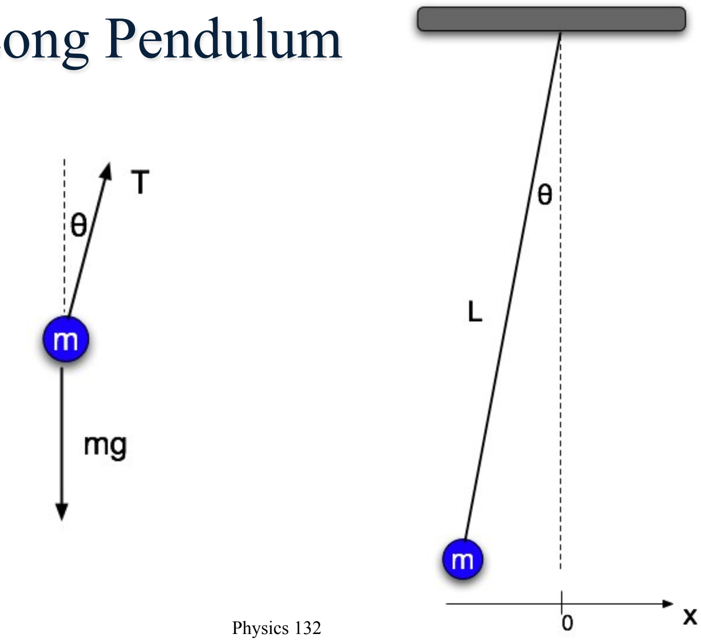
The Long Pendulum



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The small angle approximation

- Set your calculator to work <u>in radians</u>.
- What happens to cos for small angles?
- What happens to sin for small angles?

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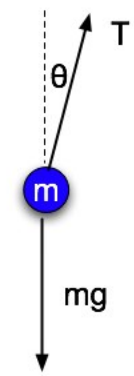
Pendulum



- Restoring force = $T \sin \theta$
- For small θ, there is very little vertical motion, so $T \approx mg$
- Restoring force $\approx mg \sin \theta$
- Restoring force $\approx mg\,(x/L)$
- Newton's 2nd Law:

$$ma = -(mg/L)x$$

The small angle approximation

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \ldots$$

$$\cos\theta = 1 - \theta^2 + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \ldots$$

$$\tan\theta = \theta + \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \ldots$$

This is how these are calculated! (Didn't you ever wonder how they did that?)

But these are often good enough.

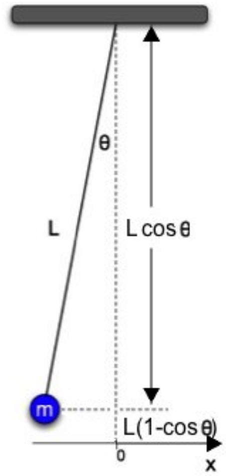
$\sin\theta \approx \theta$ *Good to 1% for $\theta < 1/4$ rad (15°)*

$\cos\theta \approx 1 - \frac{1}{2}\theta^2$ *Good to 1% for $\theta < 1/3$ rad (20°)*

$\tan\theta \approx \theta$ *Good to 1% for $\theta < 1/4$ rad (15°)*

Pendulum motion energy

$$E_0 = \tfrac{1}{2}mv^2 + mgh = \tfrac{1}{2}mv^2 + mgL(1-\cos\theta)$$
$$\cos\theta \approx 1 - \tfrac{1}{2}\theta^2$$
$$E_0 \approx \tfrac{1}{2}mv^2 + \tfrac{1}{2}[mgL]\theta^2$$
$$\theta \approx \sin\theta = \frac{x}{L}$$
$$E_0 \approx \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2 \qquad k = \frac{mg}{L}$$



Same as mass on a spring!
Just with a different $\omega_0^2 = k/m = g/L$

What's the period? Why doesn't it depend on m?

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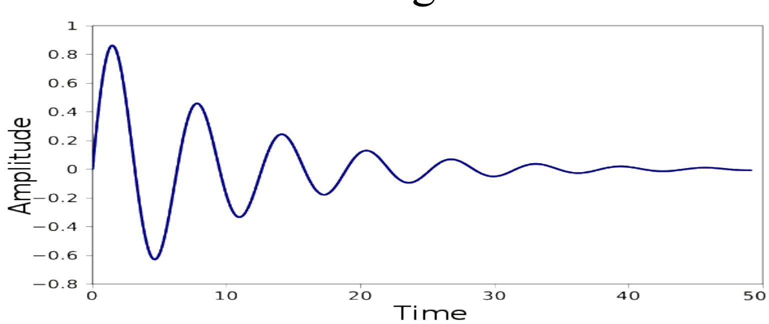
Foothold ideas: Damped oscillator

- Our toy model of an oscillator gave the result $x(t) = A\cos(\omega_0 t)$.
- As we watch, it doesn't do that. What are we missing?



Foothold ideas: Damped oscillator 1

- Amplitude of an oscillator tends to decrease. Simplest model is viscous drag.

$$ma = -kx - bv$$

$$\frac{d^2x}{dt^2} + 2\gamma\frac{dx}{dt} + \omega_0^2 x = 0 \qquad \gamma = \frac{b}{2m} \quad \omega_0 = \sqrt{\frac{k}{m}}$$

- Solution:

$$x(t) = A_0 e^{-\gamma t}\cos(\omega_1 t + \phi)$$

$$\omega_1 = \sqrt{\omega_0^2 - \gamma^2}$$

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Foothold ideas: Damped oscillator 2

- Competing time constants:

$$\gamma = \frac{1}{\tau} \qquad \frac{\omega_0}{2\pi} = \frac{1}{T} \qquad Q = \frac{\omega_0}{2\gamma} = \pi\frac{\tau}{T}$$

Decay time

Period

Tells which force dominates: restoring or damping.

- If:

$\omega_0 > \gamma$ underdamped: oscillates

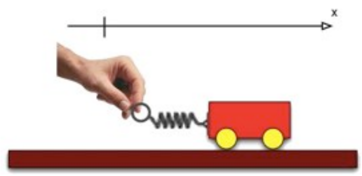
$\omega_0 = \gamma$ critically damped: no oscillation, fastest decay

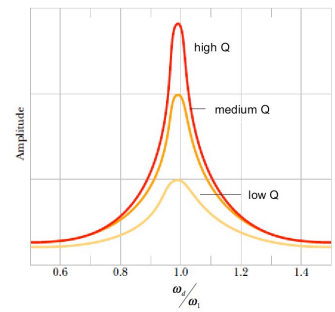
$\omega_0 < \gamma$ over damped: no oscillation, slower decay

Physics 132 **27**

Foothold ideas: Driven oscillator

- Adding an oscillating force.
- When the extra oscillating force (driver) matches the natural frequency of the oscillator you get a big displacement (**resonance**). Otherwise, not much.





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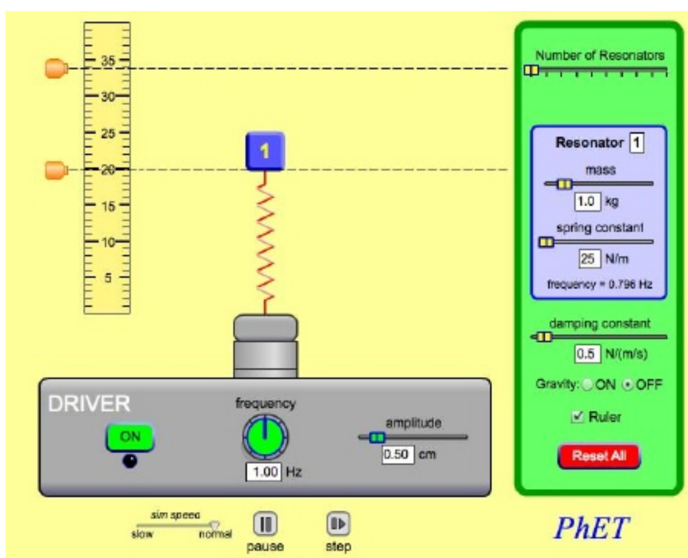
Resonance



https://phet.colorado.edu/en/simulation/legacy/resonance

https://www.youtube.com/watch?v=17tqXgvCN0E