HW #1 SOL^N — ADAM COHEN

Chapter 15 → ELECTRIC FIELDS

(3) Coulomb's law describes electric force.

- Magnitude of force is:

Coulomb's Law

$$\boxed{F = K\frac{q_1 q_2}{r^2}} = \frac{K(2e)(79e)}{(2.0\times10^{-14}\,m)^2}$$

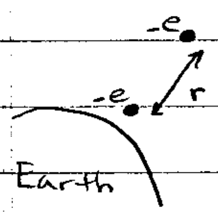
where
$$\begin{cases} K = \text{Coulomb's constant} \\ e = \text{elemental unit of charge} \\ \quad = 1.6\times10^{-19}\ C \end{cases}$$

plugging in numbers, $\boxed{F = 91\,N}$

- a force is a vector, so it must have a direction
- recall like charges repel ⇒ this force is $\boxed{\text{repulsive}}$

(8) free-body diagram:

Free-Body Diagram



$\uparrow$ Fe = electric force from other electron

● -e

$\downarrow$ mg = weight due to Earth's mass

Equilibrium means $\vec{a} = 0$

$$\sum F = m\cancel{a}^{\,0} = 0 \quad \text{since static}$$

$$F_e - m_e g = 0$$

$$\left(K\frac{e\cdot e}{r^2}\right) - m_e\,g = 0$$

m_e: mass of electron

g: acceleration due to gravity $= 9.8\ m/s^2$

by algebra : $\frac{Ke^2}{r^2} = m_e g$

rearrange to solve for r, the unknown distance between the electrons :

$$r = \sqrt{\frac{Ke^2}{m_e g}}$$

now plug in all the constants :

$K,\ e,\ m_e,\ g$

$$\Rightarrow \boxed{r = 5.08\ m}$$

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$q_1 = 6\times10^{-9}C \longleftrightarrow q_2 = -3\times10^{-9}C$

$r = 60\ cm = 0.6\ m$

where can we put $q_3 = 12\times10^{-9}\ C$ such that it is stationary?

- q_3 will feel two electric forces : one from q_1 and one from q_2, call these $\vec{F}_1$ and $\vec{F}_2$ respectively.
- we need this condition

superposition of electric forces

$$\sum F = F_1 + F_2 = m\cancel{a}^{\,0} = 0$$

- by Coulomb's law :

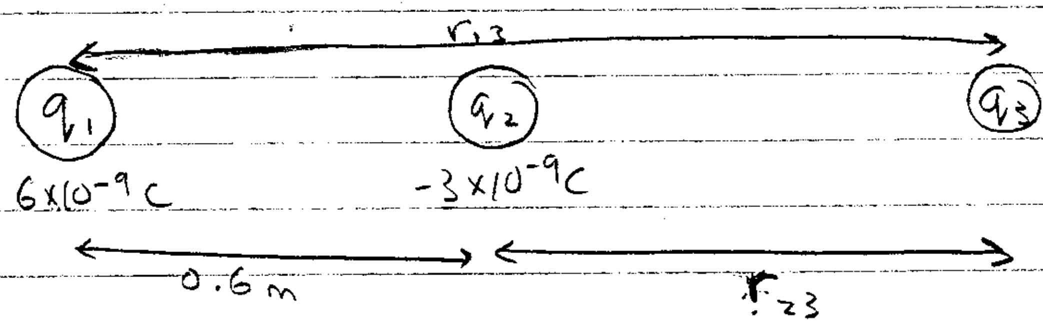
$$\frac{K q_1 q_3}{r_{13}^2} + \frac{K q_2 q_3}{r_{23}^2} = 0$$

where r_{13} = distance between q_1, q_3
r_{23} = — " — " — q_2 ; q_3

Note, we can divide by K, q_3 $\Rightarrow$

$$\frac{q_1}{r_{13}^2} + \frac{q_3}{r_{23}^2} = 0 \qquad (I)$$

- electric force is proportional to charge, so q_3 must be closer to the smaller charge (q_2) than the larger charge (q_1) in order to be in equilibrium
- then q_3 must be located to the right of q_3:



- note that we can write:

$$r_{13} = 0.6m + r_{23}$$

- so eq^n (I) can be rewritten as:

$$\frac{q_1}{(0.6 + r_{23})^2} + \frac{q_2}{r_{23}^2} = 0$$

- now solve for r_{23}, our unknown.

$$\frac{q_1}{(0.6 + r_{23})^2} = \frac{-q_2}{(r_{23})^2}$$

cross-multiply:

$$q_1 r_{23}^2 = -q_2 (0.6 + r_{23})^2$$

expand the square on the right:

$$q_1 r_{23}^2 = -q_2[0.36 + 1.2 r_{23} + r_{23}^2]$$

plugging in for q_1, q_2

$$(6 \cancel{\times 10^{-9}} C)\, r_{23}^2 = -(-3 \times \cancel{10^{-9}} C)[0.36 + 1.2 r_{23} + r_{23}^2]$$

$$2 r_{23}^2 = 0.36 + 1.2 r_{23} + r_{23}^2$$

$$r_{23}^2 - 1.2 r_{23} - 0.36 = 0$$

by quadratic eq^n:

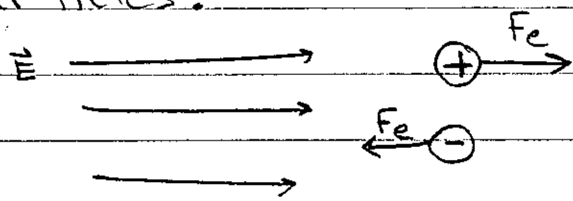
$$r_{23} = \frac{1.2 \pm \sqrt{(1.2)^2 - 4(1)(-0.36)}}{2(1)}$$

$$\boxed{r_{23} = 1.45 \text{ m}} \quad \text{OR} \quad r = \cancel{-0.25 \text{ m}}$$

reject

Reject answer between q_1, q_3 because forces can not balance.

22 A proton has positive charge. An electric field exerts a force in the direction of field for positive particles.



So, to decelerate the proton, we must apply an electric field in the direction opposite to its velocity.

I. The work-KE th^m says:

Work done by electric field = change in KE of proton

$$W_e = \Delta KE = \cancel{KE_f}^{\,0} - KE_i$$)-KE th^m

$$(\text{Force})(\text{distance}) = -3.25 \times 10^{-15}\ J$$

$\vec{F}_e = q\vec{E}$ $$(-qE)(1.25\,m) = -3.25 \times 10^{-15}\ J$$

negative since force is applied opposite to distance

here $q = 1.6 \times 10^{-19}\ C$

by algebra: $$\boxed{|E| = 1.63 \times 10^{4}\ N/C}$$

II. Alternatively, one can use Kinematic eq^ns:

def'n of KE

$$KE_i = \frac{1}{2} m v_i^2 = 3.25 \times 10^{-15}\ J$$

mass of proton $= 1.67 \times 10^{-27}$ Kg

$$\Rightarrow \begin{cases} v_i = 1.97 \times 10^{6}\ m/s \\ v_f = 0\ m/s \\ \Delta x = 1.25\ m \end{cases}$$

kinematic eq^n from 121

recall: $\cancel{v_f}^{\,2} \xrightarrow{} 0 = v_i^2 + 2 a \Delta x$

by rearranging: $\dfrac{-v_i^2}{2 \Delta x} = a$

so $a = -1.557 \times 10^{12}\ m/s^2$

and Newton's Second Law say:

Newton's 2nd Law

$$\Sigma F = ma$$

$$F_e = m_{proton}\, a$$

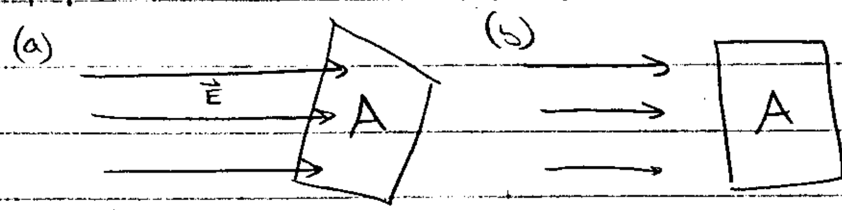
$\vec{F}_e = q\vec{E}$

$$-q_{proton} E = m_{proton}\, a$$

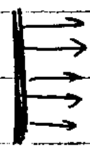
$$\Rightarrow |E| = \frac{m_{proton}\, a}{q_{proton}} = \boxed{1.63 \times 10^4\ N/c}$$

38) $|\vec{E}| = 6.2\times10^{5}\ N/C$, $A = 3.2\ m^2$

(a) (b)



electric flux can be thought of the number of field lines penetrating the surface.

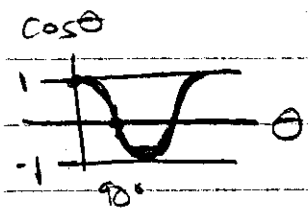


for (a) the field lines "break" the surface.

$$\Phi_E = EA\underline{\cos}\,\theta$$

this "picks" the magnitude of electric field vectors $\perp$ to surface

for (a) $\theta = 0^\circ$ since $\vec{E}$ and normal vector of A are $\parallel$

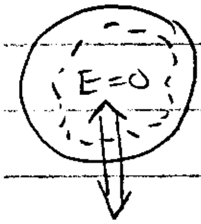


$$\Rightarrow \quad \Phi_E = EA\,\cancel{\cos 0^\circ}^{\,1} = EA = \boxed{2.0\times10^{6}\ \frac{N\cdot m^2}{C}}$$

- for (b), no field lines "break" surface
- can say $\theta = 90^\circ$, so

$$\Phi_E = EA\,\cancel{\cos 90^\circ}^{\,0} = \boxed{0}$$

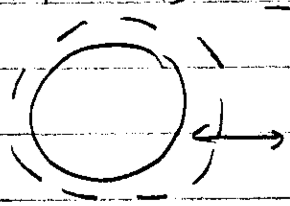
44 Gauss's law states that charges are the source of Electric fields.



$Q_{inside} = 0$

Consider a conductor. Within, $E = 0$. We can draw any closed surface within the conductor. Since $E = 0$, by Gauss's Law ($EA = \frac{Q}{\epsilon_0}$), $\frac{Q}{\epsilon_0}$ must be 0.

So $Q = 0$ inside our gaussian surface. BUT the electric-field just outside the conductor is NOT zero.



$E \neq 0 \Rightarrow Q_{inside} \neq 0$

So there must exist a net charge inside this (above) gaussian surface. This excess charge must be outside any gaussian surface within the conductor. Indeed, it is on the surface of the conductor.

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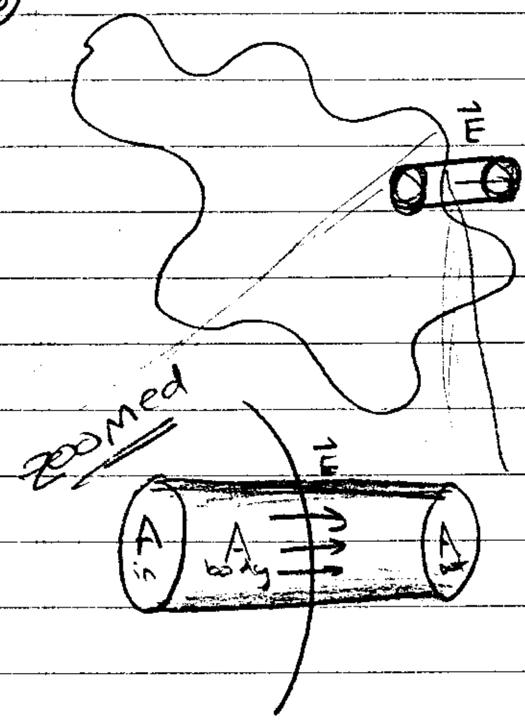


Choose the gaussian surface such that it is a small cylinder with one end inside the conductor and the other end outside.

There are 3 areas to worry about on a cylinder: the 2 end caps and the body.

As for the cap within the conductor:

$E = 0$ inside the conductor

$\Rightarrow \quad E A_{inside} = 0$

So there is no flux from this cap.

As for the body, the electric field runs parallel to it, so there is no flux (see diagram)

$\Rightarrow \quad \Phi_{body} = 0$

But for the outside cap, exactly the opposite is true. The electric field is perpendicular to surface. So

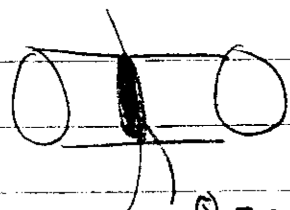
$$\Phi_{outside} = E A_{outside}$$

So, to summarize the total flux through our cylinder is:

$$\Phi_{total} = \underbrace{\Phi_{body}}_{=0} + \underbrace{\Phi_{\substack{inside \\ cap}}}_{=0} + \Phi_{\substack{outside \\ cap}}$$

$$\Phi = EA$$

Now, the total charge within our cylinder is:



$Q = \sigma A$

$$Q = \left(\frac{\text{charge}}{\text{area}}\right)(\text{area})$$

$$= (\text{charge density})(\text{area})$$

$$= \sigma A$$

So Gauss's Law says:

$$\Phi = \frac{Q_{inside}}{\epsilon_0}$$

$$E\cancel{A} = \frac{\sigma \cancel{A}}{\epsilon_0} \quad \text{(cross out same terms)}$$

$$\boxed{E = \frac{\sigma}{\epsilon_0}}$$

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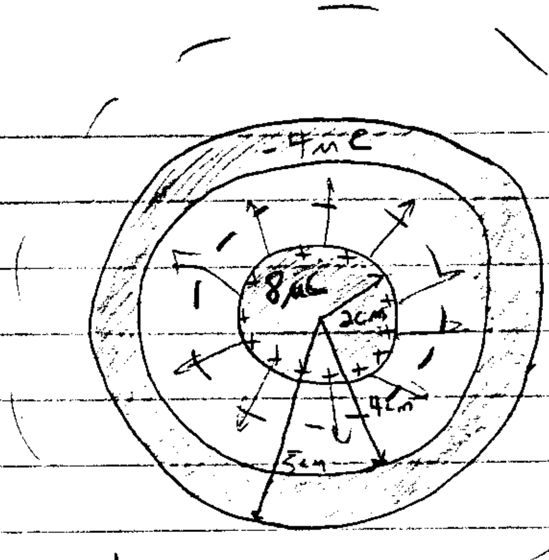

$\vec{E}$ at $r = \begin{cases} 1\text{cm} \\ 3 \\ 4.5 \\ 7 \end{cases}$

(a) $r = 1$cm

The shell is a conductor.
Conductors always arrange themselves such that $\boxed{E = 0}$ inside.

(b) $r = 3$cm

- Consider the gaussian sphere at 3cm. The charge within is the $8\,\mu C$ from the conductor. $\Rightarrow$ $Q_{inside} = 8\,\mu C = 8 \times 10^{-6}$ C
- The surface area of a sphere is
  $A_{sphere} = 4\pi r^2 = 4\pi (0.03\text{ m})^2 = 1.13 \times 10^{-2}\text{ m}^2$
- The electric field points radially outward
  $\Rightarrow \Phi = EA$
  (since every field line $\perp$ to sphere)
- So Guass's Law says:

$$EA = \frac{Q}{\epsilon_0} \Rightarrow E = \frac{(8 \times 10^{-6})}{(8.85 \times 10^{-12})(1.13 \times 10^{-2})} \frac{N}{C}$$

$$\boxed{|\vec{E}| = 7.99 \times 10^{7} \frac{N}{C}}$$

(c) At $r = 4.5$ cm, we are again within a conductor, so $\boxed{E = 0}$

(d) $r = 7$ cm

Similar to part (b).

But now, make a guassian surface at $r = 7$ cm.

The net charge inside is: $Q_{inside} = (+8 - 4)\,\mu C$
$= +4\,\mu C$

The surface area is

$$A = 4\pi r^2 = 4\pi(0.07\text{ m})^2 = \ldots$$

By Gauss, $E = \dfrac{Q}{\epsilon_0 A}$

plug in for Q, ϵ_0, A
to find:

$$\boxed{E = 7.34 \times 10^6 \text{ N/C}}$$

Again this is radially outward since the net charge is positive. (A test charge would be repelled)