TRIG IDENTITY

$\sin A + \sin B$

$$= \sin\left(\frac{A+A+B-B}{2}\right) + \sin\left(\frac{B+B+A-A}{2}\right)$$

$$= \sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) + \cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$+ \sin\left(\frac{B+A}{2}\right)\cos\left(\frac{B-A}{2}\right) + \cos\left(\frac{B+A}{2}\right)\sin\left(\frac{B-A}{2}\right)$$

Now $\cos\left(\frac{B-A}{2}\right) = \cos\left(\frac{A-B}{2}\right)$

But $\sin\frac{(B-A)}{2} = -\sin\left(\frac{A-B}{2}\right)$

Hence

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right).$$

Similarly

$$\cos A + \cos B = 2\cos\frac{(A+B)}{2}\cos\left(\frac{A-B}{2}\right).$$

So when you superpose

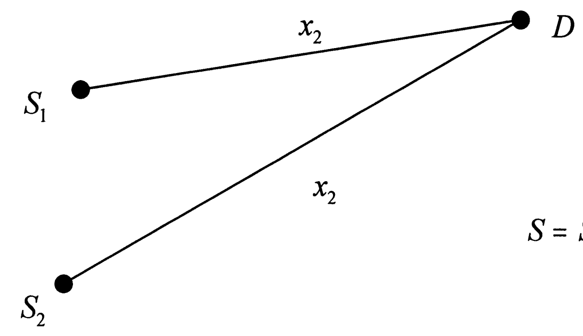
$y_1 = A\sin(kx_1 - \omega t)$ and $y_2 = A\sin(kx_2 - \omega t)$

you get $y = 2A\cos\frac{k(x_1 - x_2)}{2}\sin\left[\frac{k(x_1+x_2)}{2} - \omega t\right]$

That is, a wave whose amplitude is controlled by $(x_1 - x_2)$ leading to INTERFERENCE.

SOUND WAVES-INTERFERENCE

What happens if two sound waves start in phase but have traveled different distances before they arrive at the detector? S_1, S_2 are two sources of sound, which emit waves in phase.



$$S = S_m Sin(kx - \omega t)$$

Let us put both phases equal to zero at the starting points. Note: when a wave travels a distance λ its phase must change by 2π. Hence, the wave from S_1 is when it arrives at D will have the phase $\Phi_1 = \frac{2\pi}{\lambda} x_1$.

When this wave arrives at D there is nothing there. However, when the wave from S_2 arrives at D its phase will be $\Phi_2 = \frac{2\pi}{\lambda} x_2$ and since wave from S_1 already there, the two waves superpose. If $(\Phi_1 - \Phi_2) = 0, 2\pi, 4\pi, 6\pi$ the two waves will be in phase at D and will combine to produce a maximum at D.

We call this constructive interference. 

CONDITION FOR MAXIMUM AT *D*:

$$(\Phi_1 - \Phi_2) = 2M\pi \quad M = 0,1,2...$$

$$\frac{2\pi}{\lambda}(x_1 - x_2) = 2M\pi \qquad \text{(I)}$$

$$or \ (x_1 - x_2) = M\lambda \quad M = 0,1,2...$$

In other words, if the path DIFFERENCE is a whole # of λ's, the waves which started in phase will again be in phase at D and produce a maximum there.

However, if

$$(\Phi_1 - \Phi_2) = \pi, 3\pi, 5\pi...$$

or equivalently (II)

$$(x_1 - x_2) = (2m+1)\frac{\lambda}{2}$$

with m=0, 1, 2,...

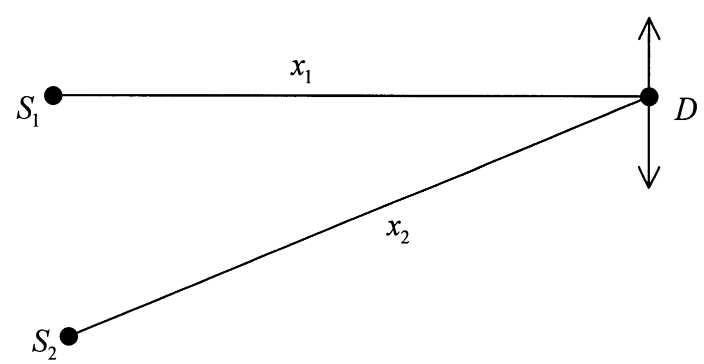
When the waves meet at D, they will be exactly out of phase
And cancel each other.

<u>DESTRUCTIVE INERFERENCE</u>

CONDITION FOR MINIMUM

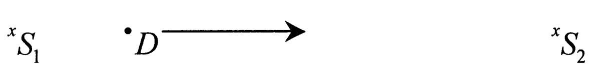
$$(x_1 - x_2) = (2m+1)\frac{\lambda}{2}, m = 0,1,2,... \qquad \text{(II)}$$

If you keep S_1 and S_2 fixed and move D up and down as shown, $(x_1 - x_2)$ will vary and you will encounter a series of maxima (loud sound) and minima (no sound) as you go alternately from Eq I to Eq II and vice versa.



<u>INTERESTING CASE</u>

S_1 and S_2 are separated by $M\lambda$ and D (your ear) moves along line joining S_1 and S_2.



a) When D is at S_1:

$x_1 = 0$

$x_2 = M\lambda$

$(x_1 - x_2) = -M\lambda$ *MAX*

b) When D is at Mid-point:

$x_1 = x_2 = \frac{M\lambda}{2}$

$(x_1 - x_2) = 0$ *MAX*

c) When D is at S_2:

$x_1 = M\lambda$

$x_2 = 0$

$(x_1 - x_2) = M\lambda$ *MAX*

In all you hear $(2M+1)$ Maxima
In between where will be $2M$ minima

<u>N.B.</u> CRUCIAL QUANTITY IS PATH DIFFERENCE!!!