

Name: ADAM COHEN

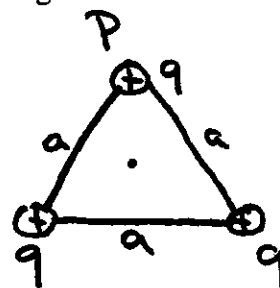
Section: 401

EXAM I

Physics 122

October 4, 2006

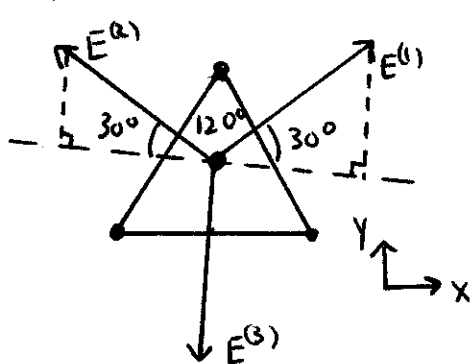
1. Three equal positive charges q are at the corners of an equilateral triangle of side a as in the figure.



10 (a) At what point in the plane (excluding ∞) is the electric field zero? Why?

15 (b) What are the magnitude and direction of the electric field at P due to the charges at the base?

a) At the center, consider the field vectors:



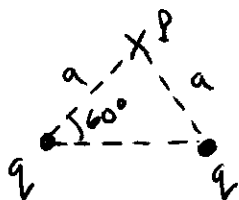
All three \vec{E} -fields have the same magnitude since $E = \frac{kq}{r^2}$ for each.

Notice the y -components $\rightarrow 0$ by symmetry.

Must also argue x -components $\rightarrow 0$.

$$E_y^{(1)} = E_y^{(2)} = E \sin(30^\circ) = \frac{1}{2} E$$

$$\Rightarrow \sum E_y = \frac{1}{2} E + \frac{1}{2} E - E = 0 \quad //$$



By symmetry $E_x(P) = 0$

$$\text{also } E_y(P) = 2 E_y^{(1)} = 2 E \sin 60^\circ$$

$$= \frac{2Kq}{a^2} \frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{3} Kq}{a^2}$$

$$\Rightarrow \vec{E}(P) = \frac{\sqrt{3}}{a^2} Kq \hat{y} \quad //$$

Name: Adam Cohen

Section: 401

EXAM I

Physics 122

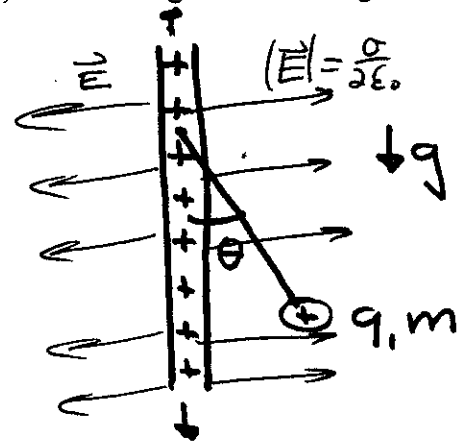
October 4, 2006

2. Consider an infinite nonconducting sheet with surface charge σ , as shown edgewise in the figure.

10 (a) In terms of σ , what is the electric field a distance r away from the sheet?

Now attach, via a massless string, a positive charge q , to the sheet. The particle has a mass m and the system is on the earth's surface. Given the data,

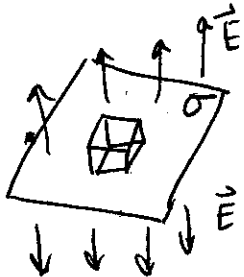
- $m = 1.2 \text{ mg}$
- $q = 19.7 \text{ nC}$
- $\theta = 27.40^\circ$



10 (b) Write equations for the various forces on the charge q .

5 (c) Use these equations to determine the surface charge density, σ .

Use Gauss's law to get \vec{E} -field around an infinite sheet:



Use a gaussian "pill box", notice the flux from the sides is zero because $\vec{E} \perp \hat{n}$ (where \hat{n} is normal unit vector)

$$\text{Similarly } \Phi_{\text{top}} = \Phi_{\text{bottom}} = EA$$

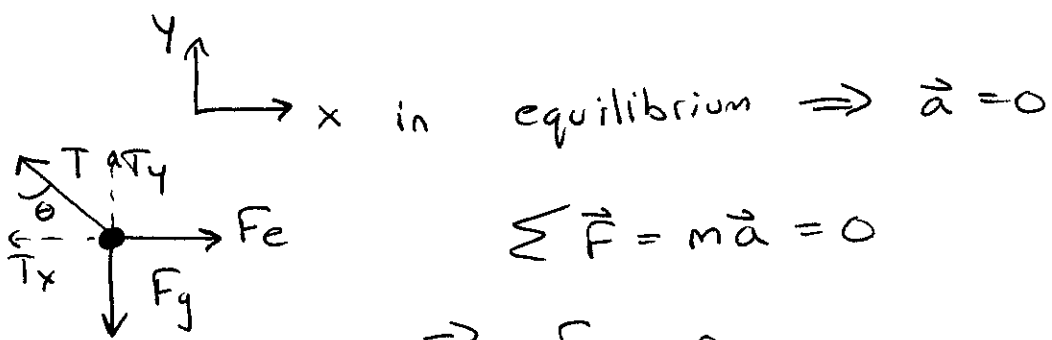
The charge within the box is: $Q_{\text{inside}} = \sigma A$

(here A is surface area of top and bottom of box)

$$\text{So } \Phi_{\text{total}} = 2EA = \frac{\sigma A}{\epsilon_0} \Rightarrow E = \frac{\sigma}{2\epsilon_0}$$

$$\vec{E} = \frac{\sigma}{2\epsilon_0} \begin{cases} +1 & \text{above plane} \\ -1 & \text{below plane} \end{cases}$$

So, electric field is independent of distance from sheet.



$$\sum \vec{F} = m\vec{a} = 0$$

$$\Rightarrow F_x = 0$$

$$F_y = 0$$

$$\Rightarrow x: F_e - T_x = 0$$

$$qE - T \sin \theta = 0$$

$$T \sin \theta = qE$$

$$y: T_y - F_g = 0$$

$$T \cos \theta - mg = 0$$

$$T \cos \theta = mg$$

We have two equations, two unknowns.

I solve by dividing the equations:

$$\frac{T \sin \theta}{T \cos \theta} = \frac{qE}{mg}$$

$$\tan \theta = \frac{qE}{mg}$$

and recall $E = \frac{\sigma}{2\epsilon_0}$

$$\tan \theta = \frac{q\sigma}{2\epsilon_0 mg}$$

$$\Rightarrow \sigma = \frac{2\epsilon_0 mg \tan \theta}{q}$$

$$\left\{ \begin{array}{l} \epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{Nm^2} \\ m = 1.2 \times 10^{-6} \text{ kg} \\ q = 19.7 \times 10^{-9} \text{ C} \\ g = 9.8 \text{ m/s}^2 \\ \theta = 27.40^\circ \end{array} \right.$$

$$\Rightarrow \sigma = 5.48 \times 10^{-9} \frac{C}{m^2}$$

Name: Adam Cohen

Section: 401

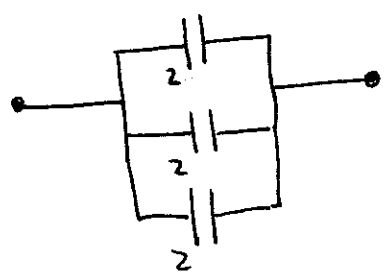
EXAM I

Physics 122

October 4, 2006

3. For three $2.0\mu\text{F}$ capacitors, sketch the arrangement that gives,
- (a) The largest equivalent capacitance. What is it?
 - (b) The smallest equivalent capacitance. What is it?
 - (c) An equivalent capacitance at $3.0\mu\text{F}$.

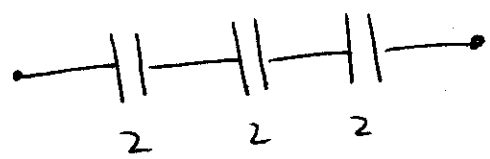
"largest"



in parallel,

$$C_{eq} = C_1 + C_2 + C_3 = 6\mu\text{F}$$

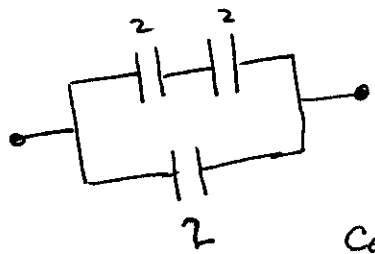
"smallest"



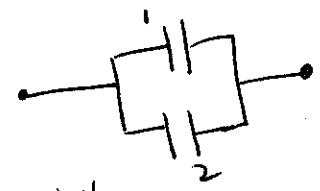
in series,

$$\begin{aligned} \frac{1}{C_{eq}} &= \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \\ &= \frac{1}{2} + \frac{1}{2} + \frac{1}{2} \\ &= \frac{3}{2} \end{aligned}$$

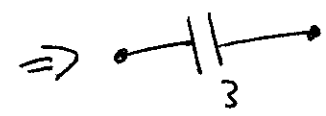
$$\Rightarrow C_{eq} = \frac{2}{3} \mu\text{F}$$



$$C_{eq} = \left(\frac{1}{2} + \frac{1}{2}\right)^{-1} = 1\mu\text{F}$$



$$C_{eq} = 1 + 2 = 3\mu\text{F}$$



Name: Adam Cohen

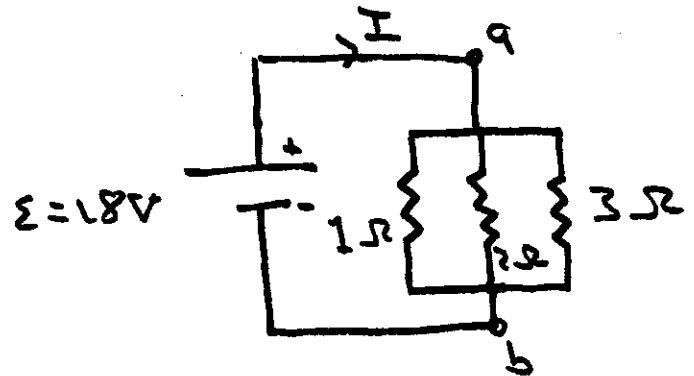
Section: 401

EXAM I

Physics 122

October 4, 2006

4. Three resistors are connected as shown. A potential difference of 18V is maintained between points a and b.



- (a) Find the current on each resistor.
(b) Calculate the power dissipated by each resistor and the total power dissipated by the three resistors.
(c) Calculate the equivalent resistance of the three resistors. From this, what is the total current I?

in parallel $\Rightarrow \Delta V_1 = \Delta V_2 = \Delta V_3 = \mathcal{E} = 18V$

Ohm's Law $\Delta V = IR \Rightarrow I = \frac{\Delta V}{R}$

$$I_1 = \frac{18V}{1\Omega} = 18A, \quad I_2 = \frac{18V}{2\Omega} = 9A, \quad I_3 = \frac{18V}{3\Omega} = 6A$$

$$P = I\Delta V \Rightarrow P_1 = I_1\Delta V_1, \quad P_2 = 162W, \quad P_3 = 108W$$
$$= (18A)(18V) = 324W$$

$$\Rightarrow P_{total} = P_1 + P_2 + P_3 = 594W$$

$$R_{eq, parallel} = \left(\frac{1}{1\Omega} + \frac{1}{2\Omega} + \frac{1}{3\Omega} \right)^{-1} = \left(\frac{6+3+2}{6\Omega} \right)^{-1} = \left(\frac{11}{6\Omega} \right)^{-1} = \boxed{\frac{6}{11}\Omega}$$

$$\Rightarrow I_{total} = \frac{(18V)}{\left(\frac{6}{11}\Omega\right)} = \boxed{33A}$$

check: $P_{total} = I_{total} \Delta V_{total}$
 $= (33A)(18V)$
 $= 594W \quad \checkmark\checkmark$

//