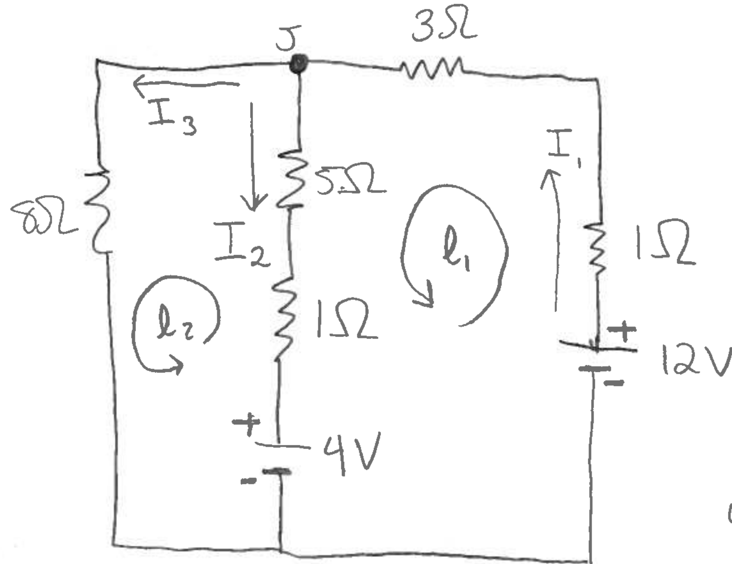
SOLUTIONS - 9

18-17 Determine the current in each branch of the circuit shown



I have defined three currents (I_1, I_2, I_3) and guessed which direction they are going. If my guesses are wrong, the values I get for the current will be negative, (but the magnitudes will be correct).

Since current is conserved @ the junction labeled "J", we can write: $\boxed{I_1 = I_2 + I_3}$ eq. 1

We have three unknowns in eq. 1, so we need two more equations which we'll get by going around the loops l_1 + l_2. "J" will be the starting point for both loops.

$$l_1: \; -I_2(5\Omega) - I_2(1\Omega) - 4V + 12V - I_1(1\Omega) - I_1(3\Omega) = 0$$

$$l_2: \; -I_3(8\Omega) + 4V + I_2(1\Omega) + I_2(5\Omega) = 0$$

(Do you understand why some terms are negative + some are positive?)*

combining $l_1 + l_2$: $I_2\, 6\Omega + 4V = I_3\, 8\Omega$

$$I_2(6\Omega) = 8V - I_1 4\Omega$$

* ON A RESISTOR, GOING ALONG I, ΔV is −ive
AGAINST I, ΔV is +ive

Plugging these into eq. 1

$$\frac{8V - I_2(6\Omega)}{4\Omega} = I_2 + \frac{I_2 6\Omega + 4V}{8\Omega}$$

collect terms $\quad I_2\left(1 + \frac{6}{8} + \frac{6}{4}\right) = \left(\frac{8}{4} - \frac{4}{8}\right)$ amps

$$I_2 = \left(\frac{\frac{16-4}{\cancel{8}}}{\frac{8+6+12}{\cancel{8}}}\right) \text{amp} = \frac{12}{26} \text{ amps} = \frac{6}{13} \text{ amps}$$

plugging back into l_2: $\quad I_3 = \frac{1}{8\Omega}\left(4V + \frac{6}{13}\text{amps}(6\Omega)\right)$

$$I_3 = \left(\frac{1}{2} + \frac{18}{4 \cdot 13}\right)\text{amps} \cong 0.846 \text{ amps}$$

Plugging back into eq 1

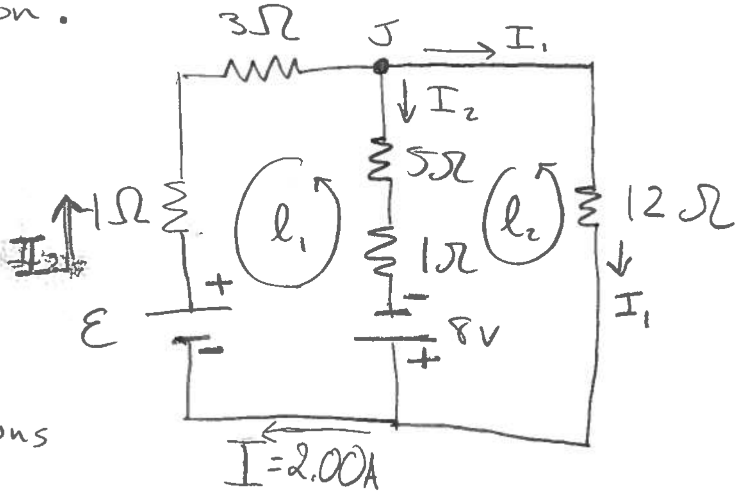
$$I_1 = \left(\frac{6}{13} + 0.846\right)\text{amps} = 1.31 \text{ amps}$$

so

$$\boxed{\begin{aligned} I_1 &= 1.31 \text{ amps} \\ I_2 &= \frac{6}{13} \text{ amps} \\ I_3 &= 0.846 \text{ amps} \end{aligned}}$$

(No negatives I guessed the right directions!)

18-21 What is the emf $\mathcal{E}$ of the battery in the circuit shown.



Again, we guessed the direction of I_1 + I_2.

We have three unknowns $\mathcal{E}, I_1, I_2$ so we'll write down three equations.

(1) At junction "J": $2.00\text{ amps} = I_1 + I_2$

(2) loop one (starting @ "J"): $+2\text{amps}(3\Omega + 1\Omega) - \mathcal{E} - 8V + I_2(1\Omega + 5\Omega) = 0$

(3) loop two (starting @ "J"): $-I_2(5\Omega + 1\Omega) + 8V + I_1\, 12\Omega = 0$

plugging (1) into (3): $-I_2 6\Omega + 8V + (2\text{ Amps} - I_2)12\Omega = 0$

$$I_2(6\Omega + 12\Omega) = 8V + 24V$$

$$I_2 = \frac{32}{18}\text{ amps}$$

plugging this into (2): $\cancel{8V} - \mathcal{E} - \cancel{8V} + \frac{32}{18} 6V = 0$

$$\boxed{\mathcal{E} = 10.7V}$$

18-31

Consider a series RC circuit for which $C = 6\,\mu F$, $R = 2 \times 10^6\,\Omega$ and $\mathcal{E} = 20V$. Find (a) The time constant of the circuit and (b) the maximum charge on the capacitor after a switch in the circuit is closed.

a) The time constant $\tau = RC = (2\times10^6\,\Omega)(6\,\mu F)$

$= (2\times10^6 \times 6\times10^{-6})\,sec$

$$\boxed{\tau = 12\,sec}$$

b) $q = Q(1 - e^{-t/RC}) \xrightarrow[t\to\infty]{} Q = q$

$V = \frac{Q}{C}$, $Q = VC = (20v)(6\,\mu F) = 120\,\mu C$

$$\boxed{q_{max} = 120\,\mu C}$$

18-30 RC

$R = \frac{V}{I}$ $\quad \frac{V}{QT^{-1}}$

$C = \frac{Q}{V}$

$RC = \frac{V}{QT^{-1}}\,\frac{Q}{V} = T^1.$

Hence RC has units of time

18-33 Consider a series RC circuit for which $R = 1\,M\Omega$, $C = 5\,\mu F$ + $\mathcal{E} = 30\,V$. Find the Charge on the capacitor after 10 sec

$$q = Q\left(1 - e^{-t/RC}\right)$$

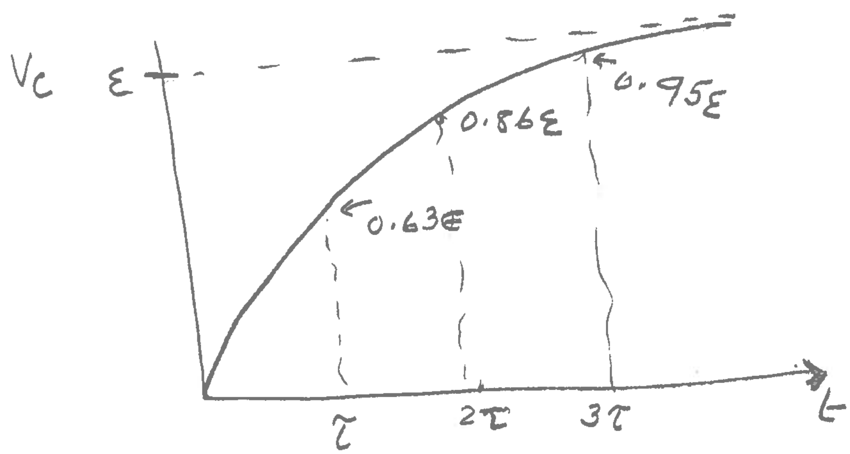
$$RC = 10^{6} \times 5 \times 10^{-6} = 5\,sec.$$

$$V = \frac{Q}{C}, \quad q = VC\left(1 - e^{-t/RC}\right)$$

$$= (30V)(5\mu F)\left(1 - e^{-\frac{10\,sec}{1M\Omega\, 5\mu F}}\right)$$

$$= (150 \times 10^{-6}\,c)\left(1 - e^{-2}\right) =$$

$$\boxed{q = 130 \times 10^{-6}\,c}$$



$$q = CV_c$$

18-36 A series RC circuit has a time constant of 0.960s. The battery has an emf of 48.0V, and the maximum current in the circuit is 0.500 mA. What are (a) the value of the capacitance and (b) the charge stored in the capacitor 1.92 s after the switch is closed?

a) The maximum current occurs at t=0 when there is no charge on the capacitor. $\mathcal{E} = i_0 R$

$$R = \frac{\mathcal{E}}{i_0} = \frac{48.0\,V}{0.500\,mA} = 96 \times 10^3\,\Omega$$

The time constant $\tau = RC = 0.960\,s$

$$C = \frac{0.960\,s}{96 \times 10^3\,\Omega} = \boxed{10^{-5}\,F}$$

b) $q = Q(1 - e^{-t/RC}) = \mathcal{E}C(1 - e^{-t/RC})$

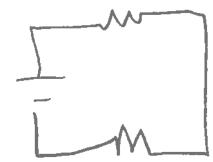
$$= (48 \times 10^{-5}\,C)(1 - e^{-\frac{1.92}{0.96}}) = \boxed{41.5 \times 10^{-5}\,C}$$

18-40 Your toaster oven + coffeemaker each dissipate 1,200 W of power. Can you operate them together if the 120V line that feeds them has a circuit breaker rated @ 15A?

$$P = I^2 R$$

$$P = IV$$

R_1



R_1

Series

in Series: $V = IR = I(R_1 + R_2) = \cancel{I}\left(\frac{P}{I^2} + \frac{P}{I^2}\right) = \frac{2P}{I}$

$I = \frac{2P}{V} = 2 \cdot 10 = 20$ amps (Too much current to be run in series)

If the resistors are in parallel, they would draw even more current!

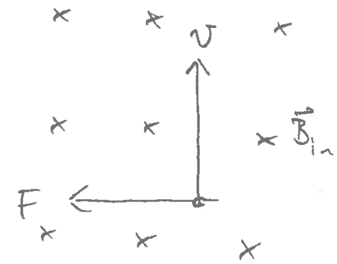
19-2 a) Find the direction of the force on a proton moving through the magnetic fields as shown.

b) repeat part (a) assuming the particle is an electron.

proton (+ive charge)

electron (-ive charge)

a)



Use right hand rule

$q\underset{\rightarrow}{V}$ ∥ Thumb

$\underset{\rightarrow}{B}$ ∥ Fingers

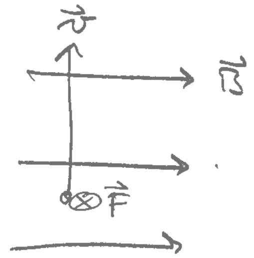
$\underset{\rightarrow}{F_B} \perp$ Palm.

$\vec{F} = q[\vec{v} \times \vec{B}]$

For the electron, we just put a minus on q and get the opposite

a) $\Longrightarrow \vec{F}$

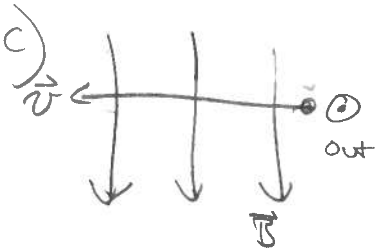
b)



F is into the page $(-\hat{z})$

b) ⊙ out of page $(+\hat{z})$

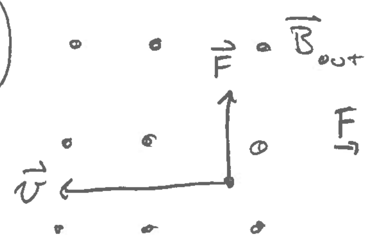
c)



⊙ out of page $(+\hat{z})$

c) ⊗ into page $(-\hat{z})$

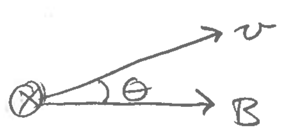
d)



$\underset{\rightarrow}{F} \parallel +\hat{y}$

d) ↓ $\underset{\rightarrow}{F} = -F\hat{y}$

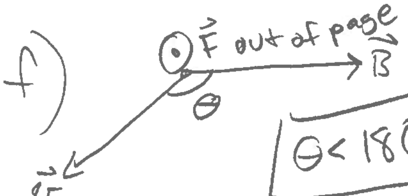
e)



$(-\hat{z})$

e) ⊙ out of page $(+\hat{z})$

f)



$\boxed{\theta < 180°}$

If $\underset{\rightarrow}{v}$ in plane $(+\hat{z})$

If $\underset{\rightarrow}{v}$ out of plane $\underset{\rightarrow}{F} \parallel \hat{y}$

f) ⊗ into page $(-\hat{z})$ If $\underset{\rightarrow}{v}$ in plane,

If $\underset{\rightarrow}{v}$ out-of-plane $\underset{\rightarrow}{F} \parallel -\hat{y}$

19-5 At the equator, near the surface of Earth, the magnetic field is approximately 50.0 μT northward, and the electric field is about 100 N/c downward in fair weather. Find the gravitational, electric and magnetic forces on an electron w/ instantaneous velocity 6.00×10^6 m/s directed to the east.

Gravity: $|\vec{F}_g| = m_e g = 9.11\times10^{-31}\,\text{kg}\,(9.8\,\text{m/s}^2) = 89\times10^{-31}\,N$

$\vec{F}_g = -89\times10^{-31}\,N\,\hat{z}$

Electric: $|\vec{F}_e| = qE = (1.6\times10^{-19}\,C)\,100\,N/C = 1.6\times10^{-17}\,N$.

$\vec{F}_E = +1.6\times10^{-17}\,N\,\hat{z}$.

Magnetic: $|\vec{F}_m| = q|\vec{v}\times\vec{B}| = qvB = (1.6\times10^{-19}\,C)\,6\times10^{6}\,\text{m/s}\;50\times10^{-6}\,T$

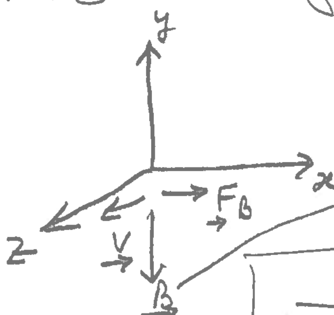
$= 48\times10^{-18}\,N = 4.8\times10^{-17}\,N$

$\vec{F}_B = +4.8\times10^{-17}\,N\,\hat{z}$

Notice $\left|\dfrac{F_e}{F_m}\right| \sim \dfrac{1}{3}$ $\left|\dfrac{F_e}{F_g}\right| \sim 10^{12}$ $\left|\dfrac{F_m}{F_g}\right| \sim 10^{12}$

Gravity is extremely weak!!

19-9 A proton moves perpendicular to a uniform magnetic field $\vec{B}$ @ 1.0×10^{7} m/s and exhibits an acceleration of 2.0×10^{13} m/s² in the +x direction when its velocity is in the +z direction. Determine the magnitude + direction of the field.



need $\underrightarrow{B}$ here!

$$\vec{F} = q\left[\vec{v}\times\vec{B}\right]$$

If $\dfrac{\vec{v}}{|\vec{v}|} = \hat{z}$ and $\dfrac{\vec{F}}{|\vec{F}|} = \hat{x}$ then $\boxed{\dfrac{\vec{B}}{|\vec{B}|} = -\hat{y}}$

because $\hat{z}\times(-\hat{y}) = \hat{x}$

$$|\vec{F}| = ma = q|\vec{v}\times\vec{B}| = qvB$$

$$B = \frac{ma}{qv} = \frac{1.\cancel{7}\times10^{-27}\,\text{kg}\,(2\times10^{13}\,\text{m/s}^2)}{1.6\times10^{-19}\,\text{C}\,(1\times10^{7}\,\text{m/s})}$$

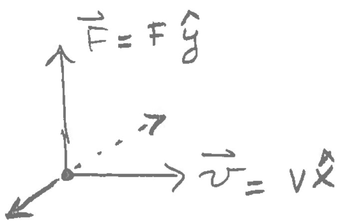
$$= \frac{(1.7)2}{1.6}\times10^{-27+13+19-7}\ \text{T}$$

$$\boxed{\vec{B} = -2.1\times10^{-2}\,\text{T}\,\hat{y}}$$

19 - 13 In the figure assume that in each case the velocity vector shown is replaced by a wire carrying a current in the direction of the velocity vector. For each case, find the direction of the magnetic field that will produce the magnetic force shown.

$$\vec{F}_I = I[\vec{\Delta \ell} \times \vec{B}] \quad \text{here } \vec{\Delta \ell} \parallel \vec{v}$$

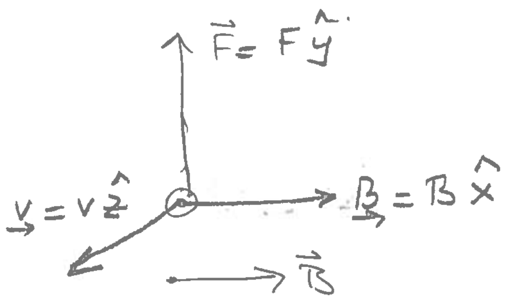
(a)



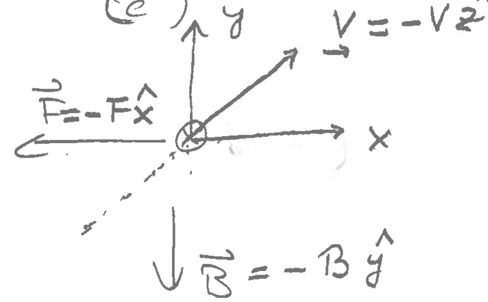
⊗ B is into the page

$\vec{B} \parallel -\hat{z}$

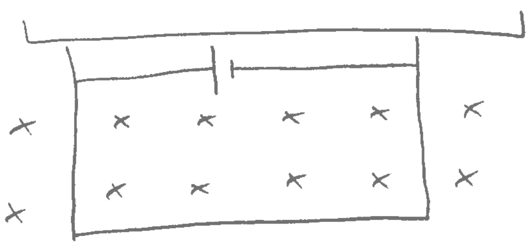
(b)



(c)



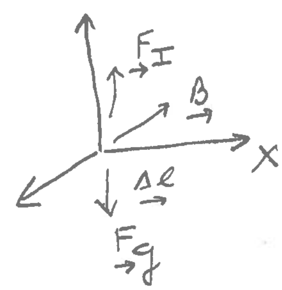
19-18 A conductor suspended by two flexible wires as shown has a mass per unit length of 0.0400 kg/m. What current must exist in the conductor for the tension in the supporting wires to be zero when the magnetic field is 3.6 T into the page? What is the required direction of the current?



We want the force on the wire due to the current to balance the gravitational force, so that there is no tension.

$\vec{F}_g = -Mg\hat{y}$

$\vec{F}_I = I\vec{\Delta l} \times \vec{B}$



$\vec{F}_g + \vec{F}_I = 0$, so $\vec{F}_I = IlB\hat{y}$

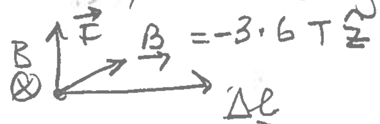
$-Mg + IlB = 0$

$mg = BIl$

$I = \dfrac{m}{l}\dfrac{g}{B}$, $\dfrac{m}{l}$ is the mass per unit length.

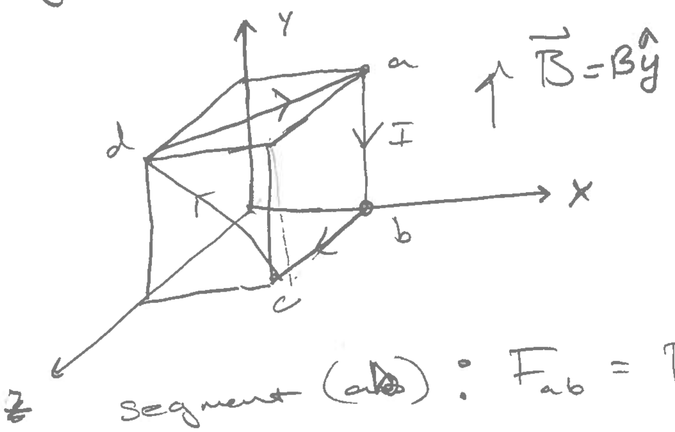
$I = \dfrac{(0.0400\,\frac{kg}{m})(9.8\,m/s^2)}{3.6\,T} = 0.109$ amps

We want the force due to the magnetic field to point along $\hat{y}$, so using the right hand rule, we determine



so the current must be to the right.

19-21 In the figure, the cube is 40 cm on each edge. Four straight segments of wire - ab, bc, cd and da form a closed loop that carries a current $I = 5$ Amps in the direction shown. A uniform magnetic field of magnitude $B = 0.0200\,T$ is in the positive y-direction. Determine the magnitude and direction of the magnetic force on each segment.



$\underset{\to}{F_I} = I \underset{\to}{\Delta l} \times \underset{\to}{B}$

$F_I = BIl\sin\theta$ (Magnitude)

segment (ab): $F_{ab} = BIl\,\cancel{\sin(180^\circ)}^{\,0} = 0$

segment (bc): $F_{bc} = BIl\sin(90^\circ) = BIl$

$= (0.02)(5)(.4)\,N = \boxed{0.04\,N = F(bc)}$

using the right hand rule $\boxed{\vec{F}_{bc} = -0.04\,N\,\hat{x}}$ $\quad \underset{I}{\hat{z}} \times \underset{B}{\hat{y}} = -\hat{x}$

segment cd: $F_{cd} = BIl\sin(45^\circ) = \dfrac{BIl}{\sqrt{2}} = \dfrac{BI\sqrt{.4^2 + .4^2}\,m}{\sqrt{2}}$

$\underset{\to}{cd} = (-0.4\hat{x} + 0.4\hat{y})\,m$

$\underset{\to}{cd} \times \underset{\to}{B} = -0.4 \times 0.02\,\hat{z}$

$= BI\,.4\,m = 0.04\,N$

$\boxed{\vec{F}_{cd} = -0.04\,N\,\hat{z}}$ (because $\underset{I_\perp}{-\hat{x}} \times \underset{B}{\hat{y}} = -\hat{z}$)

segment da: $F_{da} = BIl\sin 90^\circ = BIl$

$\underset{\to}{da} = (+0.4\hat{x} + 0.4\hat{z})\,m$

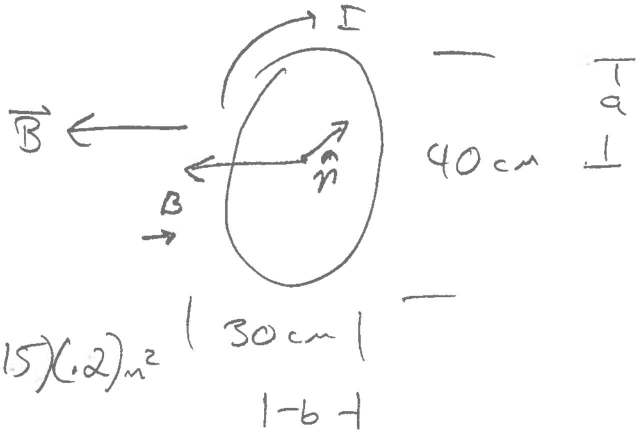
$= BI\sqrt{2(.4)^2}\,m = \sqrt{2} \cdot 0.04\,N = 0.057\,N$

$\boxed{\vec{F}_{da} = 0.057\,N\left(\dfrac{\hat{x} + \hat{z}}{\sqrt{2}}\right)}$

because $\underset{I}{\left(\dfrac{\hat{x} - \hat{z}}{\sqrt{2}}\right)} \times \underset{B}{\hat{y}} = \dfrac{\hat{x} + \hat{z}}{\sqrt{2}}$

19-23 An eight-turn coil encloses an elliptical area having a major axis of 40.0 cm and a minor axis of 30.0 cm. The coil lies in the plane of the page and has an 6 amp current flowing clockwise. If the coil is in a uniform magnetic field of 2.0×10^{-4}T directed toward the left of the page, what is the magnitude of the torque on the coil. $A = \pi ab$

\# of turns $N = 8$



$\tau = NBIA \sin\theta$

$= NBIA$

$8(2 \times 10^{-4}\,T)(6\text{ amp})\,\pi(.15)(.2)\,m^2$

$$\boxed{\tau = 9.05 \times 10^{-4}\,Nm}$$

$\vec{\tau} = N I A \hat{n} \times \vec{B}$

$\hat{n} \parallel -\hat{z}$

$\vec{B} \parallel -\hat{x}$

$\vec{\tau} \parallel +\hat{y}$

$\vec{\tau} = 9.05 \times 10^{-4}\ N\text{-}m\ \hat{\tau}$