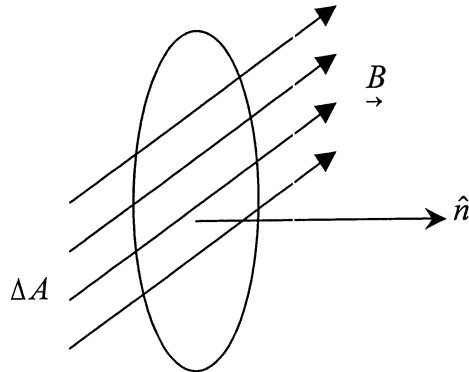


NON-COULOMB \vec{E} FIELD (INDUCTION).

We begin by considering a uniform \vec{B} -field represented by a set of parallel lines. Next, imagine an area ΔA whose normal

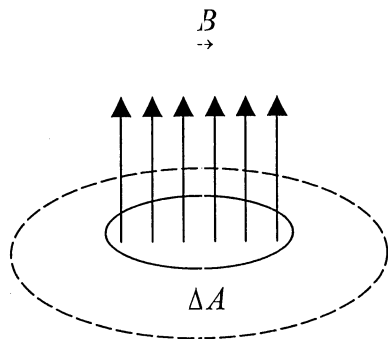


is along \hat{n} . Then, as in the case of \vec{E} , we define flux of \vec{B} through ΔA to be

$$\Delta \Phi_B = \vec{B} \cdot \Delta \vec{A} = B \Delta A \cos(\angle B, \hat{n})$$

As before, flux is maximum when $\vec{B} \parallel \hat{n}$ (\perp to area) and zero if $\vec{B} \perp \hat{n}$ (\vec{B} lines lie in plane of ΔA).

Faraday's discovery was that if Φ_B varies with time, that is, $\frac{\Delta \Phi_B}{\Delta t} \neq 0$, there will be an *emf* ε induced in every closed loop surrounding the region where $\Delta \Phi_B$ is changing. For example, if in the picture \vec{B} varies with the time, or angle of \vec{B} with \hat{n}



Note: Flux of \vec{B} can vary with time due to

3 variations:

- i) B is a function of t .*
- ii) ΔA is a function of t .*
- (iii) Angle between \vec{B} and \hat{n} is a function of t .*

varies with time, $\frac{\Delta \Phi_B}{\Delta t} \neq 0$, and an *emf* will appear in the closed loop, represented by the dotted line, surrounding ΔA . We know that if there is an *emf* in a circuit (closed loop) there must be an \vec{E} -field present at every point of the circuit such that $\varepsilon = \oint_c \vec{E} \cdot d\vec{l}$

Where the sum is over the closed loop and Δl represents displacement along the loop.

Next, we add Lenz's principle to Faraday's discovery to write

$$\sum_c \vec{E} \cdot \vec{\Delta l} = - \frac{\Delta \Phi_B}{\Delta t}$$

The minus sign on the right is crucial, it implies that the *emf* or equivalently \vec{E} -field, is such that it opposes the change in the flux of \vec{B} which generates the *emf*.

Another remarkable point to note is that the \vec{E} -field generated by a time varying Φ_B is represented by lines which close on themselves – there is no beginning and no end. This is totally different from the \vec{E} -field implied by Coulomb's law: $\vec{E} = \frac{q}{4\pi\epsilon_0 r^2} \hat{r}$ which “started” at +ive charges and “ended” at -ive charges.

We call the \vec{E} -field generated by the time varying flux of \vec{B} a NON-COULOMB \vec{E} -FIELD:

$$\sum_c \vec{E}_{NC} \cdot \vec{\Delta l} = - \frac{\Delta \Phi_B}{\Delta t}$$

Notice that for \vec{E}_{NC} Gauss' law will always give

$$\sum_c \vec{E}_{NC} \cdot \vec{\Delta \mathbf{A}} = 0.$$

Total flux of \vec{E}_{NC} through a closed surface is always equal to zero.

Note: A charge q placed in \vec{E}_{NC} experiences a force

$$\vec{F}_E = q \vec{E}_{NC}$$

exactly as for a Coulomb \vec{E} -field.

APPLICATIONS

Calculate the \vec{E}_{NC} for a solenoid at a distance r from its axis when the flux of \vec{B} is varied by

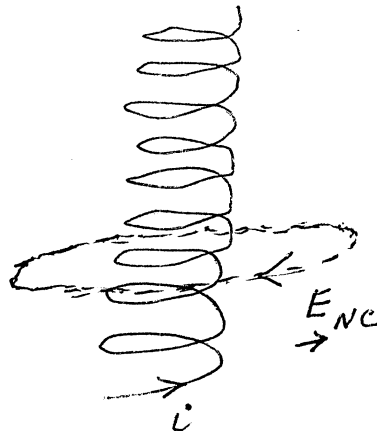
time variation of the current in the solenoid. That is, $\frac{\Delta i}{\Delta t} \neq 0 \Rightarrow \frac{\Delta B}{\Delta t} \neq 0 \Rightarrow \frac{\Delta \Phi_B}{\Delta t} \neq 0$. Consider a

solenoid wound on a tube of radius a . If there are n turns per meter and the current flow is as shown, there is a uniform \vec{B} inside it

$$\vec{B} = \mu_0 n i \hat{y}$$

and

$$\frac{\Delta \vec{B}}{\Delta t} = \mu_0 n \frac{\Delta i}{\Delta t} \hat{y}$$



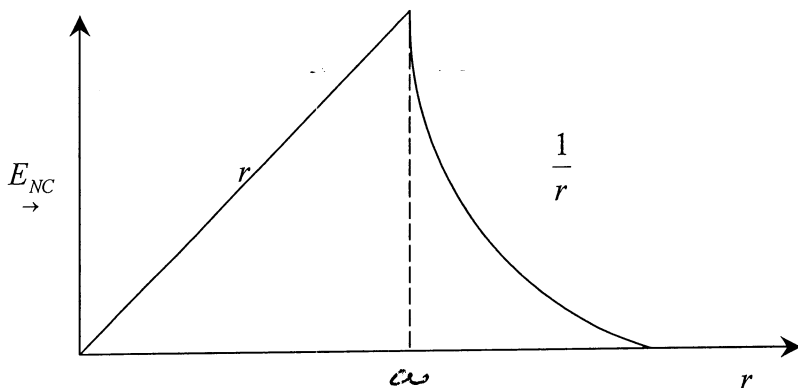
The problem has cylindrical symmetry about axis of solenoid, E_{NC} at r is a function of r only and must be azimuthal. Let i increase with time. Then flux of B along $+\hat{y}$ is increasing with time. Take a circular loop. As shown direction of E_{NC} must be clockwise (as viewed from above) to oppose increase of Φ_B .

Next,

$$\text{If } r < a \quad E_{NC} 2\pi r = -\mu_0 n \pi r^2 \frac{\Delta i}{\Delta t} \quad [\text{LOOP INSIDE SOLENOID}]$$

$$\text{If } r > a \quad E_{NC} 2\pi r = -\mu_0 n \pi a^2 \frac{\Delta i}{\Delta t} \quad [\text{LOOP OUTSIDE SOLENOID}]$$

Magnitude



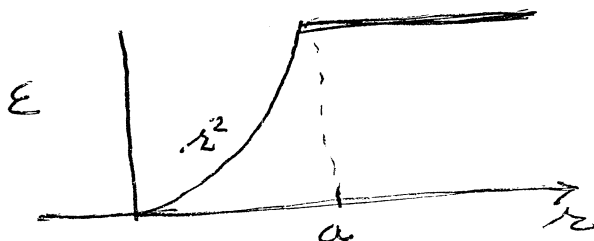
$$E_{NC} = -\frac{\mu_0 n r}{2} \frac{\Delta i}{\Delta t} \quad r < a$$

$$E_{NC} = -\mu_0 n \frac{a^2}{2r} \frac{\Delta i}{\Delta t} \quad r > a$$

It is useful to ask what is the \mathcal{E} -mf in these two loops $\mathcal{E} = \sum_{C \rightarrow} E_{NC} \Delta l$

$$r < a \quad \mathcal{E} = E_{NC} \cdot 2\pi r = -\frac{\mu_0 n \pi r^2 \Delta i}{\Delta t}$$

$$r > a \quad \mathcal{E} = -\mu_0 n \pi a^2 \frac{\Delta i}{\Delta t}$$



Inside \mathcal{E} increases as r^2
Outside it is const. for all r !