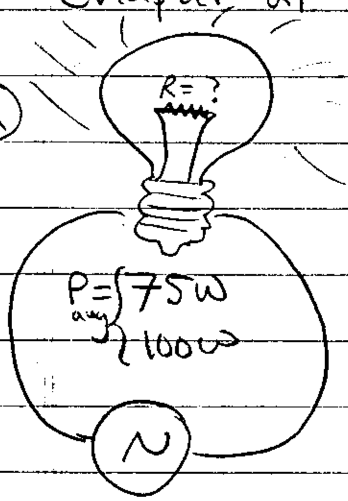
HW # 7 - SOL^N - ADAM COHEN

Chapter 21 — AC Circuits & EM waves

(2)



60 Hz, $V_{max} = 170V$

$$P_{avg} = I_{rms}^2 R$$

$$= \left(\frac{I_{max}}{\sqrt{2}}\right)^2 R, \quad \text{since } I_{rms} = \frac{I_{max}}{\sqrt{2}}$$

$$= \frac{1}{2} I_{max}^2 R$$

$$= \frac{1}{2}\left(\frac{\Delta V_{max}}{R}\right)^2 R, \quad \text{by Ohm's Law } \Delta V = IR$$

$$P_{avg} = \frac{1}{2}\frac{\Delta V_{max}^2}{R}$$

$\Rightarrow$ by rearranging

$$\boxed{R = \frac{\Delta V_{max}^2}{2 P_{avg}}}$$

If $P_{avg} = 75W \Rightarrow R = \dfrac{(170V)^2}{2(75W)} = \boxed{193\,\Omega}$

$P_{avg} = 100W \Rightarrow R = \dfrac{(170V)^2}{2(100W)} = \boxed{145\,\Omega}$

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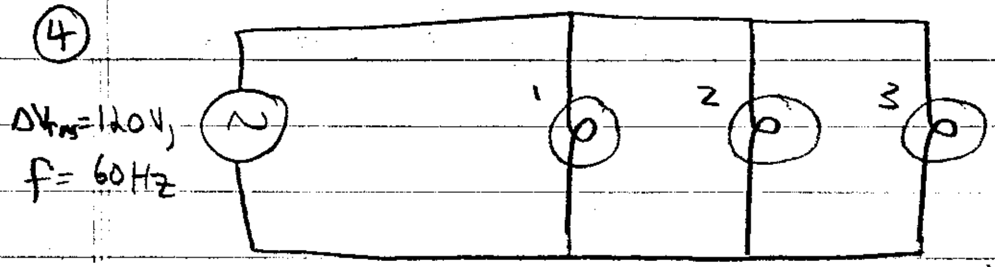

$P_{avg} = 150W, \quad 150W, \quad 100W$

- in parallel $\Rightarrow \Delta V_{rms} = \Delta V_1 = \Delta V_2 = \Delta V_3$
- by def^{n} of power, $P_{avg} = I_{rms}\, \Delta V_{rms}$

$$\Rightarrow I_{rms} = \frac{P_{avg}}{\Delta V_{rms}}$$

- by Ohm's Law, $\Delta V_{rms} = I_{rms} R$

$$\Rightarrow R = \frac{\Delta V_{rms}}{I_{rms}}$$

$$I_1 = \frac{150}{120} A = \boxed{1.25 A} \qquad R_1 = \frac{120}{1.25} \Omega = \boxed{96 \Omega}$$

$$I_2 = \frac{150}{120} A = \boxed{1.25 A} \qquad R_2 = \text{"} \quad \text{"} = \boxed{96 \Omega}$$

$$I_3 = \frac{100}{120} A = \boxed{0.833 A} \qquad R_3 = \frac{120}{0.833} \Omega = \boxed{144 \Omega}$$

7 $X_c = \dfrac{1}{\cancel{2\pi} f C}$ (don't care about prefactor $\frac{1}{2\pi}$)

$f = [Hz] = \dfrac{1}{[s]}$ $\qquad C = [F] = \dfrac{q}{\Delta V} = \dfrac{[C]}{[V]}$

so $X_c = \dfrac{1}{\left(\frac{1}{[s]} \cdot \frac{[C]}{[V]}\right)} = \dfrac{[V]}{\left([C]/[s]\right)}$

recall: $I \equiv \dfrac{q}{\Delta t} = \dfrac{[C]}{[s]} = [A]$

so $X_c = \dfrac{[V]}{[A]}$

recall, Ohm's Law: $\Delta V = IR \Rightarrow R = \dfrac{\Delta V}{I} = \dfrac{[V]}{[A]} = [\Omega]$

$\Rightarrow \boxed{X_c = [\Omega]}$ ✓

12 $\begin{cases} \omega = 120\pi \text{ rad/s} = 2\pi f \\ \Delta V_{max} = 140\text{ V} \\ L = 0.1\text{ H} \end{cases} \Rightarrow I_{rms} = ?$

$\sqrt{2} \cdot \left\{ \Delta V_{rms} = I_{rms} X_L = I_{rms} (2\pi f L) \right\} \cdot \sqrt{2}$

$\Delta V_{max} = \sqrt{2} \cdot I_{rms} \underbrace{[2\pi f]}_{=\omega} L = \sqrt{2}\, I_{rms}\, \omega L$

$\Rightarrow I_{rms} = \dfrac{\Delta V_{max}}{\sqrt{2}\,\omega L} = \dfrac{140}{\sqrt{2}\,(120\pi)(0.1)} = \boxed{2.63\text{ A}}$

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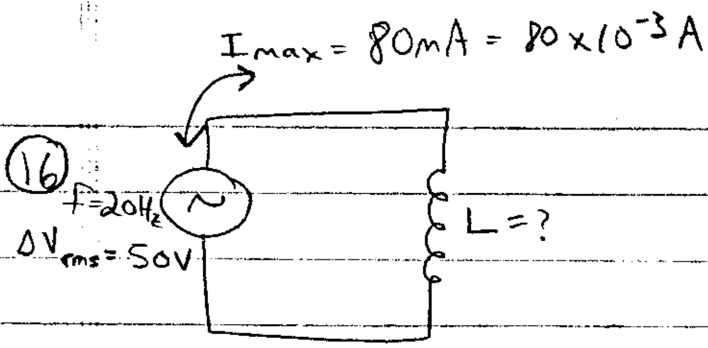

- $I_{rms} = \dfrac{I_{max}}{\sqrt{2}} = \dfrac{80 \times 10^{-3} A}{\sqrt{2}} = 0.056 A$

- $\Delta V_{rms} = I_{rms} X_L \equiv I_{rms} 2\pi f L$

$$\Rightarrow L = \frac{\Delta V_{rms}}{2\pi f \cdot I_{rms}} = \frac{50 V}{2\pi (20Hz)(0.056 A)}$$

$$\boxed{L = 7.03 H}$$

This is the minimum inductance to have at most $80 mA = I_{max}$.

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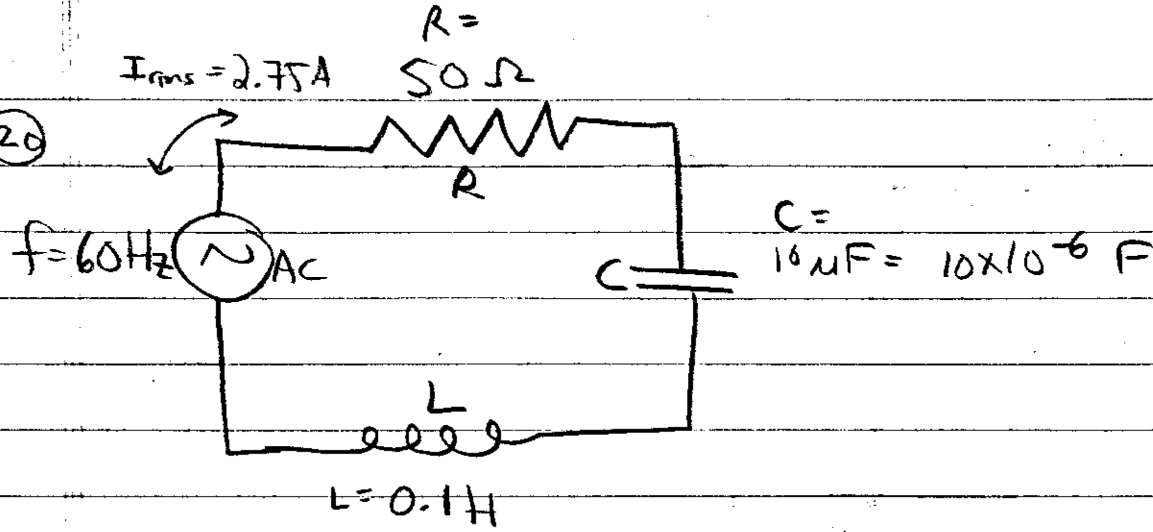


• inductive reactance $X_L \equiv 2\pi f L$

$= 2\pi(60)(0.1)\ \Omega$

$= 37.7\ \Omega$

• Conductive reactance $X_C \equiv \dfrac{1}{2\pi f C}$

$= \dfrac{1}{2\pi(60)(10\times10^{-6})}\ \Omega$

$= 265\ \Omega$

• impedence $Z \equiv \sqrt{R^2 + (X_L - X_C)^2}$

$= \sqrt{50^2 + (37.7 - 265)^2}\ \Omega$

$= 233\ \Omega$

• potential diffence accross R

$\Delta V_R = I_{rms} R = \boxed{138V}$

" " C

$\Delta V_C = I_{rms} X_C = \boxed{729V}$

" " L

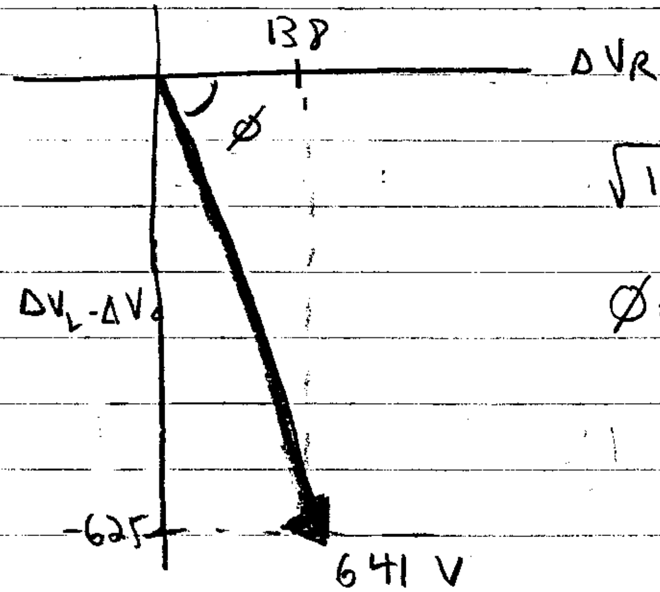
$\Delta V_L = I_{rms} X_L = \boxed{104V}$

• potential differnce accross RLC :

$\Delta V_{RLC} = I_{rms} Z = \boxed{641V}$

phasor diagram

$\Delta V_R = 138\text{ V}$, $\Delta V_L - \Delta V_C = -625\text{ V}$



$\sqrt{138^2 + 625^2} \approx 641$

$\phi = \arctan\left(\frac{625}{138}\right)$

$\left(\frac{O}{A}\right)$

$= 77.6°$

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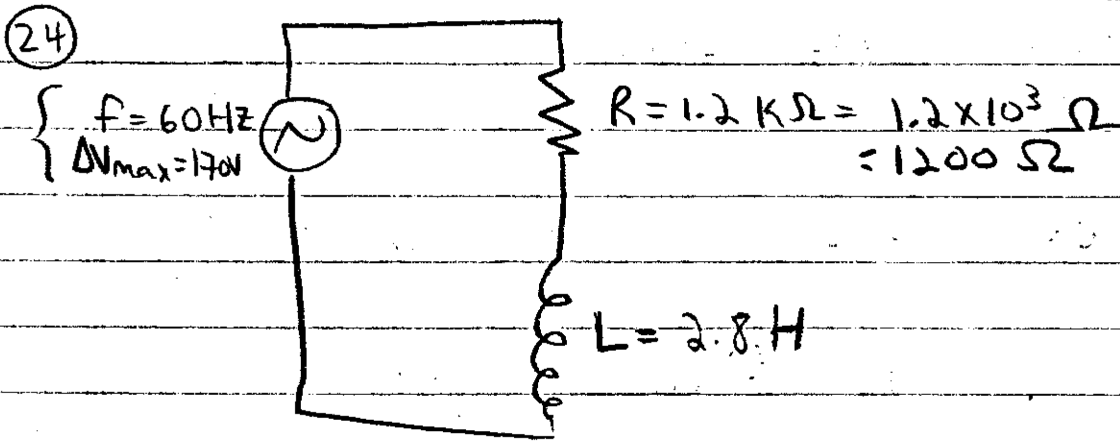


$$\begin{cases} X_L = 2\pi f L = 1056\ \Omega \\ Z = \sqrt{R^2 + (X_L - \cancel{X_C})^2} = 1598\ \Omega \end{cases}$$

$X_C \to 0$

(a)

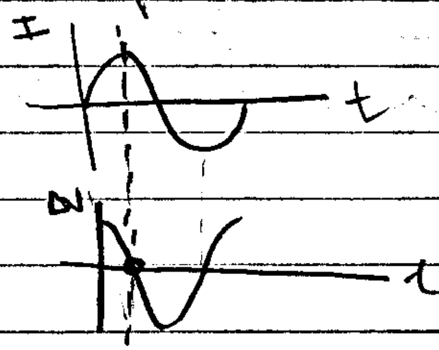
$$\Rightarrow I_{max} = \frac{\Delta V_{max}}{Z} = \frac{170\ V}{1.6 \times 10^3\ \Omega} = \boxed{0.106 A}$$

$$\begin{cases} \Delta V_R = I_{max} R = \boxed{128\ V} \\ \Delta V_L = I_{max} X_L = \boxed{112\ V} \end{cases}$$ (b)

c) if $I = I_{max}$ $\Rightarrow$ $\Delta V_R = I_{max} R$

$$\boxed{\Delta V_R = 128\ V}$$

• FACT: the instantaneous voltage across an inductor is always 90° out of phase with current:



$\Rightarrow$ $\boxed{\Delta V_L = 0}$

[at $I = I_{max}$, the instantaneous rate of change of I, $\frac{\Delta I}{\Delta t} = 0$, so

$$\Delta V_L = -L\frac{\Delta I}{\Delta t} = -L(0) = 0.]$$

- by loop rule, $\Delta V_{closed\ path} = 0$

$$\Rightarrow \Delta V_{source} + \Delta V_R + \underbrace{\cancel{\Delta V_L}}_{0} = 0$$

$$\Rightarrow \Delta V_{source} = -\Delta V_R$$

$$\boxed{\Delta V_{source} = -128V}$$

d) when $I=0$, $\Delta V_R = (0)R = \boxed{0V}$

- depending on if the rate of change of I is positive or negative:

$$\Delta V_L = \pm 110V$$

$$\Rightarrow \boxed{|\Delta V_L| = 112V}$$

by loop rule $\boxed{|\Delta V_{source}| = 112V}$

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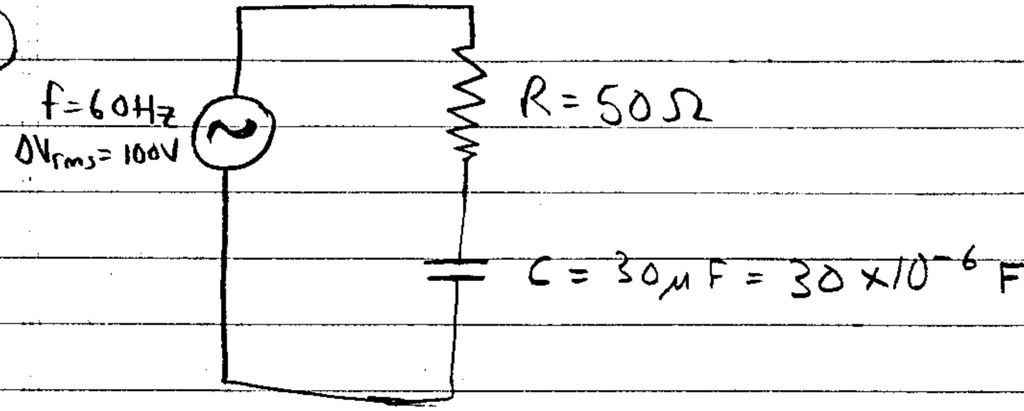

$$\begin{cases} R = 50\,\Omega \\ X_C \equiv 1/2\pi f C = \dfrac{1}{2\pi(60)(30\times10^{-6})}\,\Omega = 88.4\,\Omega \\ X_L = 0 \end{cases}$$

$\Rightarrow$ phase ∡ $\phi = \arctan\left(\dfrac{X_L - X_C}{R}\right) = \arctan\left(\dfrac{-88.4}{50}\right)$

$= -60.5^\circ$

$\Rightarrow$ power factor $\cos\phi = \boxed{0.492}$

• impedence : $Z = \sqrt{R^2 + X_c^2} = 102\,\Omega$

$\Rightarrow$ $I_{rms} = \dfrac{\Delta V_{rms}}{Z} = 0.984\,A$

so $P_{avg} = I_{rms}\,\Delta V_{rms}\cos\phi$

$= (0.984\,A)(100\,V)(0.492)$

$= \boxed{48.5\,W}$

if, instead $X_C = 0$

$$X_L \equiv 2\pi f L = 2\pi(60)(0.3)\,\Omega = 113\,\Omega$$

$$\Rightarrow Z = \sqrt{R^2 + X_L^2} = 124\,\Omega$$

$$I_{rms} = \frac{\Delta V_{rms}}{Z} = 0.809\text{ A}$$

$$\phi = \arctan\left(\frac{X_C}{R}\right) = 66.1^\circ$$

$$\text{power f}^r = \cos\phi = \boxed{0.404}$$

and $P_{avg} = I_{rms}\,\Delta V_{rms}\cos\phi = \boxed{32.7\text{W}}$

33) resonance frequency $f_0 = \frac{1}{2\pi\sqrt{LC}}$

Solve for L: $2\pi f_0 = \frac{1}{\sqrt{LC}}$

$$\frac{1}{2\pi f_0} = \sqrt{LC}$$

$$\frac{1}{4\pi^2 f_0^2} = LC$$

$$L = \frac{1}{4\pi^2 f_0^2 C}$$

plug in: $f_0 = 88.9\text{ MHz} = 88.9\times10^6\text{ Hz}$
$C = 1.4\text{ pF} = 1.4\times10^{-12}\text{ F}$

$$\Rightarrow L = 2.29\times10^{-6}\text{ H} = \boxed{2.29\,\mu\text{H}}$$

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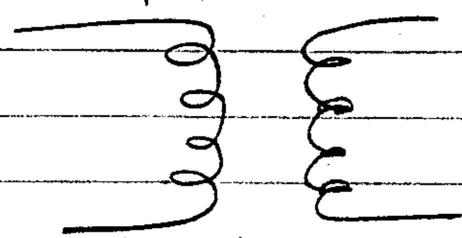
$N_1 = 240$ N_2



$\Delta V_1 = 120V$

$\begin{cases} \Delta V_2 = 9V \\ I_2 = 400mA = 400\times10^{-3}A = 0.4A \end{cases}$

for the transformer : $\Delta V_2 = \dfrac{N_2}{N_1}\Delta V_1$

$$\Rightarrow N_2 = \frac{N_1 \Delta V_2}{\Delta V_1} = \frac{(240)(9\cancel{V})}{(120\cancel{V})}$$

$$= \boxed{18 \text{ turns}}$$

for an ideal transformer

$$P_{in} = P_{out} = \Delta V_2 I_2$$

$$= (9V)(0.4A)$$

$$\text{Power} = \boxed{3.6W}$$

(45) $\begin{cases} \lambda_{red} = 660\ nm = 660 \times 10^{-9}\ m \\ \lambda_{IR} = 940\ nm = 940 \times 10^{-9}\ m \end{cases}$

for EM wave $c = f\lambda$ where c = speed of light $= 3\times10^{8}\ m/s$

so $f_{red} = \dfrac{c}{\lambda} = \dfrac{3\times10^{8}\ m/s}{660\times10^{-9}\ m} = \boxed{4.55\times10^{14}\ Hz}$

$f_{IR} = \dfrac{c}{940\times10^{-9}\ m} = \boxed{3.19\times10^{14}\ Hz}$

$$\frac{\text{Energy out}}{\text{Energy in}} = \frac{I_{out}}{I_{in}} = \frac{\left(\frac{E_{max,out}^2}{2\mu_0 c}\right)}{\left(\frac{E_{max,in}^2}{2\mu_0 c}\right)}$$

FACT: intensity is related to the square of amplitude.

$$= \frac{E_{max,out}^2}{E_{max,in}^2}$$ where E_{max} " amplitude of wave

$$\Rightarrow \frac{E_{max,out}}{E_{max,in}} = \sqrt{\left(\frac{\text{Energy out}}{\text{Energy in}}\right)}$$

$$= \sqrt{\frac{0.33}{1}}$$ since 33% of energy emitted.

$$= \sqrt{0.33}$$

$$= \boxed{0.57} = 57\% \text{ of amplitude}$$

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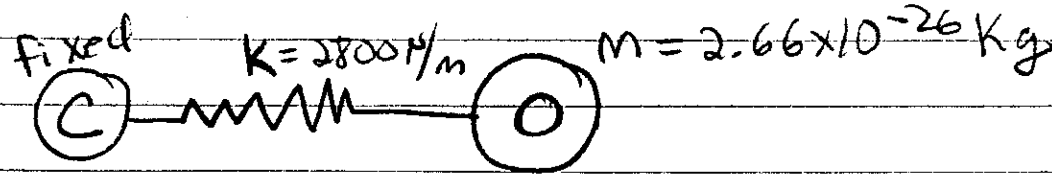

recall for a spring: $F = -Kx$

$ma = -Kx$

$\Rightarrow x = A\cos\left(\sqrt{\frac{K}{m}}\,t\right)$

so the resonant frequency

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{2800}{2.66\times10^{-26}}}\ Hz$$

$$\boxed{f = 5.2\times10^{13}\ Hz}$$

What wavelength light coorespands to this frequecy?

$$c = f\lambda \Rightarrow \lambda = \frac{c}{f} = \frac{3\times10^{8}\ m/s}{5.2\times10^{13}\ 1/s}$$

$$= 5.8\times10^{-6}\ m$$

$$= \boxed{5.8\ \mu m}$$

IR: $0.7\mu m \lesssim \lambda \lesssim 1mm$

so $5.8\mu m$ is IR ✓